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Reparametrization Invariance in Inclusive Decays of Heavy Hadrons

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Abstract

Reparametrization invariance is the invariance of the heavy mass limit under small changes of the heavy-quark four velocity. We discuss the implications of this invariance for non-local light cone operators, the matrix elements of which are relevant for the leading and subleading shape functions describing differential rates for inclusive heavy-to-light transitions.

1 Introduction

Recent investigations of the heavy-mass expansion for heavy meson decays made the role of certain non-local operators appearent. This type of operators appears in the context of differential rates for heavy-to-light decays, where e.g. the photon spectrum of $B \to X_s \gamma$ is expressed in terms of the so-called shape function $f(\omega)$ [1] which is given as

$$f(\omega) = \langle B(v)|\bar{h}_v \delta(\omega + (in \cdot D))h_v|B(v)\rangle \, , \tag{1}$$

where n is a certain light cone vector determined by the kinematics. This expression, given as a matrix element of a non-local light cone operator, corresponds to the leading term of a twist expansion of the inclusive decay rates, in analogy to deep inelastic scattering.

The subleading terms have been investigated at tree level in [2]. While the leading term can be interpreted in terms of light-like Wilson lines connecting two heavy quarks at "light-cone time" 0 and t, the subleading terms can be interpreted as light-like Wilson lines with "insertions" of additional covariant derivatives, leading to contributions suppressed by one power of $1/m_Q$.

The analysis of the subleading terms beyond tree level has not yet been performed. Including radiative corrections to the leading shape function leads schematically to a rate of the form [3]

$$d\Gamma = H \otimes J \otimes S \tag{2}$$

where $\otimes$ denotes a convolution of the functions S, J and H. Here H describes a hard, process dependent contribution, J is a "jetlike" contribution, containing also the Sudakov logarithms [4], and S describes the soft terms. For the subleading shape functions a similar pattern of the radiative corrections is expected.

In deriving the heavy mass expansion from QCD one introduces a velocity vector v which is the velocity of the hadron containing the heavy quark, $v = p_{hadron}/M_{hadron}$. The heavy quark momentum p_Q inside the heavy meson is decomposed into a large part $m_Q v$ and a residual part k, $p_Q = m_Q v + k$, and the heavy mass expansion is constructed by expanding the amplitudes in the small quantity k/m_Q. From this point of view the velocity vector v in HQET is an external variable, which is not present in full QCD, and which is only fixed up to terms of the order Λ_{QCD}/m_b. Consequently, small reparametrizations of the form $v \to v + \Delta$ with $\Delta = \mathcal{O}(\Lambda_{QCD}/m_b)$ should leave the physical results of the heavy mass expansion invariant.

This so-called reparametrization invariance is known since the early days of heavy quark effective theory (HQET) [5] and its main feature is that it connects different order of the $1/m_b$ expansion. Many applications of reparametrization invariance have been studied, the most prominent of which is the non-renormalization of the kinetic energy operator $\bar{h}_v (iD)^2 h_v$.

The purpose of the present note is to exploit the consequences of reparametrization invariance for the non-local operators appearing in the description of spectra in heavy-to-light decays. The main result is that the number of unknown functions appearing at order $1/m_b$ is reduced.

In the next section we discuss reparametrization invariance in HQET, in section 3 we discuss the light-cone operators and construct reparametrization-invariant combinations of such operators. Finally we consider applications and conclude.

2 Reparametrization Invariance

We consider two versions of HQET with two different choices of the velocity vector v and v' differing by a small quantity Δ[1]

$$v^2 = 1 \qquad v'^2 = 1 = (v+\Delta)^2 = 1 + 2v\cdot\Delta + \mathcal{O}(\Delta^2) \quad \text{thus} \quad v\cdot\Delta = 0\,. \tag{3}$$

If the change Δ in the velocity vector is of the order Λ_{QCD}/m_Q, the two versions of HQET have to be equivalent.

Constructing HQET from QCD involves a redefinition of the quark field Q of the form

$$Q = \exp(-im_Q v\cdot x) Q_v \tag{4}$$

such that the covariant derivative acts as

$$iD_\mu Q = \exp(-im_Q v\cdot x)(m_Q v + iD_\mu) Q_v \tag{5}$$

The left hand side corresponds to the full heavy quark momentum which is not changed under reparametrization. This implies for the change δ_R of the covariant derivative acting on a the quark field Q_v

$$\delta_R(iD_\mu) = -m_Q \Delta_\mu\,. \tag{6}$$

[1] In the following we shall closely follow the discussion given by Chen, second paper of [5].

In the following we have to develop a consistent scheme to count powers. Defining the action to be $\mathcal{O}(1)$, we get that static heavy quark field is $\mathcal{O}(\Lambda_{QCD}^{3/2})$. The covariant derivative as well as the variation δ_R of the covariant derivative are $\mathcal{O}(\Lambda_{QCD})$, and the variaton of the heavy quark field under reparametrization is

$$\delta_R h_v = \frac{\not{\Delta}}{2}\left[1 + \frac{i\not{D}}{2m_Q}\right] h_v + \mathcal{O}[\Lambda_{QCD}^{3/2}(\Lambda_{QCD}/m_Q)^3]\,. \tag{7}$$

Note that the leading contribution originates from the variation of the projector $P_+ = (\not{v} + 1)/2$ and is of order $\Lambda_{QCD}^{5/2}/m_Q$

Equations (3), (6) and (7) are the reparametrization transformations of all relevant quantities needed to exploit the consequences of this symmetry.

Reparametrization invariance connects terms of different orders in the $1/m_Q$ expansion. As an example we consider the HQET Lagrangian

$$\begin{aligned}\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \cdots = \bar{h}_v(iv\cdot D)h_v \qquad\qquad\qquad\qquad\qquad (8)\\ + \frac{1}{2m_Q}\bar{h}_v(iD)^2 h_v - \frac{i}{2m_Q}\bar{h}_v(iD_\mu)(iD_\nu)\sigma^{\mu\nu}h_v + \mathcal{O}(\Lambda_{QCD}^6/m^2)\end{aligned}$$

with $h_v = P_+ h_v$ where $P_+ = (1 + \not{v})/2$.

The leading order term $\mathcal{L}_0$ is of order Λ_{QCD}^4, while its variation is of order Λ_{QCD}^5/m_Q

$$\delta_R \mathcal{L}_0 = \bar{h}_v(i\Delta\cdot D)h_v + \mathcal{O}[\Lambda_{QCD}^6/m_Q^2] \tag{9}$$

Note that the leading term of the variation of the fields (7) does not contribute since

$$P_+\not{\Delta}P_+ = P_+(v\cdot\Delta) = 0 \tag{10}$$

The variation of the leading-order term is compensated by the kinetic energy term, since

$$\delta_R\left(\bar{h}_v(iv\cdot D)h_v + \frac{1}{2m_Q}\bar{h}_v(iD)^2 h_v\right) = \mathcal{O}[\Lambda_{QCD}^6/m_Q^2] \tag{11}$$

Relation (11) is preserved under renormalization which ensures that the kinetic energy piece is not renormalized [5]

In a similar way one can obtain relations between higher order terms in the Lagrangian and also for matrix elements. Again these relations do not change under renormalization from which relations between renormalization constants can be derived.

3 Light-Cone Operators

In inclusive decays the typical situation in which non-local light cone operators are necessary is when a heavy quark decays into light particles and the energy spectrum of one of the outgoing particles or a region of small invariant mass of a set of outgoing particle is considered. The relevant kinematics for the case of an energy spectrum for one outgoing particle are

$$p_b = m_Q v + k = q + p' \qquad q^2 = 0 \tag{12}$$

where q is the momentum of the light particle for which the energy spectrum is computed and p' is the momentum of the rest of the decay products. For the case of $B \to X_s\gamma$ one has at tree level only one light-quark in the final state, leading to $\delta(p'^2) = \delta[(m_Q v + k - q)^2]$ as the spectral function for the final state. For $B \to X_u \ell \bar{\nu}_\ell$ we have at tree level a neutrino and a light quark in the final state, the spectral function of which is proportional to $\Theta(p'^2) = \Theta[(m_Q v + k - q)^2]$. Thus the lepton momentum plays the same role in $B \to X_u \ell \bar{\nu}_\ell$ as the photon momentum in $B \to X_s\gamma$. Generically, the shape function becomes relevant as soon as the spectral function of the remaining particles is a step function close to $p'^2 = 0$.

In order to describe the endpoint region of such an energy spectrum, i.e. the region close to the maximal value of the energy $q \cdot v$, it is convenient to introduce light-cone vectors n and $\bar{n}$ with $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$, one of which is collinear with the momentum q

$$v_\mu = \frac{1}{2}(n_\mu + \bar{n}_\mu) \qquad q_\mu = \frac{1}{2}(n \cdot q)\bar{n}_\mu \, . \tag{13}$$

Using these relations we can write

$$m_Q v - q = \frac{m_Q}{2} n + \frac{1}{2}(m_Q - n \cdot q)\bar{n} \tag{14}$$

The endpoint region is now characterized by

$$(m_Q - n \cdot q) \sim \mathcal{O}(\Lambda_{QCD}) \tag{15}$$

and a systematic expansion in $1/m_Q$ is performed.

The expansion close to the endpoint becomes an expansion in twist and cannot be performed in terms of local operators any more; rather non-local light cone operators are needed. The leading term has been known for some time and the relevant operators are [1]

$$O_0(\omega) = \bar{h}_v \delta(\omega + (in\cdot D)) h_v \tag{16}$$
$$P_0^\alpha(\omega) = \bar{h}_v \delta(\omega + (in\cdot D)) \gamma^\alpha \gamma_5 h_v \tag{17}$$

Note that $P_+ = (1+\not{v})/2$ and $s_\mu = P_+ \gamma_\mu \gamma_5 P_+$ form a basis in the space of (two-component) spinors projected out by P_+.

At subleading order the necesary set of operators can be chosen as [2]

$$\begin{aligned}
O_1^\mu(\omega) &= \bar{h}_v \left\{(iD^\mu), \delta(\omega + (in\cdot D))\right\} h_v \\
O_2^\mu(\omega) &= i\,\bar{h}_v \left[(iD^\mu), \delta(\omega + (in\cdot D))\right] h_v \\
O_3^{\mu\nu}(\omega_1,\omega_2) &= \bar{h}_v \delta(\omega_2 + (in\cdot D)) \left\{iD_\perp^\mu\,,\, iD_\perp^\nu\right\} \delta(\omega_1 + (in\cdot D)) h_v \\
O_4^{\mu\nu}(\omega_1,\omega_2) &= i\,\bar{h}_v \delta(\omega_2 + (in\cdot D)) \left[iD_\perp^\mu\,,\, iD_\perp^\nu\right] \delta(\omega_1 + (in\cdot D)) h_v
\end{aligned} \tag{18}$$

for the “spin-independent” operators and

$$\begin{aligned}
P_1^{\mu\alpha}(\omega) &= \bar{h}_v \left\{(iD^\mu), \delta(\omega + (in\cdot D))\right\} \gamma^\alpha \gamma_5 h_v \\
P_2^{\mu\alpha}(\omega) &= i\,\bar{h}_v \left[(iD^\mu), \delta(\omega + (in\cdot D))\right] \gamma^\alpha \gamma_5 h_v \\
P_3^{\mu\nu\alpha}(\omega_1,\omega_2) &= \bar{h}_v \delta(\omega_2 + (in\cdot D)) \left\{iD_\perp^\mu\,,\, iD_\perp^\nu\right\} \delta(\omega_1 + (in\cdot D)) \gamma^\alpha \gamma_5 h_v \\
P_4^{\mu\nu\alpha}(\omega_1,\omega_2) &= i\,\bar{h}_v \delta(\omega_2 + (in\cdot D)) \left[iD_\perp^\mu\,,\, iD_\perp^\nu\right] \delta(\omega_1 + (in\cdot D)) \gamma^\alpha \gamma_5 h_v
\end{aligned} \tag{19}$$

for the “spin-dependent” ones.

The (differential) rates are expressed in terms of convolutions of ω-dependent Wilson coefficients with forward matrix elements of these operators [2]

$$\begin{aligned}
d\Gamma = &\int d\omega \left(C_0(\omega) < O_0(\omega) > + C_{0,\alpha}^{(5)}(\omega) < P_0^\alpha(\omega) >\right) \\
&+ \frac{1}{m_Q} \sum_{i=1,2} \int d\omega \left(C_{i,\mu}(\omega) < O_i^\mu(\omega) > + C_{i,\mu\alpha}^{(5)}(\omega) < P_i^{\mu\alpha}(\omega) >\right) \\
&+ \frac{1}{m_Q} \sum_{i=3,4} \int d\omega_1\, d\omega_2\, (C_{i,\mu\nu}(\omega_1,\omega_2) < O_i^{\mu\nu}(\omega_1,\omega_2) > \\
&\qquad\qquad + C_{i,\mu\nu\alpha}^{(5)}(\omega_1,\omega_2) < P_i^{\mu\nu\alpha}(\omega_1,\omega_2) >\Big) \\
&+ \cdots
\end{aligned} \tag{20}$$

where $< .. >$ denotes the forward matrix element with b-Hadron states and the ellipses denote terms originating from time-ordered products with higher order terms of the Lagrangian, which we do not need to consider here.

In the following we want to discuss the implications of reparametriation invariance for the non-local light-cone operators. Similar to the case of local operators we shall derive reparametrization-invariant combinations of operators containing different orders of the $1/m_Q$ expansion. To investigate this we shall first compute the variation of the light cone vectors under a reparametrization transformation, which means that v is varied according to (3) and q is kept fixed. Expressing the light-cone vectors in terms of q and v we get

$$n = \frac{1}{v \cdot q}[2(v \cdot q)v - q] \quad \text{and} \quad \bar{n} = \frac{1}{v \cdot q} q \tag{21}$$

from which we can derive the variation δ_R under reparamatrization

$$\begin{aligned} \delta_R n_\mu &= \frac{\partial n_\mu}{\partial v_\alpha} \Delta_\alpha = 2\Delta_\mu + \bar{n}_\mu (\bar{n} \cdot \Delta) \\ \delta_R \bar{n}_\mu &= \frac{\partial \bar{n}_\mu}{\partial v_\alpha} \Delta_\alpha = -\bar{n}_\mu (\bar{n} \cdot \Delta) \end{aligned} \tag{22}$$

Using this we can study the variation of

$$\hat{O}_0(\omega) = \bar{h}_v \frac{1}{\omega + (in \cdot D)} \Gamma h_v \tag{23}$$

which is of order Λ_{QCD}^2 since we have to count ω as $\mathcal{O}(\Lambda_{QCD})$. The imaginary part (by replacing $\omega \to \omega + i\epsilon$) of this expression is either $O_0(\omega)$ (for $\Gamma = P_+$) or $P_0^\alpha(\omega)$ (for $\Gamma = s^\alpha = P_+ \gamma^\alpha \gamma_5 P_+$). From (6) and (22) we get

$$\delta_R(in \cdot D) = -m_Q(n \cdot \Delta) + (\bar{n} \cdot \Delta)(i\bar{n} \cdot D) + 2(i\Delta \cdot D) \tag{24}$$

and thus

$$\begin{aligned} \delta_R \hat{O}_0(\omega) &= \bar{h}_v \{ \not{\Delta}, \Gamma \} \frac{1}{\omega + (in \cdot D)} h_v \\ &+ \bar{h}_v \frac{1}{\omega + (in \cdot D)} [m_Q(n \cdot \Delta) - (\bar{n} \cdot \Delta)(i\bar{n} \cdot D) - 2(i\Delta \cdot D)] \frac{1}{\omega + (in \cdot D)} \Gamma h_v \\ &+ \mathcal{O}[\Lambda_{QCD}/m_Q^2] \end{aligned} \tag{25}$$

where we have omitted terms of subleading order in $1/m_Q$ coming e.g. from the variation of the heavy quark fields.

The first term vanishes due to (10) and fact that Γ is either P_+ or $s_\mu = P_+ \gamma_\mu \gamma_5 P_+$. The second term contains a piece of order Λ_{QCD}^2 (which is of the same order as $O_0(\omega)$ itself) coming from the variation of the covariant derivative, while all other terms in (25) are of higher order.

We shall first discuss the variation of order Λ_{QCD}^2. This can be written as

$$\delta_R \hat{O}_0(\omega) = \bar{h}_v \frac{1}{\omega + (in \cdot D)} m_Q (n \cdot \Delta) \frac{1}{\omega + (in \cdot D)} \Gamma h_v + \mathcal{O}(\Lambda_{QCD}^3 / m_Q) \tag{26}$$

$$= - \left(\frac{\partial}{\partial \omega} \hat{O}_0(\omega) \right) m_Q (n \cdot \Delta) + \mathcal{O}(\Lambda_{QCD}^3 / m_Q) \tag{27}$$

which means that the $\mathcal{O}(\Lambda_{QCD}^2)$-variation can be absorbed into a shift $\omega \to \omega - m_Q(n \cdot \Delta)$.

In the following we assume that Δ does not have a light cone component, i.e. we only consider $\Delta^\perp$ for which we have $(n \cdot \Delta^\perp) = 0$. Note that this also implies $(\bar{n} \cdot \Delta^\perp) = 0$ due to (3). In this way (25) simplifies to

$$\delta_R^\perp \hat{O}_0(\omega) = \bar{h}_v \frac{-2}{\omega + (in \cdot D)} (i\Delta^\perp \cdot D) \frac{1}{\omega + (in \cdot D)} \Gamma h_v + \mathcal{O}(\Lambda_{QCD^4} / m_Q^2) \, . \tag{28}$$

Our aim is to construct a reparametrization invariant, which is equal to $O_0(\omega)$ to leading order. The variation of $O_0(\omega)$ of order unity as given in (28), and a subleading contribution is needed to compensate this variation. To construct this invaraint, we first note that

$$\delta_R^\perp \left((in \cdot D) + \frac{1}{m_Q} (iD^\perp)^2 \right) = 0 \tag{29}$$

which means that

$$\left(\frac{1}{\omega + (in \cdot D) + \frac{1}{m_Q} (iD^\perp)^2} \right) \tag{30}$$

is an exact reparametrization invariant.

Furthermore, we may include higher order terms to construct a reparametrization invariant field

$$H_v = h_v + \frac{(i\not{D})}{2m_Q} h_v + \frac{1}{4m_Q^2} (iD)^2 h_v + \cdots \, , \quad \delta_R^\perp H_v = 0 \tag{31}$$

which can be used to construct the reparametrization-invariant quantity

$$\hat{R}_0(\omega) = \bar{H}_v \left(\frac{1}{\omega + (in \cdot D) + \frac{1}{m_Q} (iD^\perp)^2} \right) \Gamma H_v \tag{32}$$

where Γ is again either P_+ or $s_\mu = P_+\gamma_\mu\gamma_5 P_+$.

This formal expression can now be expanded to obtain the reparametrization invariant combinaton of operators appearing in the twist expansion of inclusive rates. Truncating the expansion yields operators for which where reparametrization invariance holds to a certain order in the $1/m_Q$ expansion. We get

$$\hat{R}_0^{(0)}(\omega) = \bar{h}_v \frac{1}{\omega + (in\cdot D)}\Gamma h_v \tag{33}$$

$$\hat{R}_0^{(1)}(\omega) = \bar{h}_v \frac{1}{\omega + (in\cdot D)}\Gamma h_v - \frac{1}{m_Q}\bar{h}_v \frac{1}{\omega + (in\cdot D)}(iD^\perp)^2 \frac{1}{\omega + (in\cdot D)}\Gamma h_v$$

$$\begin{aligned}\hat{R}_0^{(2)}(\omega) = {} & \bar{h}_v \frac{1}{\omega + (in\cdot D)}\Gamma h_v - \frac{1}{m_Q}\bar{h}_v \frac{1}{\omega + (in\cdot D)}(iD^\perp)^2 \frac{1}{\omega + (in\cdot D)}\Gamma h_v \\ & + \frac{1}{4m_Q^2}\bar{h}_v (i\not{D}^\perp)\frac{1}{\omega + (in\cdot D)}(i\not{D}^\perp)\Gamma h_v \\ & + \frac{1}{4m_Q^2}\bar{h}_v \left\{ (iD^\perp)^2 , \frac{1}{\omega + (in\cdot D)}\right\}\Gamma h_v \\ & + \frac{1}{m_Q^2}\bar{h}_v \frac{1}{\omega + (in\cdot D)}(iD^\perp)^2 \frac{1}{\omega + (in\cdot D)}(iD^\perp)^2 \frac{1}{\omega + (in\cdot D)}\Gamma h_v\end{aligned}$$

where we have

$$\delta_R^\perp \hat{R}_0^{(k)} = \mathcal{O}(\Lambda_{QCD}^{k+3}/m_Q^{k+1}) \tag{34}$$

For the case $\Gamma = P_+$ we may reexpress $\hat{R}_0^{(1)}$ in terms of the $O_0(\omega)$ and $O_3(\omega_1, \omega_2)$

$$\hat{R}_0^{(1)}(\omega) = \int \frac{d\sigma}{\omega - \sigma} O_0(\sigma) - \frac{1}{2m_Q}\int \frac{d\sigma_1}{\omega - \sigma_1}\frac{d\sigma_2}{\omega - \sigma_2} g_{\mu\nu} O_3^{\mu\nu}(\sigma_1, \sigma_2) \tag{35}$$

and replace $\omega \to \omega + i\epsilon$ in (35) to identify

$$\begin{aligned}R_0^{(1)}(\omega) &= O_0(\omega) - \frac{1}{\pi}\mathrm{Im}\left(\frac{1}{2m_Q}\int \frac{d\sigma_1}{\omega + i\epsilon - \sigma_1}\frac{d\sigma_2}{\omega + i\epsilon - \sigma_2} g_{\mu\nu} O_3^{\mu\nu}(\sigma_1, \sigma_2)\right) \\ &= O_0(\omega) - \frac{1}{2m_Q}\int d\sigma_1\, d\sigma_2 \left(\frac{\delta(\omega - \sigma_1) - \delta(\omega - \sigma_2)}{\sigma_1 - \sigma_2}\right) g_{\mu\nu} O_3^{\mu\nu}(\sigma_1, \sigma_2) \end{aligned} \tag{36}$$

to be the (up to order Λ_{QCD}^4/m_Q^2) reparametrization-invariant light-cone operator involving the leading order operator $O_0(\omega)$.

Likewise, for $\Gamma = s^\alpha$ we get

$$Q_0^{\alpha(1)}(\omega) = P_0^\alpha(\omega) - \frac{1}{2m_Q}\int d\sigma_1\, d\sigma_2 \left(\frac{\delta(\omega-\sigma_1)-\delta(\omega-\sigma_2)}{\sigma_1-\sigma_2}\right) g_{\mu\nu} P_3^{\mu\nu\alpha}(\sigma_1,\sigma_2) \tag{37}$$

for the spin-dependent reparametrization-invariant quantity up to order Λ_{QCD}^4/m_Q^2.

The other operators of subleading order are not related to $O_0(\omega)$ or $P_0^\alpha(\omega)$. In order to investigate the behaviour $O_1^\mu(\omega)$, we split the covariant derivative according to

$$iD^\mu = \frac{1}{2}(\bar{n}^\mu - n^\mu)(in\cdot D) + iD_\perp^\mu \tag{38}$$

where we have made use of the equation of motion for the heavy quark, which implies $(in\cdot D) = -(i\bar{n}\cdot D)$ in $O_1^\mu(\omega)$ as well as in $P_1^{\mu\alpha}(\omega)$. In the same way as before we consider $\hat{O}_1^\mu(\omega)$ in which the δ function is replaced by $1/(\omega + (in\cdot D))$. According to (38) we split $\hat{O}_1^\mu(\omega)$ into $\hat{O}_{1,||}^\mu(\omega)$ and $\hat{O}_{1,\perp}^\mu(\omega)$. We get

$$\begin{aligned}\hat{O}_{1,||}^\mu(\omega) &= (\bar{n}^\mu - n^\mu)\bar{h}_v(in\cdot D)\frac{1}{\omega+(in\cdot D)}\Gamma h_v \\ &= (\bar{n}^\mu - n^\mu)\left[\bar{h}_v\Gamma h_v - \omega\bar{h}_v\frac{1}{\omega+(in\cdot D)}\Gamma h_v\right]\end{aligned} \tag{39}$$

Taking the imaginary part (after $\omega \to \omega + i\epsilon$) we get for $\Gamma = P_+$

$$O_{1,||}^\mu(\omega) = (n^\mu - \bar{n}^\mu)\omega\bar{h}_v\delta(\omega+(in\cdot D))h_v = (n^\mu - \bar{n}^\mu)\,\omega\, O_0(\omega) \tag{40}$$

which means that $O_{1,||}^\mu(\omega)$ is completely given in terms of $O_0(\omega)$. The same arguments apply for the spin-dependent operator $P_1^{\mu\alpha}(\omega)$, where $P_{1||}^{\mu\alpha}(\omega)$ is entirely given in terms of $P_0^\alpha(\omega)$

However, for the perpendicular pieces $O_{1,\perp}^\mu(\omega)$ and $P_{1\perp}^{\mu\alpha}(\omega)$ we get for the reparametrization variation

$$\delta_R^\perp O_{1,\perp}^\mu(\omega) = -2m_Q\Delta_\perp^\mu O_0(\omega) + \mathcal{O}(\Lambda_{QCD}^3/m_Q) \tag{41}$$

$$\delta_R^\perp P_{1,\perp}^{\mu\alpha}(\omega) = -2m_Q\Delta_\perp^\mu P_0^\alpha(\omega) + \mathcal{O}(\Lambda_{QCD}^3/m_Q) \tag{42}$$

which means that this variation contains a contribution of the same order as the operator itself, which would need to be compensated by some other subleading operator. However, there is no such operator, and so we conclude that reparametrization invariance requires that only $O^{\mu}_{1,||}(\omega)$ and $P^{\mu\alpha}_{1||}(\omega)$ contribute to a physical quantity.

Using the same arguments we can discuss $O^{\mu}_{2}(\omega)$ and $P^{\mu\alpha}_{2}(\omega)$. Obviously we have $O^{\mu}_{2||}(\omega) = 0 = P^{\mu\alpha}_{2||}(\omega)$. However, unlike for $O^{\mu}_{1}(\omega)$ and $P^{\mu\alpha}_{1}(\omega)$, a reparametrization transformation yields

$$\delta_R^{\perp} O^{\mu}_{2,\perp}(\omega) = \mathcal{O}(\Lambda^3_{QCD}/m_Q) \tag{43}$$
$$\delta_R^{\perp} P^{\mu\alpha}_{2,\perp}(\omega) = \mathcal{O}(\Lambda^3_{QCD}/m_Q) \tag{44}$$

since these operators involve a commutator rather than an anticommutator, and hence these operators will in general contribute.

Finally, nothing new can be obtained for $O^{\mu\nu}_{4}$ as well as $P^{\mu\nu\alpha}_{4}$ from reparametrization invariance; these operators are related through reparametrization to higher order terms, which have not yet been classified.

4 Applications

One immediate consequence of the above result concerns the matching coefficients for light cone operators. Since physical observables such as (differential) rates are reparametrization invariants, the matching coefficients $C_0(\omega)$ of $O_0(\omega)$ and the one of $O^{\mu\nu}_3(\omega_1, \omega_2)$ have to be related, such that

$$\begin{aligned} d\Gamma &= \int d\omega\, (C_0(\omega) < R_0(\omega) > + D_{0\alpha}(\omega) < P_0^{\alpha}(\omega) >) \qquad (45)\\ &= \int d\omega\, (C_0(\omega) < O_0(\omega) > + D_{0\alpha}(\omega) < P_0^{\alpha}(\omega) >) \\ &- \frac{1}{2m_Q} \int d\omega C_0(\omega) \int d\sigma_1\, d\sigma_2 \left(\frac{\delta(\omega - \sigma_1) - \delta(\omega - \sigma_2)}{\sigma_1 - \sigma_2} \right) g_{\mu\nu} < O_3^{\mu\nu}(\sigma_1, \sigma_2) > \\ &- \frac{1}{2m_Q} \int d\omega D_{0\alpha}(\omega) \int d\sigma_1\, d\sigma_2 \left(\frac{\delta(\omega - \sigma_1) - \delta(\omega - \sigma_2)}{\sigma_1 - \sigma_2} \right) g_{\mu\nu} < P_3^{\mu\nu\alpha}(\sigma_1, \sigma_2) > \end{aligned}$$

which can now be compared to (20), yielding the reparametrization-invariance relation betwen the coefficients

$$C_{3,\mu\nu}(\sigma_1, \sigma_2) = -\frac{1}{2} \int d\omega C_0(\omega) \left(\frac{\delta(\omega - \sigma_1) - \delta(\omega - \sigma_2)}{\sigma_1 - \sigma_2} \right) g_{\mu\nu} \tag{46}$$
$$C^{(5)}_{3,\mu\nu\alpha}(\sigma_1, \sigma_2) = -\frac{1}{2} \int d\omega C^{(5)}_{0,\alpha}(\omega) \left(\frac{\delta(\omega - \sigma_1) - \delta(\omega - \sigma_2)}{\sigma_1 - \sigma_2} \right) g_{\mu\nu} \tag{47}$$

Relation (46) has been shown at tree level by explicit calculation for the case of $B \to X_s\gamma$ in [2] and holds also for the case $B \to X_u \ell \bar{\nu}_\ell$ [6], but here we claim that such a relation is a consequence of reparametrization invariance and thus has to hold including radiative corrections.

In particular, it has to hold for the renormalization kernel of the subleading operators $O_3^{\mu\nu}(\omega_1, \omega_2)$ and $P_3^{\mu\nu\alpha}(\omega_1, \omega_2)$. While up to now only the renormalization kernel of the leading order term has been investigated [7, 8], a relation like (46) has to relate the kernel of $O_3^{\mu\nu}(\omega_1, \omega_2)$ with the one of $O_0(\omega)$, and the kernel of $P_3^{\mu\nu\alpha}(\omega_1, \omega_2)$ will be related to the one of $P_0^\alpha(\omega)$.

In this way, reparametrization invariance reduces the number of unknown functions parametrizing e.g. the photon spectrum of $B \to X_s\gamma$ to subleading order. Following [2], the non-vanishing matrix elements leading to independent functions are

$$\begin{aligned}
\langle B(v)|O_0(\omega)|B(v)\rangle &= 2m_B f(\omega) \\
\langle B(v)|O_3^{\mu\nu}(\omega_1,\omega_2)|B(v)\rangle &= 2m_B\, g_2(\omega_1,\omega_2) g_\perp^{\mu\nu} \\
\langle B(v)|P_{2,\alpha}^{\mu}(\omega)|B(v)\rangle &= 2m_B\, h_1(\omega)\varepsilon_{\perp,\alpha}^{\mu} \\
\langle B(v)|P_{4,\alpha}^{\mu\nu}(\omega_1,\omega_2)|B(v)\rangle &= 2m_B\, h_2(\omega_1,\omega_2)\varepsilon_{\rho\sigma\alpha\beta}\, g_\perp^{\mu\rho} g_\perp^{\nu\sigma} v^\beta \\
\langle B(v)|O_T(\omega)|B(v)\rangle &= 2m_B\, t(\omega)\,,
\end{aligned} \tag{48}$$

where we define

$$\varepsilon_\perp^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} v_\alpha n_\beta\,, \tag{49}$$

and $\varepsilon^{0123} = 1$. Furthermore, $O_T(\omega)$ is the contribution originating from the time-ordered product of the leading-order operator $O_0(\omega)$ with the $1/m_Q$ corrections to the Lagrangian; the precise definition can be found in [2].

The contributions of g_2 and h_2 can be gathered into a fucntion of a single variable

$$G_2(\sigma) = \int d\omega_1\, d\omega_2\, g_2(\omega_1,\omega_2) \left[\frac{\delta(\sigma-\omega_1)-\delta(\sigma-\omega_2)}{\omega_1-\omega_2}\right] \tag{50}$$

$$H_2(\sigma) = \int d\omega_1\, d\omega_2\, h_2(\omega_1,\omega_2) \left[\frac{\delta(\sigma-\omega_1)-\delta(\sigma-\omega_2)}{\omega_1-\omega_2}\right]. \tag{51}$$

which is at least for g_2 not surprising, since it is a consequence of reparametrization invariance. In [6] the conclusion was reached that the four universal functions $F(\omega) = f(\omega) + t(\omega)/(2m_Q)$, $G_2(\omega)$, $h_1(\omega)$ and $H_2(\omega)$ are needed to parametrize the subleading twist contributions to heavy-to-light decays.

From reparametrization invariance we conclude that the functions $F(\omega)$ and $G_2(\omega)$ have to appear always in the same combination, such that

$$\mathcal{F}(\omega) = f(\omega) + \frac{1}{2m_Q} t(\omega) - \frac{1}{m_Q^2} G_2(\omega) \tag{52}$$

is a single universal function. This has been confirmed at tree level by explicit calculation, but this should hold to all orders in $\alpha_s(m_b)$.

5 Conclusions

We have discussed the consequences of reparametrization invariance for the subleading contributions in the twist expansion for inclusive heavy meson decays. As in the case for local operators, reparametrization invariance relates different orders in the twist expansion. Looking at the first subleading terms reparametrization relates the leading order shape function to one of the subleading matrix elements, leading to identical matching coefficients for the two contributions. As a practical concequence, the spectra of inclusive heavy-to-light transitions are parametrized in terms of three unknown universal functions, once the first subleading terms are included.

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References

[1] M. Neubert, Phys. Rev. D **49**, 3392 (1994) [arXiv:hep-ph/9311325].
M. Neubert, Phys. Rev. D **49**, 4623 (1994) [arXiv:hep-ph/9312311],
I. I. Bigi, M. A. Shifman, N. G. Uraltsev and A. I. Vainshtein, Int. J. Mod. Phys. A **9**, 2467 (1994) [arXiv:hep-ph/9312359].
T. Mannel and M. Neubert, Phys. Rev. D **50**, 2037 (1994) [arXiv:hep-ph/9402288].

[2] C. W. Bauer, M. E. Luke and T. Mannel, arXiv:hep-ph/0102089.

[3] G. P. Korchemsky and G. Sterman, Phys. Lett. B **340**, 96 (1994) [arXiv:hep-ph/9407344].

[4] C. W. Bauer, S. Fleming and M. E. Luke, Phys. Rev. D **63**, 014006 (2001) [arXiv:hep-ph/0005275].

[5] M. E. Luke and A. V. Manohar, Phys. Lett. B **286**, 348 (1992) [arXiv:hep-ph/9205228].
Y. Q. Chen, Phys. Lett. B **317**, 421 (1993).

[6] C. W. Bauer, M. E. Luke and T. Mannel, to be published

[7] C. Balzereit, T. Mannel and W. Kilian, Phys. Rev. D **58**, 114029 (1998) [arXiv:hep-ph/9805297].

[8] U. Aglietti and G. Ricciardi, Nucl. Phys. B **587**, 363 (2000) [arXiv:hep-ph/0003146].