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QCD-corrected spin analysing power of jets in decays of polarized top quarks

A. Brandenburg [a,*], Z. G. Si [b,†] and P. Uwer [c]

[a] DESY-Theorie, 22603 Hamburg, Germany
[b] Institut f. Theoretische Physik, RWTH Aachen, 52056 Aachen, Germany
[c] Institut f. Theoretische Teilchenphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany

Abstract:

We present results for the differential distributions of jets from non-leptonic decays of polarized top quarks within the Standard Model, including QCD radiative corrections. Our work extends existing results which are only available for semileptonic top quark decays at the parton level. For $t(\uparrow) \to b$-jet $+2$ light jets we compute in particular the QCD-corrected top-spin analysing power of the b-quark jet and the least energetic light jet. The dependence of the results on the choice of the jet recombination scheme is found to be small. In addition we compute the spin analysing power of the thrust axis. Our results constitute a so far missing ingredient to analyse top quark production and subsequent non-leptonic decay at next-to-leading order in α_s, keeping the full information on the top quark polarization.

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1 Introduction

The detailed analysis of the dynamics of top quark production and decay is a major objective of experiments at the Tevatron, the LHC, and a possible future linear e^+e^- collider. A special feature of the top quark that makes such studies very attractive is its large decay width: In contrast to the light quarks the large top decay width $\Gamma_t \approx 1.5$ GeV serves as a cut-off for non-perturbative effects in top quark decays. As a consequence precise theoretical predictions of cross sections and differential distributions involving top quarks are possible within the Standard Model and its extensions. A confrontation of such predictions with forthcoming high-precision data will lead to accurate determinations of Standard Model parameters and maybe hints to new phenomena. In particular, observables related to the spin of the top quark can be studied and utilized to search for new interactions of the top quark [1]. In e^+e^- collisions, top quarks are produced highly polarized, especially if one tunes the polarization of the incoming beams, as possible e.g. at the TESLA collider [2]. Furthermore, even for purely QCD-induced production of top quark pairs, the spins of t and $\bar{t}$ are in general highly correlated [3].

The polarization and spin correlations of top quarks must be traced in the differential distributions of the decay products. This is possible since the information on the top polarization is transferred to the angular distribution of the decay products through its weak, parity violating decays. To be more precise: Consider a polarized ensemble of top quarks at rest with polarization vector $\mathbf{P}$, $0 \leq |\mathbf{P}| \leq 1$. The differential decay distribution with respect to the angle ϑ between $\mathbf{P}$ and the direction $\hat{\mathbf{p}}$ of a given decay product reads:

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\vartheta} = \frac{1}{2}\left(1+|\mathbf{P}|\kappa_p\cos\vartheta\right). \tag{1}$$

In Eq. (1), Γ is the partial width for the corresponding decay of unpolarized top quarks, and κ_p is the so-called *spin analysing power* of the final state particle or jet under consideration. For example, in the semileptonic decay $t \to l^+\nu_l b$, the charged lepton (b-quark) has spin analysing power $\kappa_p = +1$ (~ -0.41) at the tree level within the Standard Model. In hadronic top decays $t \to b\bar{d}u$ (where $d(u)$ stands generically for d,s (u,c)), the rôle of the charged lepton is played by the $\bar{d}$ quark. Only for $t \to b\bar{s}c$ the maximal spin analysing power of the $\bar{s}$-quark could in principle be used by tagging b and c-quark jets. However, the efficiency of charm-tagging is quite low, and one should try to use a spin analyser that is both efficiently detected and has a large analysing power. A good choice is the least energetic light (i.e. non-b-quark) jet [4], which at tree level has $\kappa_p = +0.51$. This follows from the fact that with a probability of 61% (at tree level) this jet contains the $\bar{d}$ quark.

The topic of this letter are the QCD corrections to the above tree level results for κ_p. In fact we will be a little more general and discuss corrections to the fully differential decay distribution of polarized top quarks to be defined in section 2. These corrections are one ingredient for a full analysis of top quark (pair) production and decay at next-to-leading order in α_s, both at lepton and hadron colliders. They form part of the *factorizable* corrections within the pole approximation [5, 6] for the top quark propagator(s). (For the non-factorizable contributions, see ref. [7].) QCD corrections to the production of top quark pairs, including the full information about their spins, can be found in ref. [8] for e^+e^- collisions and in ref. [9] for hadron-hadron collisons. In the case of e^+e^- collisions, also fully analytic results for the top quark polarization [10] and a specific spin correlation [11] to order α_s are available. In ref. [12], helicity amplitudes for $e^+e^- \to t\bar{t}X$ production and semileptonic decays are computed to order α_s and are used to construct an event generator. The reaction $e^+e^- \to t\bar{t} \to W^+bW^-\bar{b}$ is treated in ref. [13], including both factorizable and non-factorizable QCD corrections. The theoretical status of polarized top quark decay is as follows: A complete calculation of the angular decay distribution for $t(\uparrow) \to W^+b$ to order α_s can be found in ref. [14]. The QCD corrections for semileptonic polarized top quark decays have been computed in ref. [15]. We will compare our results, from which the semileptonic case can be easily derived, to those of ref. [15] in section 3.

The outline of this paper is as follows: In the next section we shortly review the tree level results for the decay of polarized top quarks. In section 3 we discuss the calculation of the QCD corrections. Section 4 contains our numerical results, which we discuss in section 5.

2 Kinematics and tree level results

Consider an initial state consisting of top quarks at rest with polarization $\mathbf{P}$. For the non-leptonic decay

$$t(p_t) \to b(p_b) + u(p_u) + \bar{d}(p_{\bar{d}}), \tag{2}$$

the phase space R_3 of the final state may be parametrized by two scaled energies and two angles:

$$dR_3 = \frac{m_t^2}{32(2\pi)^4} dx_{\bar{d}} dx_b d\chi d\cos\theta, \tag{3}$$

where $x_b = 2E_b/m_t,\ x_{\bar{d}} = 2E_{\bar{d}}/m_t,\ \cos\theta = \hat{\mathbf{P}} \cdot \hat{\mathbf{p}}_{\bar{d}}$, and χ is the (signed) angle between the plane spanned by $\hat{\mathbf{P}}$ and $\hat{\mathbf{p}}_{\bar{d}}$ and the plane spanned by $\hat{\mathbf{p}}_{\bar{d}}$ and $\hat{\mathbf{p}}_b$. We will neglect the masses of the light quarks u and $\bar{d}$. The differential decay rate is given by

$$d\Gamma^0 = \frac{1}{2m_t} |\mathcal{M}(p_b, p_u, p_{\bar{d}})|^2 dR_3, \tag{4}$$

where $|\mathcal{M}(p_b, p_u, p_{\bar{d}})|^2$ stands for the squared matrix element averaged over the colour of the initial state and summed over colour and spins of the final state. The fully differential distribution for reaction (2) reads at tree level:

$$\frac{d\Gamma^0}{dx_{\bar{d}} dx_b d\chi d\cos\theta} = c \frac{x_{\bar{d}}(1 - x_{\bar{d}} - z_b)}{(1 - x_b + z_b - \xi)^2 + \eta^2\xi^2} (1 + |\mathbf{P}|\cos\theta), \tag{5}$$

where

$$c = N_C |V_{ud}|^2 \frac{e^4 |V_{tb}|^2 m_t}{128(2\pi)^4 \sin^4\theta_W} = N_C |V_{ud}|^2 \frac{|V_{tb}|^2 m_t G_F^2 m_W^4}{4(2\pi)^4}, \tag{6}$$

with

$$\xi = \frac{m_W^2}{m_t^2}, \qquad \eta = \frac{\Gamma_W}{m_W}, \qquad z_b = \frac{m_b^2}{m_t^2}. \tag{7}$$

A convenient way to compute the spin analysing power κ_p defined in Eq. (1) is to evaluate the expectation value of $\cos\vartheta$:

$$\langle\cos\vartheta\rangle \equiv \langle \hat{\mathbf{p}}\cdot\hat{\mathbf{P}}\rangle = \frac{1}{\Gamma}\int d\Gamma \cos\vartheta = \frac{\kappa_p |\mathbf{P}|}{3}. \tag{8}$$

We want to compute κ_p for the following choices of $\hat{\mathbf{p}}$:

$$\begin{array}{ll} \text{(i)} & \hat{\mathbf{p}} = \hat{\mathbf{p}}_{\bar{d}}, \\ \text{(ii)} & \hat{\mathbf{p}} = \hat{\mathbf{p}}_b, \\ \text{(iii)} & \hat{\mathbf{p}} = \hat{\mathbf{p}}_u \\ \text{(iv)} & \hat{\mathbf{p}} = \hat{\mathbf{k}}_j, \\ \text{(v)} & \hat{\mathbf{p}} = \mathbf{T}. \end{array} \tag{9}$$

In (iv), $\hat{\mathbf{k}}_j$ is the direction of the light (non-b-quark) jet with the *smallest* energy. In the leading order calculation of κ_p, one can simply identify jets with the partons, and thus $\hat{\mathbf{k}}_j$ denotes the direction of the up-type quark if $E_u < E_{\bar{d}}$, and the direction of the down-type quark otherwise. Finally, in (v), $\mathbf{T}$ denotes the *thrust axis* [17]. We define the orientation of the thrust axis such that $\mathbf{T}\cdot\hat{\mathbf{p}}_b$ is positive. In leading order, the oriented thrust is given by $\hat{\mathbf{a}}\,\mathrm{sign}(\hat{\mathbf{a}}\cdot\hat{\mathbf{p}}_b)$, where $\hat{\mathbf{a}}$ denotes the direction of the parton with the largest 3-momentum.

In Table 1 we list our results. As input we use $m_t = 175$ GeV, $m_b = 5$ GeV, $m_W = 80.41$ GeV, and $\Gamma_W = 2.06$ GeV. All other constants cancel in the computation of κ_p. We also give numbers for the limiting cases $m_b = 0$ and $\Gamma_W \to 0$. The latter corresponds to the *narrow width approximation* for the W-boson, i.e. the replacement

$$\frac{1}{(k^2 - m_W^2)^2 + m_W^2\Gamma_W^2} \to \frac{\pi}{m_W\Gamma_W}\delta(k^2 - m_W^2), \tag{10}$$

where k^2 is the squared momentum of the W. One sees that the results are essentially insensitive to the bottom quark mass, while keeping the W width changes, e.g., κ_u by more than 2% as compared to the narrow width approximation. One further comment concerning the dependence of κ_p on the W boson width: In the narrow width approximation, Γ_W drops out completely. (This is also true for the QCD-corrected results for κ_p.) Keeping the Breit-Wigner form for the W propagator, the dependence of κ_p on the W width is extremely small. For example, a 10% change in Γ_W changes the top quark

Table 1: Born results for spin analysing power of $\bar{d}$, b, u , least energetic light jet and thrust axis.

	$m_b = 0, \Gamma_W \to 0$	$m_b = 0, \Gamma_W$ kept	$m_b = 5$ GeV, $\Gamma_W \to 0$	$m_b = 5$ GeV, Γ_W kept
$\kappa^0_{\bar{d}}$	1	1	1	1
κ^0_b	-0.40622	-0.40867	-0.40553	-0.40800
κ^0_u	-0.31817	-0.31091	-0.31964	-0.31236
κ^0_j	0.50774	0.51088	0.50708	0.51021
κ^0_T	-0.31712	-0.31782	-0.31597	-0.31671

hadronic decay rate Γ by about 10%, but the spin analysing powers are affected only at the permill level. Therefore we simply use, both at leading and next-to-leading order, a fixed value $\Gamma_W = 2.06$ GeV.

As already mentioned in the introduction, the case of semileptonic decays $t \to l^+ \nu_l b$ follows from the hadronic decay by the identifications $l^+ \leftrightarrow \bar{d}$, $\nu_l \leftrightarrow u$, and by leaving out the factor $N_C|V_{ud}|^2$ in Eq. (6).

3 QCD corrections

The computation of the QCD corrections to the decay $t(\uparrow) \to b\bar{d}u$ is a generalization of the corresponding computation for $t(\uparrow) \to bl^+\nu_l$ [15]: For the virtual amplitude, one has to add the $W\bar{d}u$ gluonic vertex correction (box diagrams do not contribute). The emission of real gluons from u and $\bar{d}$ does not interfere with the gluon emission from t and b and are added incoherently.

We work in $d = 4 - 2\varepsilon$ space-time dimensions to regularize both soft/collinear and ultraviolet singularities. We simplify the virtual amplitude using an anticommuting γ_5, thus respecting the chiral Ward identities. The only divergent part of the amplitude is proportional to the Born amplitude. The square of the Born amplitude does not depend on d if one keeps the W-boson polarization in 4 dimensions. Therefore, we can evaluate all necessary traces in 4 dimensions. We also choose to keep the phase space measure dR_3 in 4 dimensions. Our result for the virtual corrections reads:

$$\begin{aligned} \frac{d\Gamma^{\text{virtual}}}{dx_{\bar{d}}dx_b d\chi d\cos\theta} &= \frac{c}{(1-x_b+z_b-\xi)^2+\eta^2\xi^2}\frac{\alpha_s C_F}{4\pi}\left(\frac{4\pi\mu^2}{m_t^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)} \\ &\times \left[f_1(x_{\bar{d}},x_b,z_b)\,(1+|\mathbf{P}|\cos\theta)+f_2(x_{\bar{d}},x_b,z_b)|\mathbf{P}|\sin\theta\cos\chi\right], \end{aligned} \quad (11)$$

with

$$
\begin{aligned}
f_1(x_{\bar{d}}, x_b, z_b) &= \left\{ \frac{1}{\beta}\left[-2\left(1+\frac{1}{\varepsilon}\right)\ln(\omega) + \ln^2(\omega) + 4\ln(\omega)\ln\left(\frac{x_b(1-\omega)}{1+\omega-x_b\omega}\right) \right.\right. \\
&+ \left.\left. 4\mathrm{Li}_2\left(\frac{\omega(1+\omega-x_b)}{1+\omega-x_b\omega}\right) - 4\mathrm{Li}_2\left(\frac{1+\omega-x_b}{1+\omega-x_b\omega}\right)\right] \right. \\
&- \left. \frac{4}{\varepsilon} - 8 + \ln(z_b) + g^{Wu\bar{d}} \right\} x_{\bar{d}}(1-x_{\bar{d}}-z_b) \\
&+ \left[\frac{\ln(\omega)}{\beta} - \ln(z_b)\right]\left[(1-x_{\bar{d}}-z_b)^2 + z_b(1-z_b)\right] \\
&+ \frac{2z_b\ln(\omega)}{x_b\beta}(2z_b - x_b + 2x_{\bar{d}} - x_{\bar{d}}x_b) + O(\varepsilon),
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
f_2(x_{\bar{d}}, x_b, z_b) &= \frac{\beta\sin\theta_{\bar{d}b}x_{\bar{d}}(1-x_{\bar{d}}-z_b)}{2(1-x_b+z_b)}\left\{ x_b\left[\frac{\ln(\omega)}{\beta} - \ln(z_b)\right]\right. \\
&+ \left. 2z_b\left[\frac{1-x_b+z_b}{1-x_{\bar{d}}-z_b} - 1\right]\frac{\ln(\omega)}{\beta}\right\} + O(\varepsilon),
\end{aligned}
\tag{13}
$$

where

$$\beta = \sqrt{1-4z_b/x_b^2}, \tag{14}$$

$$\omega = \frac{1-\beta}{1+\beta}, \tag{15}$$

$$g^{Wu\bar{d}} = 2\left(\frac{m_t^2}{k^2}\right)^{\varepsilon}\left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \pi^2\right], \tag{16}$$

and $\theta_{\bar{d}b}$ is the angle between the $\bar{d}$ and the b in the top quark rest frame. One can easily recover the case of semileptonic decays from the above result by leaving out a factor $N_C|V_{ud}|^2$ and the additional incoherent contribution (16) from the QCD-corrected $Wu\bar{d}$ vertex. Thus we can perform an analytic comparison to the results given in Eqs. (2.7), (2.8) of ref. [15]. We find complete agreement by using the well-known correspondence

$$\frac{1}{\varepsilon}\left(\frac{4\pi\mu^2}{m_t^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)} \to \ln\left(\frac{\lambda^2}{m_t^2}\right) \tag{17}$$

between dimensional regularization in $d = 4-2\varepsilon$ dimensions at a scale μ and regularization by a small gluon mass λ used in ref. [15].

In addition to the virtual corrections we have to include the real corrections which are given by the process with an additional gluon:

$$t(p_t) \to b(p_b) + u(p_u) + \bar{d}(p_{\bar{d}}) + g(p_g). \tag{18}$$

This contribution can also be split into two separate parts, which do not interfere with each other: the case where the gluon is emitted from a heavy quark (t or b), and the contribution where the gluon is emitted from a light quark. The calculation of the corresponding amplitudes is straightforward. As always in the case of a jet-calculation we have to address the question how to cancel the infrared and collinear singularities. For the contribution where the gluon is emitted from the secondary fermion line we use the dipole formalism [16]. Dropping overall factors the subtraction term for this process is given by

$$-\frac{V_{ug,\bar{d}}}{2(p_u\cdot p_g)}|\mathcal{M}(p_b,\tilde{p}_{ug},p_{\bar{d}})|^2-\frac{V_{\bar{d}g,u}}{2(p_{\bar{d}}\cdot p_g)}|\mathcal{M}(p_b,p_u,\tilde{p}_{\bar{d}g})|^2 \tag{19}$$

with $\mathcal{M}(p_b,p_u,p_{\bar{d}})$ being the matrix element at leading order and $\tilde{p}$, V are defined in Eq. (5.3) and Eq. (5.7) of ref. [16]. The subtraction term integrated over the singular region is also given in ref. [16] and reads:

$$\frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2}{k^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\left\{\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}+10-\pi^2+O(\varepsilon)\right\}|\mathcal{M}(p_b,p_u,p_{\bar{d}})|^2. \tag{20}$$

Comparing with Eq. (16) one obtains immediately the cancellation of the soft and collinear singularities. For the case where the gluon is emitted from the heavy quark line only a soft singularity is present. The collinear singularity is regulated by the finite quark masses. To extract the soft singularity we slice the phase space as follows:

$$1=\Theta(2(p_t\cdot p_g)-x_{\min}m_b m_t)+\Theta(x_{\min}m_b m_t-2(p_t\cdot p_g)). \tag{21}$$

This splits the phase space into a 'resolved' and an 'unresolved' region. The contribution from the resolved region is obtained from a numerical integration in 4 dimensions. In the unresolved region one can use the soft factorization to approximate the matrix element and integrate out the soft gluon. The result is given by:

$$\begin{aligned}
&\frac{1}{2m_t}\int dR_4^d|\mathcal{M}(p_b,p_u,p_{\bar{d}},p_g)|^2\Theta(x_{\min}m_b m_t-2(p_t\cdot p_g))\\
&=\frac{\alpha_s C_F}{2\pi}\left(\frac{4\pi\mu^2}{x_{\min}^2 m_t^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\frac{1}{2m_t}\left\{\left[\frac{1}{\varepsilon}-\ln(z_b)\right]\left[2+\frac{1}{\beta}\ln(\omega)\right]\right.\\
&\left.-\frac{1}{\beta}(-2\beta+2\mathrm{Li}_2(1-\omega)+\ln(\omega)+\frac{1}{2}\ln(\omega)^2)\right)|\mathcal{M}(p_b,p_u,p_{\bar{d}})|^2 dR_3^d.
\end{aligned} \tag{22}$$

In order to be consistent with our definition of the phase space measure dR_3 for the virtual corrections, we have to evaluate the above formula for $dR_3^{d=4}$. Comparing with Eq. (12) it is straightforward to see that the infrared singularities cancel. The numerical implementation of the subtraction term given in Eq. (19) and the slicing given in Eq. (21) does not impose any problem. One should keep in mind that the method presented above allows the calculation of arbitrary infrared save observables.

4 Numerical results

While for the three parton final state one can simply identify the partons with the jets, in the case of real gluon emission a definition of jets in terms of partons is needed which fulfills the condition of infrared-safeness. There are many variations of such definitions, and in general the NLO result for κ_p will depend on the chosen definition. In order to study this dependence, we choose two different jet clustering schemes: The E-algorithm and the Durham-algorithm. The first step is to compute for all pairs (i,j) of the momenta of the final state partons the jet measure y_{ij}, which reads

$$y_{ij} = \frac{(p_i+p_j)^2}{m_t^2} \quad \text{E-algorithm,} \tag{23}$$

$$y_{ij} = \frac{2\min\{E_i^2,E_j^2\}}{m_t^2}(1-\cos\theta_{ij}) \quad \text{Durham-algorithm,} \tag{24}$$

where θ_{ij} is the angle between parton i and parton j in the top quark rest frame. The second step is to recombine the two partons with the smallest y_{ij} into a pseudoparticle with momentum $p_k = p_i + p_j$. Then the direction of the b-jet and the light quark jet with the smallest energy can be readily obtained from p_k and the remaining two momenta. For $b\bar{s}cg$ events also the direction of the charm jet can be defined if the charm is tagged. The construction of the "$\bar{s}$-jet" from $b\bar{s}cg$ events is not straightforward: Only events where both a b-jet and a c-jet are tagged can be used (in particular, the (rare) events where b and c are clustered into a single jet have to be discarded), and for those events the remaining third jet is defined to be the "$\bar{s}$-jet". This definition includes cases where the $\bar{s}$ is recombined with the b- or c-quark and the remaining third jet consists of a hard gluon rather than an $\bar{s}$-quark.

Note that no jet resolution parameter enters in the above definitions; this is not necessary, since the leading order process is free from soft and collinear singularities.

In an experiment which produces top quarks that decay into jets, the first step in the analysis is to identify the signal by using a jet finding algorithm and by applying a number of cuts. For example, at the Tevatron a cone jet algorithm is used to classify the events according to the number of jets. To be more specific, consider top quark pairs where the t decays semileptonically and the $\bar{t}$ decays into jets. Then the event contains at least two b-jets and two light jets. For those events with 4 or more jets originating from the $\bar{t}$, our algorithm should be used in addition to the production-specific jet algorithm, thus leaving only events with exactly 3 jets from the $\bar{t}$ decay. In principle the value for κ_p can depend on the details of the "pre-clustering". This should be studied in Monte-Carlo simulations.

We can also compute κ_p for bare $b, \bar{d}$ $(\bar{s})$ and u (c) quarks, since the directions of a quark and a quark plus a collinear or soft gluon are identical and thus the condition of infrared/collinear safeness is fulfilled. This serves as a benchmark for the realistic case of κ_p for jets. Note however that recombination is needed to define the direction of the least energetic light jet. In the case of the thrust axis the concept of a jet is not needed.

We write our results to order α_s in the following form

$$\kappa_p = \frac{\Gamma^0\kappa_p^0 + \alpha_s\Gamma^1\kappa_p^1}{\Gamma^0 + \alpha_s\Gamma^1} = \kappa_p^0 + \alpha_s\frac{\Gamma^1}{\Gamma^0}\left(\kappa_p^1 - \kappa_p^0\right) + O(\alpha_s^2) \equiv \kappa_p^0[1+\delta_p^{QCD}] + O(\alpha_s^2), \quad (25)$$

where κ_p^0 denotes the Born result. Table 2 gives our results for κ_p and δ_p^{QCD}, where we use the expanded form of Eq. (25). The strong coupling constant is set to $\alpha_s(m_t) = 0.108$.

Table 2: QCD-corrected results for spin analysing powers.

	partons	jets, E-alg.	jets, D-alg.
$\kappa_{\bar{d}}$	$0.9664(7)$	$0.9379(8)$	$0.9327(8)$
$\delta_{\bar{d}}^{QCD}$ [%]	-3.36 ± 0.07	-6.21 ± 0.08	-6.73 ± 0.08
κ_b	$-0.3925(6)$	$-0.3907(6)$	$-0.3910(6)$
δ_b^{QCD} [%]	-3.80 ± 0.15	-4.24 ± 0.15	-4.18 ± 0.15
κ_u	$-0.3167(6)$	$-0.3032(6)$	$-0.3054(6)$
δ_u^{QCD} [%]	$+1.39 \pm 0.19$	-2.93 ± 0.19	-2.22 ± 0.19
κ_j	–	$0.4736(7)$	$0.4734(7)$
δ_j^{QCD} [%]	–	-7.18 ± 0.13	-7.21 ± 0.13
κ_T	$-0.3083(6)$	–	–
δ_T^{QCD} [%]	-2.65 ± 0.19	–	–

5 Discussion and conclusions

Our results listed in Table 2 show that the top-spin analysing powers of the final states in non-leptonic top quark decays receive QCD corrections in the range $+1.4\%$ to -7.2%. This has to be contrasted with the spin analysing power of the charged lepton in decays $t(\uparrow) \to bl^+\nu_l$: the QCD corrected result (for $m_b = 0$) reads [15] $\kappa_l = 1 - 0.015\alpha_s$, i.e. the correction is at the permill level. The QCD corrections to the spin analysing power of a given quark are much larger due to hard gluon emission from that quark. The spin analysing power of jets is smaller than that of the corresponding bare quarks. This effect is largest for the "$\bar{s}$-jet". We find only a small (at most 0.7%) dependence of the results on the jet algorithm. In practice the most important spin analysers are, as far as non-leptonic top decays are concerned, the b-quark jet and the least energetic light (non-b-quark) jet. The QCD corrected results are $\kappa_b \approx -0.39$ and $\kappa_j \approx 0.47$. For the b-jet the difference between the parton level result and the jet result is small. The oriented thrust axis, for which $\kappa_T \approx -0.31$, may also serve as a good spin analyser, since it is easily measurable.

In summary, we have computed the QCD corrections to the top-spin analysing power of jets and the thrust axis in non-leptonic polarized top quark decays. Our results can be used in conjunction with the known NLO QCD results for the production of polarized top quarks both at lepton and hadron colliders.

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