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Gluonic Penguins in $B \to \pi\pi$ from QCD Light-Cone Sum Rules

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Abstract

The $B \to \pi\pi$ hadronic matrix element of the chromomagnetic dipole operator O_{8g} (gluonic penguin) is calculated using the QCD light-cone sum rule approach. The resulting sum rule for $\langle \pi\pi | O_{8g} | B \rangle$ contains, in addition to the $O(\alpha_s)$ part induced by hard gluon exchanges, a contribution due to soft gluons. We find that in the limit $m_b \to \infty$ the soft-gluon contribution is suppressed as a second power of $1/m_b$ with respect to the leading-order factorizable $B \to \pi\pi$ amplitude, whereas the hard-gluon contribution has only an α_s suppression. Nevertheless, at finite m_b, soft and hard effects of the gluonic penguin in $B \to \pi\pi$ are of the same order. Our result indicates that soft contributions are indispensable for an accurate counting of nonfactorizable effects in charmless B decays. On the phenomenological side we predict that the impact of gluonic penguins on $\bar{B}_d^0 \to \pi^+\pi^-$ is very small, but is noticeable for $\bar{B}_d^0 \to \pi^0\pi^0$.

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1 Introduction

Intensive experimental studies of $B \to \pi\pi$ and other charmless two-body hadronic B decays are currently under way at B factories [1]. The major goal is to complement the already observed CP-violation in the $B \to J/\psi K$ channel by measuring other CP-violating effects and quantifying them in terms of the angles α and γ of the CKM unitarity triangle (for a recent comprehensive review, see, e.g. Ref. [2]). The extraction of these fundamental parameters from experimental data on $B \to \pi\pi, K\pi, K\bar{K}$ demands reliable theoretical estimates of hadronic matrix elements of the effective weak Hamiltonian between the initial B and the final two-light-meson states. These matrix elements are not yet accessible in lattice QCD, and thus other QCD methods of their calculation are actively being developed.

The factorization ansatz in hadronic two-body B decays has recently been put on a more solid ground. It has been shown [3], to $O(\alpha_s)$, that in the limit $m_b \to \infty$ the amplitude for $B \to \pi\pi$ factorizes into a product of the $B \to \pi$ form factor and the pion decay constant. The nonfactorizable effects due to hard gluon exchanges are directly calculated combining perturbative QCD with certain nonperturbative inputs. The latter include the $B \to \pi$ form factor and the light-cone distribution amplitudes (DA) of pions and B meson. According to QCD factorization, various soft nonfactorizable contributions to $B \to \pi\pi$, such as exchanges of low virtuality gluons between the "emission" pion and the remaining $B \to \pi$ part of the process, vanish in the $m_b \to \infty$ limit. An open phenomenological problem for QCD factorization is to obtain quantitative estimates of soft nonfactorizable effects at a finite, physical b quark mass. Among these effects, the contributions of current-current operators in the penguin topology (e.g., the "charming penguins") could be important, as was argued in Ref. [4].

Recently, a new method to calculate the $B \to \pi\pi$ hadronic matrix elements from QCD light-cone sum rules (LCSR) [5, 6, 7] has been suggested by one of us [8]. The main advantage of LCSR is that the hadronic matrix elements of heavy-to-light transitions can be calculated including simultaneously soft and hard effects. Thus, the $B \to \pi$ form factor including both soft (end-point) contributions and hard-gluon exchanges was obtained from LCSR [9, 10, 11, 12] providing the main input for the factorizable $B \to \pi\pi$ amplitude. Furthermore, in Ref. [8] it was shown that with LCSR one achieves a quantitative control over $1/m_b$ effects of soft nonfactorizable gluons in $B \to \pi\pi$. The soft-gluon effect in the matrix element of the current-current operator in the emission topology turns out to be of the same size as the hard-gluon nonfactorizable contributions obtained from QCD factorization. It is important therefore to investigate other nonfactorizable effects, related to the penguin and dipole operators and/or to non-emission topologies for the current-current operators. Especially interesting is to compare the size of soft- and hard-gluon contributions.

To continue a systematic study in this direction we apply in this paper the LCSR method to the $B \to \pi\pi$ hadronic matrix element of the chromomagnetic dipole operator O_{8g} (gluonic penguin). Apart from being phenomenologically interesting by itself, the $\langle \pi\pi | O_{8g} | B \rangle$ matrix element provides a very useful study case for the LCSR approach. Indeed, as we shall see, the hard-gluon contribution is given by one-loop diagrams and is therefore relatively simple. The soft-gluon effect corresponding to a tree-level diagram is also easily calculable. One is therefore able to estimate both hard and soft gluonic-penguin contributions using one and the same method and input.

The paper is organized as follows. In section 2 we introduce the relevant three-point correlation function and calculate it using operator-product expansion (OPE) near the light-cone and taking into account both hard- ($O(\alpha_s)$) and soft-gluon contributions. In section 3, following the procedure described in Ref. [8] we match the result of this calculation to the dispersion relations in π and B channels combined with quark-hadron duality, and obtain the sum rule for the $\langle\pi\pi|O_{8g}|B\rangle$ matrix element. In section 4 we investigate the $m_b \to \infty$ limit of the sum rule. Section 5 contains the numerical analysis. Furthermore, in section 6 our prediction is discussed from the phenomenological point of view and compared, in section 7, with the corresponding result of QCD factorization. We conclude in section 8. The appendix contains some useful formulae: the decompositions of the vacuum-to-pion matrix elements in terms of the light-cone DA of various twist.

2 Correlation function

Following Ref. [8] we start with defining a vacuum-pion correlation function:

$$F_\alpha(p,q,k) = -\int d^4x\, e^{-i(p-q)x} \int d^4y\, e^{i(p-k)y} \langle\, 0 \mid T\{j^{(\pi)}_{\alpha 5}(y) O_{8g}(0) j^{(B)}_5(x)\} \mid \pi^-(q)\rangle\,, \quad (1)$$

where the quark currents $j^{(\pi)}_{\alpha5} = \bar{u}\gamma_\alpha\gamma_5 d$ and $j^{(B)}_5 = m_b\bar{b}i\gamma_5 d$ interpolate π and B mesons, respectively, and the quark-gluon chromomagnetic dipole operator O_{8g} relevant for $B\to\pi\pi$ has the following standard expression:

$$O_{8g} = \frac{m_b}{8\pi^2}\bar{d}\sigma^{\mu\nu}(1+\gamma_5)\frac{\lambda^a}{2}g_s G^a_{\mu\nu} b\,. \quad (2)$$

This operator enters the $\Delta B = 1$ effective weak Hamiltonian

$$H_W = \frac{G_F}{\sqrt{2}}\left\{\lambda_u\left[\left(c_1(\mu)+\frac{c_2(\mu)}{3}\right)O_1(\mu) + 2c_2(\mu)\tilde{O}_1(\mu)\right] + ... + \lambda_t c_{8g}(\mu)O_{8g}(\mu)\right\}\,, \quad (3)$$

where $\lambda_q = V_{qb}V^*_{qd}$, $q = u, t$ are the combinations of CKM parameters, and we have singled out the relevant current-current operators:

$$O_1 = (\bar{d}\Gamma_\mu u)(\bar{u}\Gamma^\mu b)\,, \quad \tilde{O}_1 = (\bar{d}\Gamma_\mu\frac{\lambda^a}{2}u)(\bar{u}\Gamma^\mu\frac{\lambda^a}{2}b)\,, \quad (4)$$

denoting by ellipses all remaining operators which we do not consider in this paper. In Eqs. (2)-(4), $\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$, $c_{1,2,8g}$ are the Wilson coefficients, $\mu \sim m_b$ is the normalization scale and $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$. Furthermore, the operator $O_2 = (\bar{u}\Gamma_\mu u)(\bar{d}\Gamma^\mu b)$ has been Fierz transformed: $O_2 = O_1/3 + 2\tilde{O}_1$.

The correlator (1) is a function of three independent momenta, which are chosen to be q, $p-k$, k, and, for simplicity, $p^2 = k^2 = 0$. As explained in Ref. [8], the momentum k

plays an auxiliary role and will not appear in the sum rule for the hadronic matrix element. The pion is on shell, $q^2 = m_\pi^2$. We will work in the chiral limit and set $m_\pi = 0$ everywhere, except in the enhanced combination $\mu_\pi = m_\pi^2/(m_u + m_d)$. The Lorentz-decomposition of the correlation function (1) contains four invariant amplitudes:

$$F_\alpha = (p-k)_\alpha F + q_\alpha \widetilde{F}_1 + k_\alpha \widetilde{F}_2 + \epsilon_{\alpha\beta\lambda\rho} q^\beta p^\lambda k^\rho \widetilde{F}_3 \,, \tag{5}$$

depending on three kinematical invariants $(p-k)^2$, $(p-q)^2$ and $P^2 = (p-q-k)^2$. In what follows only the amplitude F is needed.

In the region of large virtualities: $(p-k)^2, (p-q)^2, P^2 < 0$, $|(p-k)^2|, |(p-q)^2|, |P^2| \gg \Lambda_{QCD}^2$, the correlation function (1) is calculated using light-cone OPE, in a form of the sum of short-distance coefficient functions (hard amplitudes) convoluted with the pion DA of growing twist and multiplicity. Only a few first terms of this expansion are usually retained, the components with higher twist/multiplicity are neglected being suppressed by inverse powers of large virtualities.

The gluon emitted from the vertex O_8 in the correlation function contributes in two different ways. First, a highly virtual (hard) gluon is absorbed by one of the quark lines at a short light-cone separation from the emission point. The corresponding one-loop diagrams (see Fig. 1) are then calculated perturbatively, as an $O(\alpha_s)$ part of the hard amplitude. The second possibility is that the emitted gluon together with the quark-antiquark pair forms the quark-antiquark-gluon DA of the pion (see Fig. 2a). We will call these low-virtuality gluons *soft*. Thus, in the correlation function the hard- and soft-gluon contributions belong to different terms of the light-cone OPE and can be calculated separately. In what follows, we take into account all $O(\alpha_s)$ hard-gluon effects of twist 2 and 3, and, in addition the effects of quark-antiquark-gluon DA of twist 3 and 4 in zeroth order in α_s. This approximation corresponds to the accuracy of the light-cone expansion adopted for the LCSR calculation of the $B \to \pi$ form factor.

The higher twist terms corresponding to the soft-gluon contributions are subleading in the light-cone OPE, being suppressed by inverse powers of large virtualities. However, in the correlation function (1) the lowest twist contribution is of $O(\alpha_s)$, therefore both hard and soft contributions can be equally important. We may again refer to the LCSR for the $B \to \pi$ form factor where the $O(\alpha_s)$ twist 2 and the zeroth-order in α_s twist 4 contributions are at the same level numerically.

In the following three subsections the calculation of the various contributions to the correlation function (1) is presented.

2.1 Hard gluon contribution

We begin with calculating the leading $O(\alpha_s)$ contribution to the correlator (1) generated by hard gluons. To obtain the relevant diagrams one has to contract the b-quark fields and, in addition, the d- and $\bar{d}-$quark fields from the operators $j_{\alpha 5}^{(\pi)}$ and O_{8g}, respectively, inserting the free-quark propagators for both contractions. The remaining on-shell $\bar{u}$ and d fields form the quark-antiquark DA of the pion. The decomposition of the relevant

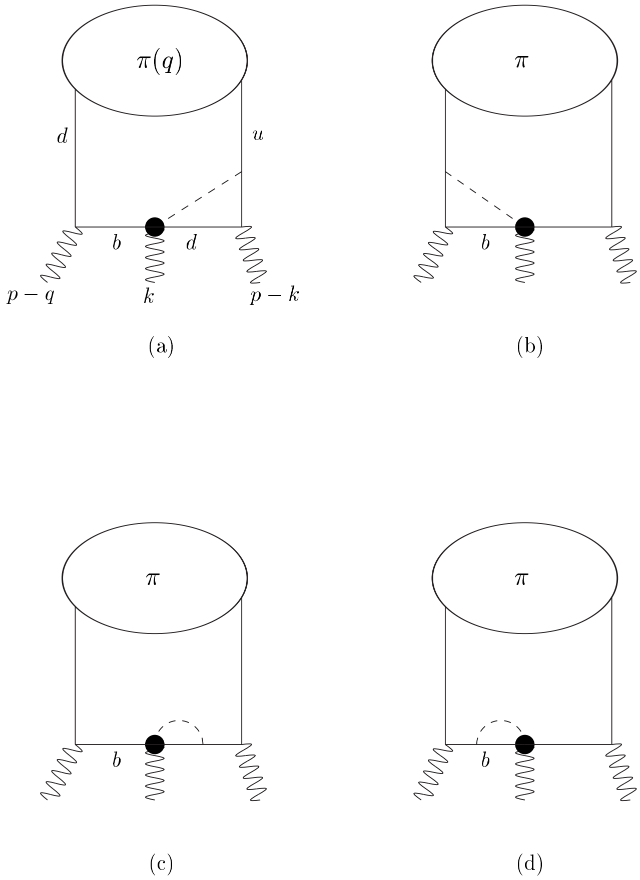

Figure 1: *Diagrams corresponding to the perturbative $O(\alpha_s)$ contributions to the correlation function (1). Solid, dashed, wavy lines and ovals represent quarks, gluons, external currents and pseudoscalar meson DA, respectively. Thick points indicate the gluonic penguin vertex.*

vacuum-pion matrix elements in terms of DA of the lowest twists 2 and 3 is presented in the appendix. Twist 4 components are neglected, because the $O(\alpha_s)$ twist-4 effects are beyond the accuracy of our calculation. The gluon emitted by O_{8g} is absorbed by one of the four quark lines leading to the four diagrams shown in Fig. 1. Note that we disregard the possibility to contract the $\bar{d}$ quark from O_{8g} with the d quark from $j_5^{(B)}$. That type of contraction leads to "penguin-annihilation" diagrams where the heavy-light loop is connected by a single gluon with the pion part of the correlation function. An additional gluon has to be added in this case to obey color conservation, making this contribution either $O(\alpha_s^2)$, with both gluons hard, or $O(\alpha_s)$, with one hard gluon and one soft gluon, the latter entering the pion DA. Both effects are beyond the adopted approximation for the correlation function and presumably very small.

The one-loop diagrams are calculated employing dimensional regularization. Among other tools we used the FORM program [13]. The invariant amplitude F is obtained by taking the coefficient at $(p-k)_\alpha$. The result for F is ultraviolet divergent; however, this divergence does not play any role in the resulting sum rule because the $1/\epsilon$ terms vanish after Borel transformations. On the other hand, we find that the amplitude F does not contain infrared-collinear divergences. This is in accordance with the fact that the perturbative expansion for the correlation function starts, in twists 2 and 3, with $O(\alpha_s)$. If one takes into account an additional perturbative gluon correction in the correlation function, the resulting two-loop diagrams will yield infrared-collinear divergences which should be absorbed by the evolution of the pion DA, as in the case of the $O(\alpha_s)$ correction to the LCSR for the $B \to \pi$ form factor [10, 11].

According to the procedure of the sum rule derivation explained in Ref. [8] we have to retain in the final answer for the amplitude F only those terms which, in the limit of large $|P^2| \sim m_B^2$, have a nonvanishing double imaginary part in the variables $(p-k)^2$ and $(p-q)^2$, taken in the duality region $0 < (p-k)^2 < s_0^\pi$, $m_b^2 < (p-q)^2 < s_0^B$. We find that only two diagrams, in Fig. 1a,b, contain nonvanishing terms. Retaining only these terms we obtain the following analytical expression for the hard-gluon contribution to the amplitude F:

$$F^{hard}(s_1,s_2,P^2) = F^{tw2}(s_1,s_2,P^2) + F^{tw3}(s_1,s_2,P^2)\,, \tag{6}$$

where the twist 2 and 3 parts are, respectively

$$F^{tw2}(s_1,s_2,P^2) = \frac{\alpha_s C_F m_b^2 f_\pi}{16\pi^3}\int\limits_0^1 du\ \varphi_\pi(u)$$
$$\times\left\{\frac{s_2}{m_b^2-\bar{u}s_2}\ln\left(\frac{-s_1}{\mu^2}\right) + \frac{(s_1+s_2-P^2)(m_b^2-s_2)^2}{u(-\bar{u}P^2-us_1)s_2^2}\ln\left(\frac{m_b^2-s_2}{m_b^2}\right)\right\}, \tag{7}$$

and

$$F^{tw3}(s_1,s_2,P^2) = \frac{\alpha_s C_F m_b^3 f_\pi \mu_\pi}{32\pi^3}\Bigg[\int\limits_0^1 du\ \varphi_p(u)\Bigg\{\frac{3}{m_b^2-\bar{u}s_2}\ln\left(\frac{-s_1}{\mu^2}\right)$$
$$+\frac{(s_1-3s_2-P^2)(m_b^2-s_2)}{(-\bar{u}P^2-us_1)s_2^2}\ln\left(\frac{m_b^2-s_2}{m_b^2}\right)\Bigg\}$$
$$+\frac{1}{6}\int\limits_0^1 du\ \varphi_\sigma(u)\Bigg\{\left(\frac{2}{\bar{u}(m_b^2-\bar{u}s_2)}+\frac{s_2}{(m_b^2-\bar{u}s_2)^2}\right)\ln\left(\frac{-s_1}{\mu^2}\right)$$
$$-\frac{(m_b^2-s_2)}{s_2^2}\left(\frac{2(s_1+s_2-P^2)}{u(-\bar{u}P^2-us_1)}+\frac{(s_1-P^2)(s_1+5s_2-P^2)}{(-\bar{u}P^2-us_1)^2}\right)\ln\left(\frac{m_b^2-s_2}{m_b^2}\right)\Bigg\}\Bigg]. \tag{8}$$

In the above, we denote $s_1 = (p-k)^2$, $s_2 = (p-q)^2$ and $\bar{u} = 1-u$ for brevity. The terms proportional to $\ln(-s_1/\mu^2)$ and $\ln((m_b^2-s_2)/m_b^2)$ originate from the diagrams in Fig. 1a and 1b, respectively. Note that the logarithmic dependence on the dimensional regularization scale μ is unimportant because it vanishes after Borel transformation in the variable s_1.

2.2 Soft gluon contribution

In order to reach the same accuracy in the light-cone expansion of the correlation function (1) as in the calculation of the $B \to \pi$ form factor one has to take into account the contributions of quark-antiquark-gluon three-particle DA. Being a regular part of the light-cone expansion, these contributions correspond to a nonperturbative effect of zeroth order in α_s which is qualitatively different from hard-gluon exchanges. The on-shell gluon field emitted by the penguin operator is directly absorbed in the three-particle pion DA yielding a single tree-level diagram shown in Fig. 2a. The corresponding expression is obtained by contracting b and d quark fields and using the decomposition of the $\langle 0|\bar{u}(y)G_{\mu\nu}(0)d(x)|\pi\rangle$ matrix element in terms of twist 3 and 4 quark-antiquark-gluon DA given in the appendix. We find that the contribution of twist 3 DA vanishes in the amplitude F and the answer contains only twist 4 DA :

$$F^{soft}(s_1, s_2, P^2) = \frac{m_b^2 f_\pi}{8\pi^2} \int \frac{\mathcal{D}\alpha_i}{(m_b^2 - (1-\alpha_1)s_2)(-\alpha_2 P^2 - (1-\alpha_2)s_1)}$$
$$\times \{2(s_1 - s_2 - P^2)[\varphi_\perp(\alpha_i) + \widetilde{\varphi}_\perp(\alpha_i))] + (s_1 + s_2 - P^2)[\varphi_{||}(\alpha_i) + \widetilde{\varphi}_{||}(\alpha_i)]\}\,, \tag{9}$$

where $\alpha_i \equiv \{\alpha_1, \alpha_2, \alpha_3\}$ and $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta\,(1-\alpha_1-\alpha_2-\alpha_3)$.

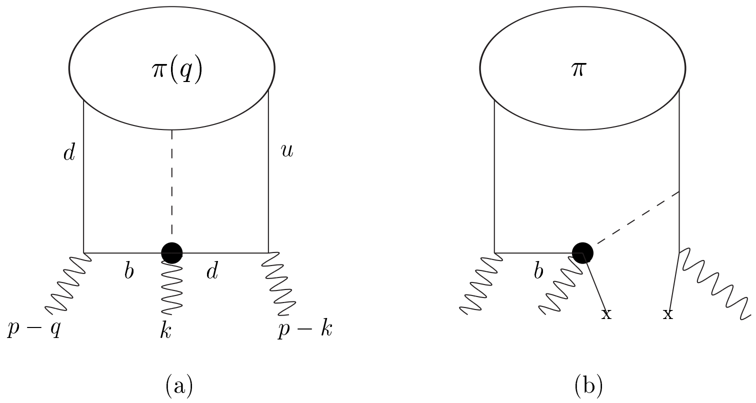

Figure 2: *Diagrams corresponding to the soft-gluon (a) and factorizable 4-quark (b) contributions to the correlation function (1).*

2.3 Factorizable 4-quark contribution

Continuing the light-cone expansion of the correlation function (1) one encounters also four-particle Fock states. In particular, four-quark ($\bar{q}q\bar{q}q$) states emerge when b quark and hard gluon propagate at short light-cone separations and four light-quark fields are on-shell, yielding $O(\alpha_s)$ contribution. One of the corresponding diagrams is shown in Fig. 2b. The Fock states with ≥ 3 multiplicity contribute in two different ways. First, various multiparticle DA of higher twist can be formed. Usually, in LCSR applications their contributions are expected to be strongly suppressed (see, e.g. a more detailed discussion in Ref. [14]) and are therefore neglected, also in our analysis. Second, the configurations of multiple quark and gluon fields sandwiched between the vacuum and pion state, can in general be splitted into a product of two operators of lower multiplicity, one of them being vacuum averaged, and the other one forming a vacuum-pion matrix element. In order to avoid double counting, these factorizable contributions have to be subtracted from genuine multiparticle DA. In our case, the four-quark configurations emerging from the correlator (1) contain a factorizable piece, a product of vacuum-averaged quark-antiquark fields and a low-twist (2,3) quark-antiquark DA. We will take the four-quark factorizable contributions into account approximating the vacuum average of quark and antiquark fields with the standard quark condensate of dimension 3. Combined with the twist 2 and 3 DA, these contributions are effectively of twist 5 and 6 respectively, and are analogous to the factorizable twist 6 terms taken into account in the LCSR for the pion electromagnetic form factor derived in Ref. [14]. Other factorizable multiparticle contributions, such as $\bar{q}qGG$ or $\bar{q}q\bar{q}qG$ which can be parameterized in terms of dimension 4 (gluon) and 5 (quark-gluon) condensates are neglected. The purpose of taking into account the quark-condensate contribution is twofold. First, it is enhanced by the numerically large parameter μ_π, therefore the size of the quark-condensate contribution to LCSR serves as an upper limit for all neglected multiparticle contributions. Second, the QCD factorization answer for the gluonic penguin amplitude contains a certain counterpart of the quark-antiquark condensate term, with which our result will be compared.

Turning to the actual calculation, we find that only one diagram (Fig. 2b) provides nonvanishing contributions to the sum rule. The short-distance part of this diagram consists of b- and u- quark propagators. The momenta of the vacuum averaged $\bar{d}d$-quarks are neglected in the adopted condensate approximation and the vacuum averaging is done in a usual way: $\langle 0|\bar{d}^i_\xi d^j_\omega|0\rangle = \delta^{ij}\delta_{\omega\xi}\langle\bar{q}q\rangle/12$, where $\langle\bar{q}q\rangle$ is the quark condensate density, $q = u, d$, and i, j, and ξ, ω are the colour and spinor indices, respectively. The diagram in Fig. 2b yields:

$$F^{\langle\bar{q}q\rangle}(s_1, s_2, P^2) = -\frac{\alpha_s C_F m_b^3 \langle\bar{q}q\rangle f_\pi}{12\pi}\int\limits_0^1 \frac{du}{s_1(-\bar{u}P^2 - us_1)(m_b^2 - \bar{u}s_2)}$$
$$\times\Bigg\{2(s_1 - P^2)\varphi_\pi(u) + \frac{\mu_\pi}{2m_b}\Big[3\bar{u}(s_1 - s_2 - P^2) - 2s_1\Big]\varphi_p(u)$$

$$+\frac{\mu_\pi}{12m_b}\left[15(s_1-P^2)+9s_2+\frac{(s_1-P^2)(4s_1-3\bar{u}s_2)-5\bar{u}(s_1-P^2)^2+6s_1s_2}{-\bar{u}P^2-us_1}\right.$$
$$\left.+\frac{5\bar{u}s_2(s_1-P^2)+3\bar{u}s_2^2+2s_1s_2}{m_b^2-\bar{u}s_2}\right]\varphi_\sigma(u)\Bigg\}, \qquad (10)$$

where only the terms which contribute to the sum rule are retained. Adding together the contributions presented in Eqs. (8)-(10), we obtain our final result for the amplitude F:

$$F(s_1,s_2,P^2)=F^{hard}(s_1,s_2,P^2)+F^{soft}(s_1,s_2,P^2)+F^{\langle\bar{q}q\rangle}(s_1,s_2,P^2) \qquad (11)$$

which will be used to derive LCSR.

3 Dispersion relation and sum rule

The procedure of deriving the sum rule for the $B\to\pi\pi$ matrix element from the correlation function (1) is independent of the operator O_{8g}. Therefore we refer to the paper [8] where this procedure was explained in detail for a generic case. For completeness, let us summarize the main steps of this derivation:

1. The calculated correlation function (the amplitude F in Eq. (11)) is matched to the dispersion relation in the pion channel, in the variable $s_1=(p-k)^2$, at large $s_1<0$.

2. The Borel transformation, $s_1\to M_1^2$, is performed and quark-hadron duality is used to approximate the contribution of excited pseudoscalar and axial states (with an effective threshold s_0^π) yielding, at large spacelike $P^2\sim -m_b^2$, an expression for the matrix element of the operator product $O_{8g}j_B$ between pion states.

3. The matrix element $\langle\pi|O_{8g}j_B|\pi\rangle$ is expanded in powers of the small ratio $s_0^\pi/(-P^2)$. retaining the leading term of this expansion. In the resulting expression the analytic continuation from large spacelike P^2 to large timelike $P^2=m_B^2$ is performed. Importantly, no imaginary part is recovered in the adopted approximation for the matrix element $\langle\pi\pi|O_{8g}j_B|0\rangle|_{P^2=m_B^2}$.

4. The expression obtained for $\langle\pi\pi|O_{8g}j_B|0\rangle$ is then equated to the dispersion relation in the B-channel, in the variable $s_2=(p-q)^2$. The auxiliary momentum k vanishes in the ground state B-meson contribution. The latter contains the hadronic matrix element of O_{8g} between B and $\pi\pi$ states multiplied by the B-meson decay constant f_B.

5. The second Borel transformation, $s_2\to M_2^2$, is applied to the resulting relation and quark-hadron duality is employed again, with the corresponding threshold s_0^B, to approximate the contribution of the excited B states. Finally, the desired sum rule for the $\langle\pi\pi|O_{8g}|B\rangle$ matrix element is obtained.

For convenience, we present the resulting LCSR in a form of the sum of three separate contributions:

$$A^{(O_{8g})}(\bar{B}_d^0 \to \pi^+\pi^-) \equiv \langle \pi^-(p)\pi^+(-q)|O_{8g}|\bar{B}_d^0(p-q)\rangle = A^{(O_{8g})}_{hard} + A^{(O_{8g})}_{soft} + A^{(O_{8g})}_{\langle\bar{q}q\rangle}\,, \quad (12)$$

where

$$A^{(O_{8g})}_{hard} = i\frac{\alpha_s C_F}{2\pi} m_b^2 \left(\frac{1}{4\pi^2 f_\pi}\int\limits_0^{s_0^\pi} ds\; e^{-s/M_1^2}\right)\left(\frac{m_b^2 f_\pi}{2m_B^2 f_B}\int\limits_{u_0^B}^{1}\frac{du}{u}\; e^{m_B^2/M_2^2 - m_b^2/uM_2^2}\right.$$
$$\times\left[\frac{\varphi_\pi(u)}{u} + \frac{\mu_\pi}{2m_b}\left\{3\varphi_p(u) + \frac{\varphi_\sigma(u)}{3u} - \frac{\varphi'_\sigma(u)}{6}\right.\right.$$
$$\left.\left.\left. - \;\varphi_p(1)\bar{u}\left(1+\frac{3m_b^2}{um_B^2}\right) + \frac{\varphi'_\sigma(1)\bar{u}}{6}\left(\frac{5m_b^2}{um_B^2} - 1\right)\right\}\right]\right), \quad (13)$$

$$A^{(O_{8g})}_{soft} = -im_b^2\left(\frac{1}{4\pi^2 f_\pi}\int\limits_0^{s_0^\pi} ds\; e^{-s/M_1^2}\right)\left(\frac{f_\pi}{m_B^2 f_B}\int\limits_{u_0^B}^{1}\frac{du}{u}\; e^{m_B^2/M_2^2 - m_b^2/uM_2^2}\right.$$
$$\left.\times\quad\left(1+\frac{m_b^2}{um_B^2}\right)\left[\varphi_\perp(1-u,0,u) + \widetilde{\varphi}_\perp(1-u,0,u)\right]\right), \quad (14)$$

$$A^{(O_{8g})}_{\langle\bar{q}q\rangle} = i\frac{\alpha_s C_F}{3\pi} m_b^2\left(\frac{-\langle\bar{q}q\rangle}{f_\pi m_b}\right)\left(\frac{m_b^2 f_\pi}{2m_B^2 f_B}\int\limits_{u_0^B}^{1}\frac{du}{u}\; e^{m_B^2/M_2^2 - m_b^2/uM_2^2}\right.$$
$$\left.\times\left[\frac{\varphi_\pi(u)}{u} + \frac{\mu_\pi}{4m_b}\left\{3\left(1+\frac{m_b^2}{um_B^2}\right)\varphi_p(u) + \left(5 - \frac{3m_b^2}{um_B^2}\right)\left(\frac{\varphi_\sigma(u)}{3u} - \frac{\varphi'_\sigma(u)}{6}\right)\right\}\right]\right). \quad (15)$$

In the above, $\varphi'_\sigma(u) = \partial\varphi_\sigma(u)/\partial u$, $u_0^B = m_b^2/s_0^B$ and f_B is the B-meson decay constant defined as $m_b\langle 0|\bar{q}i\gamma_5 b|B\rangle = m_B^2 f_B$. Note that the first bracket in Eqs. (13) and (14) is approximately equal to f_π if one uses the SVZ sum rule [15]:

$$f_\pi^2 = \frac{1}{4\pi^2}\int\limits_0^{s_0^\pi} ds\; e^{-s/M_1^2} + \ldots\,, \quad (16)$$

retaining only the leading-order quark-loop term and neglecting condensate contributions denoted by ellipses. The second and third lines in Eq. (13) contain the contributions of the diagrams in Fig. 1a and 1b, respectively. In the latter contribution only twist 3 DA remain, the twist 2 term proportional to the small ratio s_0^π/m_B^2 is neglected. For the same reason, in the soft-gluon contribution (14) only two DA, $\varphi_\perp$ and $\widetilde{\varphi}_\perp$ contribute.

To minimize theoretical uncertainties we will calculate the ratio of the gluonic penguin matrix element (12) to the factorizable $B \to \pi\pi$ hadronic amplitude. The latter, as shown in Ref. [8], simply coincides with the matrix element of the operator O_1 in the emission topology:

$$A_E^{(O_1)}(\bar{B}_d^0 \to \pi^+\pi^-) \equiv \langle\pi^-(p)\pi^+(-q)|O_1|\bar{B}_d^0(p-q)\rangle_E = im_B^2 f_\pi f^+_{B\pi}(0)\,, \quad (17)$$

if one neglects the $O(s_0^\pi/m_B^2)$ terms and the two-gluon nonfactorizable exchanges which are beyond the adopted approximation. In the above, $f^+_{B\pi}(0)$ is the LCSR result for the $B\to\pi$ form factor at zero momentum transfer. For completeness we write down this sum rule [9, 10, 11]:

$$\begin{aligned} f^+_{B\pi}(0) &= \frac{f_\pi m_b^2}{2m_B^2 f_B}\int\limits_{u_0^B}^{1}\frac{du}{u}e^{m_B^2/M_2^2-m_b^2/uM_2^2}\left(\varphi_\pi(u)+\frac{\mu_\pi}{m_b}\left[u\varphi_p(u)+\frac{\varphi_\sigma(u)}{3}-\frac{u\varphi'_\sigma(u)}{6}\right]\right)\\ &+\; O(q\bar{q}\mathrm{tw4})+O(q\bar{q}G)+O(\alpha_s)\,, \end{aligned} \tag{18}$$

where, for brevity, the subleading contributions of the twist-4 quark-antiquark DA, twist-3,4 quark-antiquark-gluon DA and the $O(\alpha_s)$ correction to the twist 2 term are not shown explicitly but taken into account in the numerical analysis. The complete expression for LCSR (18) can be found, e.g. in the review [16]. The recently calculated [12] $O(\alpha_s)$ correction to the twist-3 part of the sum rule (18) is very small and we neglect it here.

4 Heavy quark mass limit

The sum rule (12) is derived in full QCD, with the finite b-quark mass. To obtain the heavy-quark mass limit of the matrix element $A^{(O_{8g})}(\bar{B}_d^0\to\pi^+\pi^-)$, we substitute in LCSR the expansions of all m_b-dependent quantities:

$$m_B=m_b+\bar{\Lambda}\ ,\quad s_0^B=m_b^2+2m_b\omega_0\ ,\quad M_2^2=2m_b\tau\ ,\quad f_B=\hat{f}_B/\sqrt{m_b}, \tag{19}$$

where $\bar{\Lambda}$, ω_0, τ, and $\hat{f}_B$ are the parameters independent of the heavy-mass scale. One also has to take into account that, at $m_b\to\infty$, $u_B^0\to 1-\omega_0/m_b$ and only the end-point regions of the momentum fraction u contribute in the integrals in Eqs. (13)-(15). Using a convenient integration variable $\rho=\bar{u}m_b$, and taking into account the end-point behavior of DA, the following substitutions have to be done in these integrals, to the leading order in $O(1/m_b)$:

$$\begin{aligned} &\int\limits_{u_B^0}^{1}du=(1/m_b)\int\limits_0^{2\omega_0}d\rho,\quad \varphi_{\pi,\sigma}(u)=-(\rho/m_b)\varphi'_{\pi,\sigma}(1)\,,\quad \varphi_p(u)=1,\\ &\qquad\varphi_\perp(1-u,0,u)=\widetilde{\varphi}_\perp(1-u,0,u)\simeq-(\rho/m_b)\widetilde{\varphi}'_\perp(0,0,1)\,, \end{aligned} \tag{20}$$

where $\widetilde{\varphi}'_\perp(0,0,1)=[\partial\widetilde{\varphi}_\perp(1-u,0,u)/\partial u]_{u=1}$. With these substitutions it is easy to reproduce, e.g. the leading $1/m_b^{3/2}$ behavior [7] of the $B\to\pi$ form factor from the sum rule (18) at $m_b\to\infty$:

$$f^+_{B\pi}(0)\Big|_{m_b\to\infty}=\frac{\hat{f}^+_{B\pi}(0)}{m_b^{3/2}}+O(1/m_b^{5/2})\,, \tag{21}$$

where [17]

$$\hat{f}^+_{B\pi}(0)=\frac{f_\pi}{2\hat{f}_B}e^{\overline{\Lambda}/\tau}\int\limits_0^{2\omega_0}d\rho\; e^{-\frac{\rho}{2\tau}}\left[-\rho\varphi'_\pi(1)+\mu_\pi\left(\varphi_p(1)-\frac{1}{6}\varphi'_\sigma(1)\right)\right] \tag{22}$$

is an effective, m_b-independent form factor. Consequently, the heavy-quark mass limit for the factorizable amplitude obtained from Eq. (17) is

$$A_E^{(O_1)}(\bar{B}_d^0 \to \pi^+\pi^-)\Big|_{m_b\to\infty} = i\sqrt{m_b} f_\pi \hat{f}^+_{B\pi}(0). \tag{23}$$

Applying Eqs. (19) and (20) to the sum rule (12) one obtains the limiting behavior for the separate contributions:

$$A_{hard}^{(O_{8g})}\Big|_{m_b\to\infty} = i\frac{\alpha_s C_F}{2\pi}\sqrt{m_b}\left(\frac{1}{4\pi^2 f_\pi}\int\limits_0^{s_0^\pi} ds\; e^{-s/M_1^2}\right)$$
$$\times\left(\frac{f_\pi}{2\hat{f}_B}e^{\overline{\Lambda}/\tau}\int\limits_0^{2\omega_0} d\rho e^{-\frac{\rho}{2\tau}}\left[-\rho\varphi_\pi'(1)+\mu_\pi\left(\frac{3}{2}\varphi_p(1)-\frac{1}{12}\varphi_\sigma'(1)\right)\right]\right), \tag{24}$$

$$A_{soft}^{(O_{8g})}\Big|_{m_b\to\infty} = i\sqrt{m_b}\left(\frac{1}{4\pi^2 f_\pi}\int\limits_0^{s_0^\pi} ds\; e^{-s/M_1^2}\right)\left(\frac{4f_\pi}{\hat{f}_B m_b^2}e^{\overline{\Lambda}/\tau}\int\limits_0^{2\omega_0} d\rho e^{-\frac{\rho}{2\tau}}\rho\,\tilde{\varphi}_\perp'(0,0,1)\right), \tag{25}$$

$$A_{\langle\bar{q}q\rangle}^{(O_{8g})}\Big|_{m_b\to\infty} = i\frac{\alpha_s C_F}{3\pi}\sqrt{m_b}\left(\frac{-\langle\bar{q}q\rangle}{f_\pi m_b}\right)$$
$$\times\left(\frac{f_\pi}{2\hat{f}_B}e^{\overline{\Lambda}/\tau}\int\limits_0^{2\omega_0} d\rho e^{-\frac{\rho}{2\tau}}\left[-\rho\varphi_\pi'(1)+\mu_\pi\left(\frac{3}{2}\varphi_p(1)-\frac{1}{12}\varphi_\sigma'(1)\right)\right]\right), \tag{26}$$

where only the leading in $1/m_b$ term in each of the three contributions is retained. We see that the hard-gluon contribution (24) has only $O(\alpha_s)$ suppression with respect to the factorizable amplitude (23). Note that in Eq. (24) only the diagram in Fig. 1a contributes, because the twist 3 contribution of the Fig. 1b diagram in Eq. (13) has an additional $1/m_b$ suppression. Importantly, the soft gluon contribution (25), where the pion DA is proportional to the dimensionful parameter δ_π^2, is suppressed by two powers of $1/m_b$. Finally, the quark-condensate contribution (26) is both $O(\alpha_s)$ and $O(1/m_b)$ suppressed. Thus, in the heavy-quark mass limit the sum rule (12) behaves similar to the LCSR for the $B\to\pi$ form factor: the twist 2 and 3 DA survive in the asymptotic limit, whereas the higher-twist contributions are suppressed by powers of $1/m_b$.

Furthermore, we use the relation between the twist 3 DA: $\varphi_\sigma'(u) = 6(1-2u)\varphi_p(u)$, which follows [21] from QCD equations of motion in the approximation where the quark-antiquark-gluon DA are neglected. Hence, at $u\to 1$, $\varphi_p(1) = -\varphi_\sigma'(1)/6$, and the expression in the second line in Eq. (24) (and the same in Eq. (26)) is simply equal to $\hat{f}^+_{B\pi}(0)$ given by Eq. (22). As a result, at $m_b\to\infty$, the gluonic penguin amplitude is approximately equal to the heavy-quark limit (23) of the factorizable amplitude multiplied by an $O(\alpha_s)$ factor:

$$A^{(O_{8g})}(\bar{B}_d^0\to\pi^+\pi^-)\Bigg|_{m_b\to\infty} = A_{hard}^{(O_{8g})}\Bigg|_{m_b\to\infty} = i\frac{\alpha_s C_F}{2\pi}\sqrt{m_b}\left(\frac{1}{4\pi^2 f_\pi}\int\limits_0^{s_0^\pi} ds\; e^{-s/M_1^2}\right)\hat{f}^+_{B\pi}(0)$$
$$\simeq \frac{\alpha_s C_F}{2\pi}A_E^{(O_1)}(\bar{B}_d^0\to\pi^+\pi^-)\Bigg|_{m_b\to\infty}, \tag{27}$$

where the approximate relation (16) is used. Note that the above factorization will be violated by $O(1/m_b)$ corrections to Eq. (24). The latter corrections can be easily obtained expanding Eq. (13) to the next-to-leading order in the inverse heavy quark mass. Interestingly however, the quark-condensate contribution (26), being $1/m_b$ suppressed with respect to Eq. (24), nevertheless reveals the same factorization property:

$$A^{(O_{8g})}_{\langle\bar{q}q\rangle}\Big|_{m_b\to\infty} = i\frac{\alpha_s C_F}{6\pi}\left(\frac{\mu_\pi}{m_b}\right)\sqrt{m_b}f_\pi\hat{f}^+_{B\pi}(0) = \frac{\alpha_s C_F}{6\pi}\left(\frac{\mu_\pi}{m_b}\right)A^{(O_1)}_E(\bar{B}^0_d\to\pi^+\pi^-)\Big|_{m_b\to\infty}, \quad (28)$$

where the PCAC relation $\mu_\pi = -2\langle\bar{q}q\rangle/f_\pi^2$ is used to replace the quark-condensate density by μ_π. Naturally, the soft-gluon contribution (25) also violates the factorization relation (27). Although this contribution is of order $1/m_b^2$ with respect to $A^{(O_{8g})}_{hard}\Big|_{m_b\to\infty}$, it has no α_s suppression and therefore should be considered separately from the $O(\alpha_s/m_b^2)$ corrections stemming from the heavy-mass expansion of hard-gluon and quark-condensate terms in LCSR beyond $O(1/m_b)$. A direct evaluation of the latter corrections from Eqs. (13) and (15) will, however, be incomplete because, in deriving LCSR, we have already neglected small (calculable) s_0^π/m_B^2 terms of the same order.

5 Numerical results

Let us return to the sum rule (12) at the finite b-quark mass and use it to obtain a numerical estimate for the ratio

$$r^{(O_{8g})}(\bar{B}^0_d\to\pi^+\pi^-) = A^{(O_{8g})}(\bar{B}^0_d\to\pi^+\pi^-)/A^{(O_1)}_E(\bar{B}^0_d\to\pi^+\pi^-)\,, \quad (29)$$

which determines (up to the known Wilson coefficient c_{8g}) the gluonic-penguin correction to the factorizable $B\to\pi\pi$ decay amplitude. The ratio (29) has an important advantage of being less sensitive to the input parameters than the individual matrix elements $A^{(O_8)}$ and $A^{(O_1)}_E$. Moreover, $r^{(O_{8g})}$ is independent of f_B, since the latter cancels in the ratio of the sum rules (12) and (18).

The parameters for the pion channel are $f_\pi = 132$ MeV and $s_0^\pi = 0.7$ GeV^2, the latter determined from the two-point SVZ sum rule [15] for f_π. The corresponding Borel parameter interval is $M^2 = 0.5 \div 1.5$ GeV^2, accommodating also the range used in the LCSR for the pion form factor [14].

The inputs for the B channel are taken from the earlier analysis of LCSR (18) for the $B\to\pi$ form factor in Refs. [9, 10]. In particular, to be consistent with the latter sum rule, we identify m_b with the one-loop pole mass and adopt the interval $m_b = 4.7\pm0.1$ GeV which is in accordance with the most recent average of the $\overline{\rm MS}$ mass $\bar{m}_b(\bar{m}_b) = 4.24\pm0.11$ GeV [18] obtained from various model-independent determinations (including QCD sum rules for the Υ system). Fixing the m_b interval one determines the corresponding duality threshold $s_0^B = 35\mp2$ GeV^2 from the QCD sum rule for f_B taken with $O(\alpha_s)$ accuracy. Furthermore we take the range $M_2^2 = 8\div12$ GeV^2 for the second Borel parameter and

$\mu_b = \sqrt{m_B^2 - m_b^2} \simeq 2.4$ GeV for the related scale at which the pion DA are normalized. The same scale is adopted for the α_s normalization. For the latter we also use the two-loop running with $\bar{\Lambda}^{(4)} = 280$ MeV. However, one should have in mind that in the new sum rule (12) two independent Borel parameters M_1 and M_2 are present, and the scale $M_1 \sim 1$ GeV is independent of m_b. To investigate the sensitivity to the normalization scale, we will vary it in a rather wide interval, between $\mu_b/2$ and $2\mu_b$. For the quark condensate density we take $\langle \bar{q}q \rangle(1\text{GeV}) = (-240 \pm 10 \text{ MeV})^3$, or, equivalently, $\mu_\pi(1\text{GeV}) = 1.59 \pm 0.2$ GeV. The normalization parameter of the twist 4 DA $\delta_\pi^2(1\text{GeV}) = 0.17 \pm 0.05 \text{ GeV}^2$ is determined from the two-point QCD sum rules [19] (see also Ref. [20]).

For our initial numerical illustration all pion light-cone DA are taken in the asymptotic form, that is, the coefficients $a_2, f_{3\pi}, \omega_{3\pi}, \epsilon_\pi$ are put to zero in the expressions for DA presented in the appendix. The numerical results for the ratio $r^{(O_{8g})}$, together with the hard-, soft-gluon and quark-condensate contributions, calculated at the central values of the input parameters and for the asymptotic form of the pion DA are plotted in Fig. 3 as a function of Borel variables. We observe a nice stability in both M_1, M_2. The most important feature of our calculation which is clearly seen in Fig. 3 is that the soft-gluon part of the gluonic penguin amplitude, while being suppressed by two powers of m_b, numerically amounts to about 50% of the hard-gluon contribution and has an opposite sign. A relatively large magnitude of the soft-gluon term (14) can be traced back to the large numerical factor of 20 originating from the contributing twist-4 DA and compensating the suppression factor δ_π^2/m_b^2. The quark-condensate contribution, on the other hand, is subleading also numerically, not larger than 30% of the hard-gluon contribution.

To take into account deviations from the asymptotic form, we have repeated our calculation with the nonasymptotic part of the pion DA taken to the next-to-leading order in the conformal spin [21]. In this case we adopt in the pion twist-2 DA $a_2(1\text{GeV}) = 0.24 \pm 0.14 \pm 0.08$ obtained [20] by fitting the LCSR for the pion form factor to the experimental data. The relevant parameters in the twist 3,4 DA have been estimated at the scale of ~ 1 GeV using two-point QCD sum rules: $f_{3\pi} = 0.0035$, $\omega_{3\pi} = -2.88$ [22] and $\epsilon_\pi = 0.5$ [21] . These estimates have an uncertainty of $\pm 30\%$. The corresponding LO anomalous dimensions can be found in the appendix.

The difference between the values of $r^{(O_{8g})}$ obtained with nonasymptotic and asymptotic DA is inessential for the hard-gluon and quark-condensate contributions. On the contrary, in the soft-gluon term, the effect of the nonasymptotic part determined by the parameter ϵ_π is substantial. The reason is that in the soft-gluon term described by the diagram in Fig. 2a the gluon carries the dominant fraction $\alpha_3 = u$ ($u_0^B < u < 1$) of the pion momentum, the d-quark carries $\alpha_1 = 1 - u$, whereas the $\bar{u}$-quark momentum fraction is restricted to small values: $\alpha_2 \sim s_0^\pi/m_B^2$. The integration over the variables α_i in this subspace of the integration region $\sum_i \alpha_i = 1$ is strongly influenced by the presence of the second polynomial in the conformal expansion of the twist 4 DA $\varphi_\perp(\alpha_i)$ and $\tilde{\varphi}_\perp(\alpha_i)$. As a result, the soft-gluon contribution, being -50% of the hard-gluon part in the case of asymptotic DA, drops to $-10\% \div +20\%$ at $\epsilon_\pi = 0.35 \div 0.65$.

Finally, we get the following numerical estimate for the ratio of the gluonic-penguin and factorizable amplitudes including nonasymptotic effects:

$$r^{(O_{8g})}(\bar{B}_d^0 \to \pi^+\pi^-) = 0.035 \pm 0.015. \tag{30}$$

In obtaining the above range, the uncertainties caused by the variation of all parameters within their allowed intervals are added linearly. The uncertainty in Eq. (30) is mainly due to the sensitivity to the nonperturbative parameters ϵ_π, $\langle \bar{q}q \rangle$ and δ_π^2, whereas the choice of the b-quark mass, Borel parameters and a_2 has a smaller impact. There is an additional uncertainty of approximately $\pm 20\%$ from varying the normalization scale in the limits $\mu_b/2 \div 2\mu_b$. The LCSR prediction for the $B \to \pi$ form factor with the same input and the same conservative procedure of estimating the uncertainty is $f^+_{B\pi} = 0.28 \pm 0.06$.

One can also investigate the quality of the $m_b \to \infty$ limit for the ratio $r^{(O_{8g})}$. Dividing Eq. (23) by Eq. (27) we find that, within uncertainties of our input, $r^{(O_{8g})}$ varies between 0.6 and 1.1 of its heavy quark limit. Future improvements of nonperturbative parameters determining the vacuum-pion matrix elements of twist 3,4 will allow to decrease the uncertainty of our numerical estimates, in particular, of the subleading in $1/m_b$ terms.

6 Gluonic penguins and the decay amplitude

We are now in a position to assess the role of the gluonic-penguin matrix element in $\bar{B}^0 \to \pi^+\pi^-$. The decay amplitude can be represented in the following form:

$$\begin{aligned}
\mathcal{A}(\bar{B}_d^0 \to \pi^+\pi^-) &\equiv \langle \pi^+\pi^- | H_W | \bar{B}_d^0 \rangle \\
&= i\frac{G_F}{\sqrt{2}} f_\pi f^+_{B\pi}(0)\ m_B^2 \left\{ \lambda_u \left[c_1(\mu) + \frac{c_2(\mu)}{3} + 2c_2(\mu) r_E^{(\widetilde{O}_1)}(\bar{B}_d^0 \to \pi^+\pi^-) \right] \right. \\
&+ \left. \ldots + \lambda_t c_{8g}(\mu) r^{(O_{8g})}(\bar{B}_d^0 \to \pi^+\pi^-) \right\},
\end{aligned} \tag{31}$$

where we denote by ellipses all terms that are not relevant for the present discussion: the hadronic matrix elements of $O_{1,2}$ in the annihilation, penguin and penguin-annihilation topologies; the contributions generated by current-current operators with c-quarks ("charming penguins") and, finally, all effects of quark- and electroweak-penguin operators (for a model-independent classification of all contributions and topologies see Ref. [23]). The second line in Eq. (31) contains the factorizable amplitude and the nonfactorizable correction due to the operator $\widetilde{O}_1$. The latter is parametrized by the ratio

$$r_E^{(\widetilde{O}_1)}(\bar{B}_d^0 \to \pi^+\pi^-) \equiv A_E^{(\widetilde{O}_1)}(\bar{B}_d^0 \to \pi^+\pi^-)/A_E^{(O_1)}(\bar{B}_d^0 \to \pi^+\pi^-)\,, \tag{32}$$

where $A_E^{(\widetilde{O}_1)}$ is the $\bar{B}_d^0 \to \pi^+\pi^-$ matrix element of $\widetilde{O}_1$ in the emission topology. Finally, in the third line in Eq. (31) we added the gluonic penguin correction calculated above.

The matrix element of $\widetilde{O}_1$ contains the hard $O(\alpha_s)$ and soft $O(1/m_b)$ contributions, so that

$$r_E^{(\widetilde{O}_1)}(\bar{B}_d^0 \to \pi^+\pi^-) = r_E^{(\widetilde{O}_1,\ hard)}(\bar{B}_d^0 \to \pi^+\pi^-) + \frac{\lambda_E(\bar{B}_d^0 \to \pi^+\pi^-)}{m_B}\,, \tag{33}$$

where the soft contribution was calculated from LCSR in Ref. [8] with an estimate $\lambda_E(\bar{B}_d^0 \to \pi^+\pi^-) = 0.1 \pm 0.05$ GeV. The hard nonfactorizable correction $r_E^{(\widetilde{O}_1,hard)}$ is also accessible with LCSR but demands two-loop calculation. Following Ref. [8] we simply use the QCD factorization estimate: $r_E^{(\widetilde{O}_1,hard)}(\bar{B}_d^0 \to \pi^+\pi^-) = \alpha_s C_F F/(8\pi N_c)$, where the expression for F is given in Ref. [3]. Since we are only interested in an order-of-magnitude estimate, we neglect the twist-3 part of $r_E^{(\widetilde{O}_1,hard)}$ which in QCD factorization [3] has to be regularized introducing some additional parameter. For consistency we use the same normalization scale μ_b and twist-2 pion DA as in LCSR. We obtain:

$$r_E^{(\widetilde{O}_1)}(\bar{B}_d^0 \to \pi^+\pi^-) = [-(0.002 \div 0.016) - 0.045i] + [0.01 \div 0.03]\,, \tag{34}$$

where the first and second brackets contain the hard-gluon (QCD factorization) and soft-gluon (LCSR) contributions. The relatively large uncertainty of the hard-gluon part is due to the variation of $a_2(1\text{GeV})$ in the adopted range $0.02 \div 0.46$. Substituting the estimates (30) and (34) in Eq. (31) and using also the LCSR estimate for the $B \to \pi$ form factor we obtain:

$$\mathcal{A}(\bar{B}_d^0 \to \pi^+\pi^-) = i\frac{G_F}{\sqrt{2}} f_\pi\, m_B^2 (0.28 \pm 0.05) \Big\{ \lambda_u \Big(1.03$$
$$+ [0.001 \div 0.009 + 0.025i] - [0.005 \div 0.015] \Big) + ... - \lambda_t [0.003 \div 0.008] \Big\}\,, \tag{35}$$

where, for consistency, the Wilson coefficients [24] are taken at the scale μ_b. We use $c_1(\mu_b) = 1.124$ and $c_2(\mu_b) = -0.272$ in NLO and in the NDR scheme and, correspondingly, $c_{8g}(\mu_b) = -0.166$ in LO. In Eq. (35) the first (second) bracket in the term proportional to λ_u is due to the hard (soft) parts of $r_E^{(\widetilde{O}_1)}$. Note that some of the uncertainties in the $B \to \pi$ form factor and in the nonfactorizable contributions are correlated.

Both nonfactorizable corrections in Eq. (35) turn out to be very small, not exceeding a one-percent level. However, before one can claim that $\bar{B}_d^0 \to \pi^+\pi^-$ has a predominantly factorizable amplitude, it is necessary to calculate all remaining contributions, having in mind that the sum of all nonfactorizable contributions may accumulate into a noticeable effect. As both analyses of Ref. [8] and this paper show, in this calculation the soft-gluon effects are indispensable.

Note that the situation with the colour-suppressed $\bar{B}_d^0 \to \pi^0\pi^0$ channel is quite different. Here the factorizable part is proportional to the small coefficient $c_2(\mu_b) + c_1(\mu_b)/3 \simeq 0.1$, therefore the contribution of the $\widetilde{O}_1$ matrix element multiplied by $2c_1$ is equally important. Also the gluonic-penguin correction in this channel can reach a few % of the total amplitude. One may expect that in some other penguin-dominated channels of charmless B decays, such as $B \to K\pi$, the effects of gluonic penguins may even be more important.

7 Comparison with QCD factorization

It is interesting to compare our estimate of the hadronic matrix element $\langle\pi\pi|O_{8g}|B\rangle$ expressed via the ratio $r^{(O_{8g})}$ with the same ratio derived from QCD factorization. We can only compare the $O(\alpha_s)$ contributions in our calculation because, in the QCD factorization scheme, the soft-gluon effects have not yet been investigated quantitatively.

To simplify the comparison, we set to zero all Wilson coefficients in H_W except c_{8g}. In this case, the hard-scattering kernel in $O(\alpha_s)$ calculated within QCD factorization approach contains a single diagram with a hard gluon emitted by the O_{8g} vertex and converted into a quark-antiquark pair. This diagram (the sixth one in Fig. 2 of the second paper in Ref. [3]) is simply reproduced from the one in Fig. 1a of this paper if one puts on shell the quarks emitted by the B-meson and pion currents. With the same procedure of putting on shell the quark lines applied to the Fig. 1b diagram, one gets a diagram which describes a hard-gluon exchange between the gluonic-penguin vertex and the spectator quark in the B-meson, with a subsequent hadronization of the light-quark pair into a pion pair. The long-distance part of this process can be described by a two-pion DA, an object different from the one-pion DA used before. In the framework of QCD factorization this particular diagram has not been taken into account. A convincing reason for that is that the corresponding contribution is $1/m_b$ suppressed, which is also confirmed by the heavy-mass expansion of LCSR discussed above. We conclude, that, apart from soft-gluon effects, at finite m_b there are certain hard-gluon effects in LCSR, such as the contribution of the Fig. 1b diagram, which do not appear in the QCD factorization.

After this qualitative discussion we turn to the quantitative comparison with the gluonic penguin contribution in the QCD factorization. In our toy scenario with only $c_{8g} \neq 0$ it is easy to extract this contribution from the expression for the effective coefficients a_i obtained in Refs. [3] for $B \to \pi\pi$. The only effective coefficients where c_{8g} contributes are a_4 and a_6, so that the ratio $r^{(O_{8g})}$ in the QCD factorization scheme is :

$$r^{(O_{8g})}(\bar{B}_d^0 \to \pi^+\pi^-)_{QCDfact.} = \frac{\alpha_s C_F}{2\pi N_c}\left(\int_0^1 du \frac{\varphi_\pi(u)}{1-u} + \frac{2\mu_\pi}{m_b}\right) , \tag{36}$$

together with the $O(\mu_\pi/m_b)$ correction which is the only $1/m_b$ effect retained in QCD factorization [3] because of the large numerical value of μ_π. At $m_b \to \infty$ the above ratio, for the asymptotic DA $\varphi_\pi = 6u(1-u)$, coincides with the heavy-quark mass limit (27) obtained from LCSR, if in the latter the approximation (16) is used. Thus, the LCSR prediction for the gluonic-penguin matrix element reproduces the QCD factorization result for the same matrix element in the limit of infinitely heavy b quark.

The two methods, however, substantially differ in subleading terms. This difference manifests itself already at the $O(\alpha_s/m_b)$ level. In addition to the penguin-spectator exchange effect corresponding to the diagram in Fig. 2b and discussed above, there is a factor of 2 difference between the second, subleading term in Eq. (36) and the heavy quark mass limit (28) of the quark-condensate contribution in LCSR. Furthermore, the fact that the soft gluon corrections in LCSR are suppressed by two powers of $1/m_b$ is in a general agreement with the expectations of QCD factorization. However, as we already noted, this effect which is absent in the QCD factorization prediction, turns out to be essential at finite m_b.

To complete our comparison with other methods let us parenthetically note that LCSR predictions do not support the PQCD approach [25] to charmless B decays. In particular, according to LCSR and contrary to PQCD, the soft contributions dominate in the $B \to \pi$ form factor.

8 Conclusion

In this paper, we have presented the first calculation of the hadronic $B \to \pi\pi$ matrix element of the gluonic penguin operator O_{8g} using QCD LCSR. This matrix element is a very suitable study object for the sum rule approach because both hard- and soft-gluon contributions are calculable within one procedure using the same input. The results obtained for $\langle \pi^+\pi^-|O_{8g}|\bar{B}_d^0\rangle$ from LCSR clearly indicate that at finite m_b the soft-gluon and other $1/m_b$ suppressed effects are important motivating further investigation in this direction, in particular, calculating the penguin matrix elements of current-current operators. Importantly, in the heavy-quark mass limit the LCSR prediction is consistent with the QCD factorization. Similar to the nonfactorizable emission, the gluonic-penguin contributions to $B \to \pi\pi$ are suppressed either by α_s or by powers of $1/m_b$. On the phenomenological side, our result clearly indicates that gluonic penguins play an insignificant role in the $\bar{B}_d^0 \to \pi^+\pi^-$ decay amplitude but become noticeable in $\bar{B}_d^0 \to \pi^0\pi^0$. Our prediction can also be used in the future analyses of penguin-dominated $B \to K\pi$ modes.

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Appendix: Vacuum-pion matrix elements and distribution amplitudes

The following convenient decomposition of the vacuum-pion matrix element near the light-cone is used in the calculation of the correlation function (1):

$$\langle 0|\bar{u}^i_\xi(x_2)d^j_\omega(x_1)|\pi^-(q)\rangle = -\frac{i\delta^{ij}}{12}f_\pi\int_0^1 du\, e^{-iuqx_1-i\bar{u}qx_2}\Big[(\not{q}\gamma_5)_{\omega\xi}\varphi_\pi(u)$$
$$+(\gamma_5)_{\omega\xi}\mu_\pi\varphi_p(u) - \frac{1}{6}(\sigma_{\beta\tau}\gamma_5)_{\omega\xi}q_\beta(x_1-x_2)_\tau\mu_\pi\varphi_\sigma(u)\Big] + \text{twist} \geq 4\,, \quad \text{(A1)}$$

where the quark fields are taken near the light-cone ($x_i = u_i x$, $x^2 = 0$), $i, j = 1, 2, 3$ and $\xi, \omega = 1 \div 4$ are the quark color and spinor indices, respectively. Only the twist-2 DA φ_π and the twist 3 DA φ_p and φ_σ are retained. The familiar definitions of these DA are obtained by multiplying both parts of this equation by the corresponding combinations of γ matrices and taking Dirac and color traces. For the twist-2 pion DA, in the next-to-leading accuracy in the conformal spin [21] adopted in this paper, only the first nonasymptotic term is retained in the expansion in Gegenbauer polynomials $C_{2n}^{3/2}$:

$$\varphi_\pi(u,\mu) = 6u\bar{u}\left[1 + a_2(\mu)C_2^{3/2}(u-\bar{u})\right] . \tag{A2}$$

The LO scale-dependence of the nonasymptotic part is determined by

$$a_2(\mu_2) = [L(\mu_2,\mu_1)]^{\gamma_2^{(0)}/\beta_0}\, a_2(\mu_1) , \tag{A3}$$

where $L(\mu_2,\mu_1) = \alpha_s(\mu_2)/\alpha_s(\mu_1)$, $\beta_0 = 11 - \frac{2}{3}N_F$, and $\gamma_2^{(0)} = 50/9$.

The analogous decomposition for the quark-antiquark-gluon matrix element in terms of the pion three-particle DA reads :

$$\begin{aligned}
\langle 0|\bar{u}_\xi^i(x_2) g_s G_{\mu\nu}^a(x_3) d_\omega^j(x_1)|\pi^-(q)\rangle &= \frac{\lambda_{ji}^a}{32}\int \mathcal{D}\alpha_i e^{-iq(\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3)} \\
&\times \Big[i f_{3\pi}(\sigma_{\lambda\rho}\gamma_5)_{\omega\xi}(q_\mu q_\lambda g_{\nu\rho} - q_\nu q_\lambda g_{\mu\rho})\varphi_{3\pi}(\alpha_i) \\
-f_\pi(\gamma_\lambda\gamma_5)_{\omega\xi}\Big\{(q_\nu g_{\mu\lambda} - q_\mu g_{\nu\lambda})\varphi_\perp(\alpha_i) &+ \frac{q_\lambda(q_\mu x_\nu - q_\nu x_\mu)}{(q\cdot x)}\Big(\varphi_\parallel(\alpha_i) + \varphi_\perp(\alpha_i)\Big)\Big\} \\
+\frac{if_\pi}{2}\epsilon_{\mu\nu\delta\rho}(\gamma_\lambda)_{\omega\xi}\Big\{(q^\rho g^{\delta\lambda} - q^\delta g^{\rho\lambda})\widetilde{\varphi}_\perp(\alpha_i) &+ \frac{q_\lambda(q^\delta x^\rho - q^\rho x^\delta)}{(q\cdot x)}\Big(\widetilde{\varphi}_\parallel(\alpha_i) + \widetilde{\varphi}_\perp(\alpha_i)\Big)\Big\}\Big] .
\end{aligned} \tag{A4}$$

In the above

$$\varphi_{3\pi} = 360\alpha_1\alpha_2\alpha_3^2\left[1 + \frac{\omega_{3\pi}}{2}(7\alpha_3 - 3)\right] \tag{A5}$$

is the twist-3 quark-antiquark-gluon DA, with nonperturbative parameters $f_{3\pi}$ and ϵ_π defined via matrix elements of the following local operators:

$$\langle 0|\bar{u}\sigma_{\mu\nu}\gamma_5 G_{\alpha\beta} d|\pi^-(q)\rangle = i f_{3\pi}\Big[(q_\alpha q_\mu g_{\beta\nu} - q_\beta q_\mu g_{\alpha\nu}) - (q_\alpha q_\nu g_{\beta\mu} - q_\beta q_\nu g_{\alpha\mu})\Big] , \tag{A6}$$

$$\langle 0|\bar{u}\sigma_{\mu\lambda}\gamma_5[D_\beta, G_{\alpha\lambda}]d - \frac{3}{7}\partial_\beta\bar{u}\sigma_{\mu\lambda}\gamma_5 G_{\alpha\lambda} d|\pi^-(q)\rangle = \frac{3}{14} f_{3\pi}\omega_{3\pi} q_\alpha q_\beta q_\mu . \tag{A7}$$

The scale dependence of the twist 3 parameters is given by:

$$\mu_\pi(\mu_2) = [L(\mu_2,\mu_1)]^{-\frac{4}{\beta_0}}\,\mu_\pi(\mu_1) , \quad f_{3\pi}(\mu_2) = [L(\mu_2,\mu_1)]^{\frac{1}{\beta_0}\left(\frac{7C_F}{3}+3\right)} f_{3\pi}(\mu_1) , \tag{A8}$$

$$(f_{3\pi}\omega_{3\pi})(\mu_2) = [L(\mu_2,\mu_1)]^{\frac{1}{\beta_0}\left(\frac{7C_F}{6}+10\right)}(f_{3\pi}\omega_{3\pi})(\mu_1) , \tag{A9}$$

The corresponding expressions for the twist-3 quark-antiquark DA are:

$$\varphi_p(u) = 1 + 30\frac{f_{3\pi}}{\mu_\pi f_\pi} C_2^{1/2}(u-\bar{u}) - 3\frac{f_{3\pi}\omega_{3\pi}}{\mu_\pi f_\pi} C_4^{1/2}(u-\bar{u}),$$
$$\varphi_\sigma(u) = 6u(1-u)\left(1 + 5\frac{f_{3\pi}}{\mu_\pi f_\pi}\left(1-\frac{\omega_{3\pi}}{10}\right) C_2^{3/2}(u-\bar{u})\right). \tag{A10}$$

Finally, the expressions for the four twist-4 DA entering the decomposition (A4) are:

$$\varphi_{\|}(\alpha_i) = 120\delta_\pi^2\epsilon_\pi(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3,$$
$$\varphi_\perp(\alpha_i) = 30\delta_\pi^2(\alpha_1-\alpha_2)\alpha_3^2\left[\frac{1}{3}+2\epsilon_\pi(1-2\alpha_3)\right],$$
$$\widetilde{\varphi}_{\|}(\alpha_i) = -120\delta_\pi^2\alpha_1\alpha_2\alpha_3\left[\frac{1}{3}+\epsilon_\pi(1-3\alpha_3)\right],$$
$$\widetilde{\varphi}_\perp(\alpha_i) = 30\delta_\pi^2\alpha_3^2(1-\alpha_3)\left[\frac{1}{3}+2\epsilon_\pi(1-2\alpha_3)\right], \tag{A11}$$

where the nonperturbative parameters δ_π^2 and ϵ_π are defined as

$$\langle 0|\bar{u}\widetilde{G}_{\alpha\mu}\gamma^\alpha d|\pi^-(q)\rangle = -i\delta_\pi^2 f_\pi q_\mu, \tag{A12}$$

and (up to twist 5 corrections):

$$\langle 0|\bar{u}[D_\mu,\widetilde{G}_{\nu\xi}]\gamma^\xi d - \frac{4}{9}\partial_\mu\bar{u}\widetilde{G}_{\nu\xi}\gamma^\xi d|\pi^-(q)\rangle = -\frac{8}{21}f_\pi\delta_\pi^2\epsilon_\pi q_\mu q_\nu, \tag{A13}$$

with the scale-dependence:

$$\delta_\pi^2(\mu_2) = [L(\mu_2,\mu_1)]^{\frac{8C_F}{3\beta_0}}\,\delta_\pi^2(\mu_1), \quad (\delta_\pi^2\epsilon_\pi)(\mu_2) = [L(\mu_2,\mu_1)]^{\frac{10}{\beta_0}}\,(\delta_\pi^2\epsilon_\pi)(\mu_1). \tag{A14}$$

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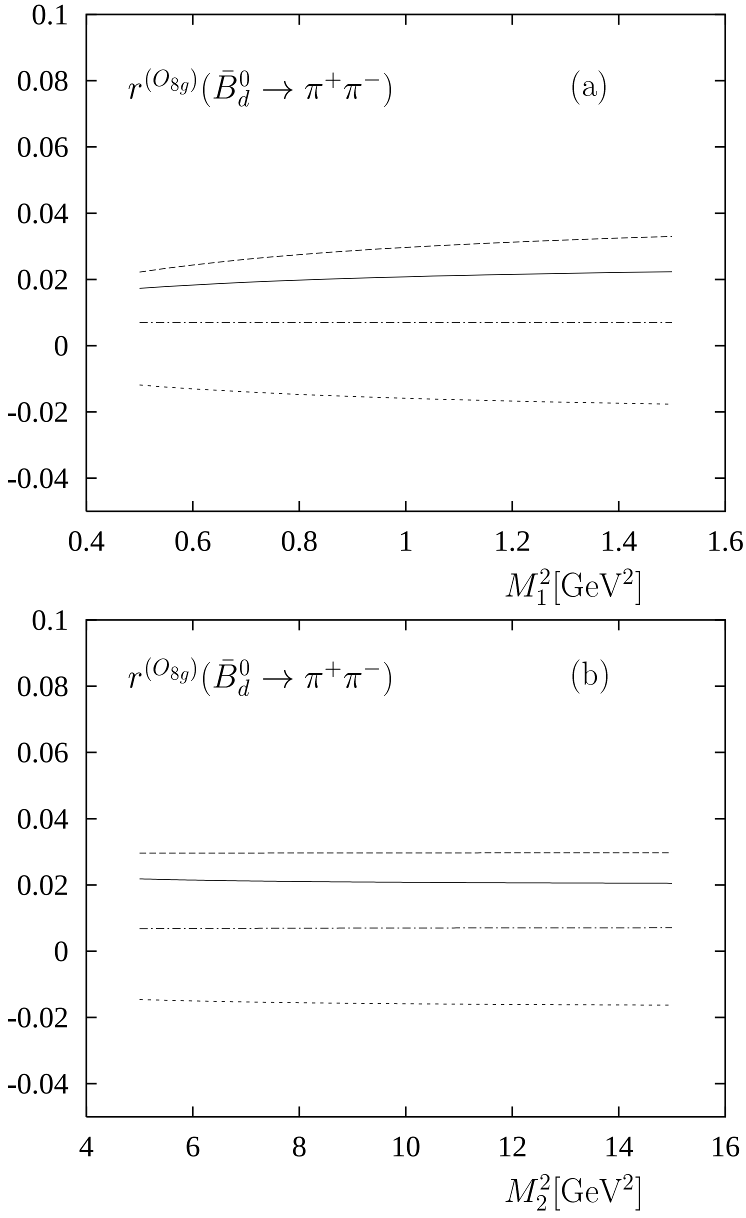

Figure 3: *The ratio of the* $\langle \pi^+\pi^-|O_{8g}|\bar{B}_d^0\rangle$ *and* $\langle \pi^+\pi^-|O_1|\bar{B}_d^0\rangle$ *matrix elements calculated from LCSR (solid line) as a function of the Borel parameters* M_1 *in the pion channel (a) and* M_2 *in the* B *channel (b). Long-dashed, short-dashed and dash-dotted lines are the hard-gluon, soft-gluon and quark-condensate contributions to the sum rule, respectively.*