TTP02-14

Light-Cone Sum Rule Results for Semileptonic Heavy-to-Light Decays

Alexander Khodjamirian [1]

Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany

Abstract

I summarize the current status of the QCD light-cone sum rule predictions for the $B \to \pi$ form factor $f^+_{B\pi}$ that is needed to extract $|V_{ub}|$ from experimental data on $B \to \pi l \nu$ decays.

presented at the Workshop on the CKM Unitarity Triangle, CERN, February 2002, to be published in the Workshop report

[1] *on leave from Yerevan Physics Institute, 375036 Yerevan, Armenia*

QCD light-cone sum rules (LCSR) [1, 2] provide estimates of various heavy-to-light transition form factors. In particular, $B \to P, V$ form factors ($P = \pi, K$ and $V = \rho, K^*, \phi$) have been calculated at small and intermediate momentum transfers, typically at $0 < q^2 \leq m_b^2 - 2m_b\Lambda_{QCD}$. The upper part of this interval overlaps with the region accessible to the lattice calculations of the same form factors, allowing one to compare the results of two methods. In what follows we will concentrate on the LCSR prediction for the $B \to \pi$ form factor $f^+_{B\pi}$ [3, 4, 5]. Its accuracy has been recently improved in Ref. [6]. For the LCSR $B \to V$ form factors we refer to the NLO calculation in Ref. [7] and to the resulting parametrization in Ref. [8].

The form factor $f^+_{B\pi}$ entering the $B \to \pi$ hadronic matrix element, $\langle \pi(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = 2f^+_{B\pi}(q^2)p_\mu + O(q_\mu)$, is at $0 < q^2 < (m_B - m_\pi)^2$ the *only* hadronic parameter one needs to extract $|V_{ub}|$ from the measured $B \to \pi l\nu$ semileptonic width:

$$\frac{d\Gamma(B \to \pi l\bar{\nu})}{dq^2} = \frac{G^2|V_{ub}|^2}{24\pi^3}(E_\pi^2 - m_\pi^2)^{3/2}[f^+_{B\pi}(q^2)]^2 \,. \quad (1)$$

The LCSR approach to calculate $f^+_{B\pi}$ employs a specially designed theoretical object, the vacuum-to-pion correlation function

$$F_\mu(p,q) = i\int d^4x e^{iqx}\langle \pi^+(p)\mid T\{\bar{u}\gamma_\mu b(x), m_b\bar{b}i\gamma_5 d(0)\}\mid 0\rangle = F((p+q)^2, q^2)p_\mu + O(q_\mu)\,, \quad (2)$$

where the $b \to u$ weak current is correlated with the quark current which has the B-meson quantum numbers, and $p^2 = m_\pi^2$. Writing down the dispersion relation for the invariant amplitude F:

$$F((p+q)^2, q^2) = \frac{2f_B f^+_{B\pi}(q^2)m_B^2}{m_B^2 - (p+q)^2} + \sum_{B_h} \frac{2f_{B_h} f^+_{B_h\pi}(q^2)m_{B_h}^2}{m_{B_h}^2 - (p+q)^2}\,, \quad (3)$$

one represents the correlation function (2) in terms of hadronic degrees of freedom in the B channel. The ground-state contribution in Eq. (3) contains a product of the B meson decay constant f_B and the form factor $f^+_{B\pi}(q^2)$ we are interested in, whereas the sum over B_h accounts for the contributions of excited and continuum B states.

The dispersion relation is then matched to the result of QCD calculation of $F((p+q)^2, q^2)$ at large virtualities, that is, at $\mid (p+q)^2 - m_b^2 \mid \gg \Lambda_{QCD}^2$ and $q^2 \ll m_b^2$. In this region the operator-product expansion (OPE) near the light-cone $x^2 = 0$ is employed:

$$F((p+q)^2, q^2) = \sum_{t=2,3,4} \int Du_i \sum_{k=0,1} \left(\frac{\alpha_s}{\pi}\right)^k T_k^{(t)}((p+q)^2, q^2, u_i, m_b, \mu)\varphi_\pi^{(t)}(u_i, \mu)\,. \quad (4)$$

This generic expression is a convolution of calculable short-distance coefficient functions $T_k^{(t)}$ and universal pion light-cone distribution amplitudes (DA) $\varphi_\pi^{(t)}(u_i, \mu)$ of twist t. Here, m_b is the one-loop b-quark pole mass, μ is the factorization scale and the integration goes over the pion momentum fractions $u_i = u_1, u_2, ...$ distributed among quarks and gluons, so that $Du_i \equiv du_1 du_2 ... \delta(1 - \sum_i u_i)$. In particular, $\varphi_\pi^{(2)}(u_1, u_2, \mu) = f_\pi \varphi_\pi(u, \mu)$,

($u_1 = u, u_2 = 1-u$) where φ_π is the lowest twist 2, quark-antiquark pion DA normalized to unit:

$$\varphi_\pi(u,\mu) = 6u(1-u)\left(1+\sum_n a_{2n}(\mu)C_{2n}^{3/2}(2u-1)\right) . \tag{5}$$

In the above, C_{2n} are Gegenbauer polynomials and the coefficients $a_2(\mu) > a_4(\mu) > a_6(\mu)$.... determine the deviation of $\varphi_\pi(u)$ from its asymptotic form. Importantly, the contributions to Eq. (4) corresponding to higher twist and/or higher multiplicity pion DA are suppressed by inverse powers of the b-quark virtuality ($(p+q)^2 - m_b^2$), allowing one to retain a few low twist contributions in this expansion. Furthermore, one uses quark-hadron duality to approximate the sum over B_h in Eq. (3) by a dispersion integral over the quark-gluon spectral density, introducing a threshold parameter s_0^B. The final step involves a Borel transformation $(p+q)^2 \to M^2$, where the scale of the Borel parameter M^2 reflects the characteristic virtuality at which the correlation function is calculated.

The resulting sum rule relation obtained by matching Eqs. (3) and (4) can be cast in the following form:

$$f_B f_{B\pi}^+(q^2) = \frac{1}{m_B^2}\exp\left(\frac{m_B^2}{M^2}\right)\sum_{t=2,3,4}\;\sum_{k=0,1}\left(\frac{\alpha_s}{\pi}\right)^k \mathcal{F}_k^{(t)}(q^2, M^2; m_b, s_0^B, \mu; \{\mathrm{DA}\}^{(t)}) , \tag{6}$$

where the double expansion (in twists and in α_s) and the dependence on the relevant parameters are made explicit. In particular, $\{DA\}^{(t)}$ denotes the nonperturbative normalization constant and nonasymptotic coefficients for each given twist component, e.g., for φ_π: $\{DA\}^{(2)} = \{f_\pi, a_i\}$. The sum rule (6) includes all zeroth order in α_s, twist 2,3,4 contributions containing quark-antiquark and quark-antiquark-gluon DA of the pion. The perturbative expansion has NLO accuracy, including the $O(\alpha_s)$ corrections to the twist 2 [4] and twist 3 coefficient functions, the latter recently calculated in Ref. [6]. More details on the derivation of LCSR (6) and the explicit expressions can be found in the review papers [9, 10, 11].

For the B-meson decay constant entering LCSR (6) one usually employs the conventional SVZ sum rule [12] for the two-point correlator of $\bar{b}i\gamma_5 q$ currents with $O(\alpha_s)$ accuracy (a recent update of this sum rule is discussed in these proceedings [13]):

$$f_B = \sum_{d=0,3\div 6}\;\sum_{k=0,1}\left(\frac{\alpha_s}{\pi}\right)^k C_k^{(d)}(\overline{M}^2, m_b, s_0^B, \mu)\langle 0|\Omega_d(\mu)|0\rangle , \tag{7}$$

where the expansion contains the perturbarive term with dimension $d=0$ ($\Omega_0 = 1$), and, at $d \geq 3$, goes over condensates, the vacuum averages of operators $\Omega_d = \bar{q}q, G^a_{\mu\nu}G^{a\mu\nu}, ...$, multiplied by calculable short-distance coefficients $C_k^{(d)}$. The Borel parameter $\overline{M}$ is correlated with M. The LCSR prediction for the $B\to\pi$ form factor is finally obtained dividing Eq. (6) by Eq. (7):

$$f_{B\pi}^+(q^2) = (f_B f_{B\pi}^+(q^2))_{LCSR}/(f_B)_{2ptSR} . \tag{8}$$

In order to demonstrate that the expansion in this relation works quite satisfactory in both twist and α_s, we present the approximate percentage of various contributions to the resulting form factor (8) in the following table:

twist	DA	*LO*	$O(\alpha_s)$ NLO
2	$\bar{q}q$	~ 50 %	$\sim 5\%$
3	$\bar{q}q$	~ 40 %	$\sim 1\%$
4	$\bar{q}q$		
		~ 5 %	-
3+4	$\bar{q}qG$		

The input parameters used in the numerical analysis of the sum rules (6) and (7) have a limited accuracy. The theoretical uncertainty is estimated by varying these inputs within the allowed regions and adding up linearly the separate uncertainties induced by these variations in the numerical prediction for $f^+_{B\pi}$. The resulting total uncertainties are given below, together with the parametrizations of the form factor. A detailed "theoretical error" analysis can be found in Ref. [14]. To summarize it briefly, one source of uncertainty is the value of the b-quark one-loop pole mass. The two most recent LCSR analyses use $m_b = 4.7 \pm 0.1$ GeV[14] and $m_b = 4.6 \pm 0.1$ GeV [6]. In both studies, the threshold s_0^B is not an independent parameter, being determined by stabilizing f_B calculated from Eq. (7) at a given b-quark mass. The uncertainty induced by varying the factorization scale μ (adopted simultaneously as the normalization scale for α_s) is very small, firstly, because the NLO approximation is implemented for both dominant twist 2 and 3 terms, and, secondly, because the relatively large $O(\alpha_s)$ corrections to the twist 2 contribution and to the f_B sum rule cancel in the ratio (8). Another source of uncertainty is our limited knowledge of the nonasymptotic part in the pion DA (determined by the coefficients a_{2n} and the analogous coefficients in twist 3,4 DA). In Ref. [14] these coefficients were varied from a certain nonasymptotic ansatz of DA (motivated by QCD sum rules) to purely asymptitic DA. Such a substantial variation covers the existing constraints on nonasymptotic coefficients obtained from LCSR for pion form factors. The latter constraints have been used in Ref. [6]. In fact, LCSR involve integration over normalized DA, therefore it is natural that the results only moderately depend on the nonasymptotic coefficents. Finally, to assess the reliability of the LCSR procedure one has to comment on the use of quark-hadron duality, which is the most sensitive point in the sum rule approach. We expect that the sensitivity to the duality approximation is substantially reduced: 1) by restricting the Borel parameter at not too large values and 2) by dividing out the f_B sum rule which depends on the same threshold. The fact that the QCD sum rule prediction for f_B is in a good agreement with the lattice results (see e.g. [13]) indicates that quark-hadron duality is indeed valid in the B channel.

For a convenient use in the experimental analysis, the LCSR results for $B \to \pi$ form factor are usually fitted to simple parametrizations. One of them, suggested in Ref. [14] employs the ansatz [15] based on the dispersion relation for $f^+_{B\pi}(q^2)$. The latter is fitted to the LCSR predictions for $f^+_{B\pi}$ in its validity region $0 < q^2 \leq 14 - 16$ GeV^2. For the B^*-pole term the $B^*B\pi$ coupling [16] is used determined from the same correlation function (4).

The result is:

$$f^+_{B\pi}(q^2) = \frac{0.23 \div 0.33}{(1-q^2/m^2_{B^*})(1-\alpha_{B\pi}q^2/m^2_{B^*})}\,. \tag{9}$$

where the values of the slope parameter correlated with the lower and upper limits of the interval for $f^+_{B\pi}(0)$ are almost equal: $\alpha_{B\pi} = 0.39 \div 0.38$. A different parametrization was suggested recently in Ref. [6]:

$$\begin{aligned} f^+_{B\pi}(q^2) &= \frac{0.26 \pm 0.06 \pm 0.05}{1-a(q^2/m_B^2)+b(q^2/m_B^2)^2}, \quad q^2 < q_0^2, \\ &= \frac{c}{1-q^2/m^2_{B^*}}, \quad q^2 > q_0^2\,, \end{aligned} \tag{10}$$

where the LCSR result is extrapolated to large q^2 using the B^*-pole form. In Eq. (10) the ranges of fitted parameters, $a = 2.34 \div 1.76$, $b = 1.77 \div 0.87$, $c = 0.384 \div 0.523$, and $q_0^2 = 14.3 \div 18.5$ GeV^2, are correlated with the first error in $f^+_{B\pi}(0)$, whereas the second error is attributed to the uncertainty of the quark-hadron duality approximation. Note that all values within the uncertainty intervals in Eqs. (9) and (10) have to be considered as equally acceptable theoretical predictions, without any "preferred central value". The numerical differences between the form factors (9) and (10) are smaller than the estimated uncertainties and are caused by slightly different inputs and by the small $O(\alpha_s)$ correction to the twist 3 term taken into account in Eq. (10) but not in Eq. (9) (where an additional uncertainty was attributed to this missing correction).

Having at hand the form factor one may predict the $B \to \pi l\nu$ decay distribution using Eq. (1). It is shown in Fig. 1 for the form factor (9). The corresponding integrated semileptonic width is

$$\Gamma(B^0 \to \pi^- l^+ \nu_l) = (7.3 \pm 2.5)|V_{ub}|^2 \text{ps}^{-1}\,, \tag{11}$$

where the indicated error is mainly caused by the uncertainty of $f^+_{B\pi}(0)$, whereas the uncertainty of the form factor shape is insignificant. This prediction was recently used by Belle collaboration in their preliminary analysis of $B \to \pi l\nu$ decay yielding [19]: $|V_{ub}| = (4.09 \pm 0.17(stat) \pm 0.33(syst.) \pm 0.76(th)) \cdot 10^{-3}$. A close estimate was obtained in Ref. [14] using the older CLEO measurement [20] of the $B \to \pi l\nu$ width.

The advantage of LCSR is that one can easily switch from the $B \to \pi$ to $D \to \pi$ form factor replacing b quark by c quark in the underlying correlation function (4). The LCSR prediction for $f^+_{D\pi}$ obtained in Ref. [14] and parametrized in the form analogous to Eq. (9) yields a D^*-pole dominance:

$$f^+_{D\pi} = \frac{065 \pm 0.11}{1-q^2/m^2_{D^*}}\,, \tag{12}$$

at $0 < q^2 < (m_D - m_\pi)^2$. The corresponding semileptonic width $\Gamma(D^0 \to \pi^- e^+ \nu_e)/|V_{cd}|^2 = 0.13 \pm 0.05$ ps^{-1} calculated with the known value of $|V_{cd}|$ is, within errors, in agreement with the experimental number 0.177 ± 0.038 ps^{-1} [21]. To make this comparison more decisive it would be very important to have new, more accurate measurements of the decay distribution and integrated width of $D^0 \to \pi^- e^+ \nu_e$.

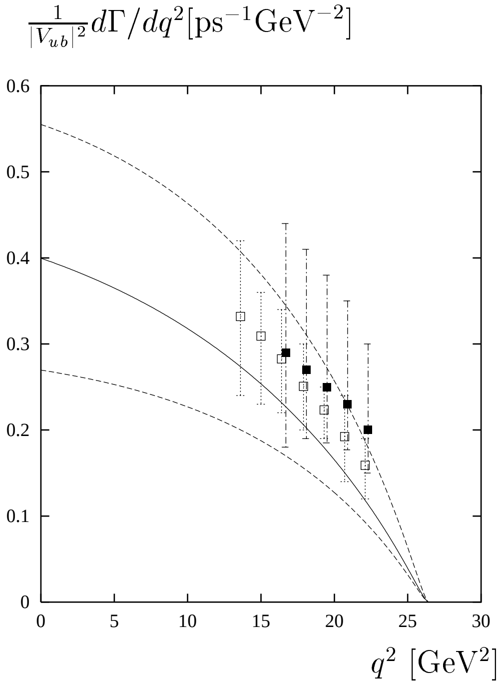

Figure 1: *LCSR prediction for the $B \to \pi l \nu$ decay distribution [14] at the nominal values of inputs (solid), with the interval of theoretical uncertainties (dashed), compared with some of the recent lattice calculations taken from Refs. [17] (solid points) and [18] (open points).*

Are further improvements of the LCSR result for $f^+_{B\pi}$ and $f^+_{D\pi}$ possible? As we have seen, the accuracy of OPE for the correlation function is quite sufficient. The $O(\alpha_s^2)$ level recently achieved in the sum rule for f_B [13] is certainly not an immediate task for LCSR, being also technically very difficult. More important is to improve the accuracy of the input parameters by 1) narrowing the interval of the b quark mass and 2) gaining a better control over the parameters of pion DA. For the latter, in particular, one needs more precise data on pion form factors, especially on $\gamma^*\gamma \to \pi^0$ (the latter form factor can in principle be measured at the same e^+e^- B-factories) and, eventually, lattice QCD simulations of $\varphi_\pi(u)$ and other DA. A better control over duality approximation in the B and D channels can be achieved if radially excited B and D states are accurately identified with their masses and widths. Optimistically, one may hope to reduce the overall accuracy of the LCSR prediction for $f^+_{B\pi}$ and other heavy-to-light form factors to the level of $\pm 10\%$ which is a natural limit for any QCD sum rule prediction.

Concluding let us emphasize that, in addition to approximate estimates of the form factors themselves, LCSR help in understanding important physical aspects of heavy-to-light transitions. First of all, LCSR allow to *quantitatively* assess the role of the soft (end-point) vs hard (perturbative gluon exchange) contributions to the form factors, because both contributions are taken into account in this approach. Secondly, with LCSR one is able to predict [2] the $m_b \to \infty$ limit, $f^+_{B\pi}(0) \sim 1/m_b^{3/2}$, which is used in some lattice extrapolations. Last but not least, LCSR can be expanded in powers of $1/m_b$ and $1/E_\pi$ assessing the size of $1/m_b$ and $1/E$ corrections to various relations predicted in effective theories for heavy-to-light decays.

This work is supported by BMBF (Bundesministerium für Bildung und Forschung).

References

[1] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. **B312** (1989) 509; V. M. Braun and I. E. Filyanov, Z. Phys. **C44** (1989) 157.

[2] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B345** (1990) 137.

[3] V. M. Belyaev, A. Khodjamirian and R. Rückl, Z. Phys. **C60** (1993) 349.

[4] A. Khodjamirian, R. Rückl, S. Weinzierl and O. Yakovlev, Phys. Lett. **B410** (1997) 275;
E. Bagan, P. Ball and V. M. Braun, Phys. Lett. **B417** (1998) 154.

[5] P. Ball, JHEP **9809** (1998) 005.

[6] P. Ball and R. Zwicky, JHEP **0110** (2001) 019.

[7] P. Ball and V. M. Braun, Phys. Rev. D **58** (1998) 094016.

[8] A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D **61** (2000) 074024.

[9] A. Khodjamirian and R. Ruckl, in *Heavy Flavors*, 2nd edition, eds., A.J. Buras and M. Lindner, World Scientific (1998), p. 345, hep-ph/9801443.

[10] V. M. Braun, hep-ph/9911206.

[11] P. Colangelo and A. Khodjamirian, hep-ph/0010175, in the *Boris Ioffe Festschrift 'At the Frontier of Particle Physics / Handbook of QCD'*, ed. M. Shifman (World Scientific, Singapore, 2001), p.1495.

[12] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147**, 385, 448 (1979).

[13] M. Jamin, talk at this Workshop

[14] A. Khodjamirian, R. Ruckl, S. Weinzierl, C. W. Winhart and O. Yakovlev, Phys. Rev. **D62** (2000) 114002.

[15] D. Becirevic and A. B. Kaidalov, Phys. Lett. B **478** (2000) 417.

[16] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Ruckl, Phys. Rev. D **51** (1995) 6177;
A. Khodjamirian, R. Ruckl, S. Weinzierl and O. Yakovlev, Phys. Lett. B **457** (1999) 245.

[17] K. C. Bowler *et al.* [UKQCD Collaboration], Phys. Lett. B **486** (2000) 111.

[18] A. Abada, D. Becirevic, P. Boucaud, J. P. Leroy, V. Lubicz and F. Mescia, Nucl. Phys. B **619** (2001) 565.

[19] H. Ishino [Belle Collaboration], talk at XXXVII Rencontres de Moriond, March 2002.

[20] J. P. Alexander *et al.* [CLEO Collaboration], Phys. Rev. Lett. **77** (1996) 5000.

[21] D. E. Groom *et al.* [Particle Data Group Collaboration], Eur. Phys. J. C **15** (2000) 1.