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COUNTING CONTACT TERMS IN $B \to V\gamma$ DECAYS

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We clarify the origin and cancellation of contact terms in the weak annihilation amplitudes contributing to $B \to V\gamma$. It is demonstrated that the photon emission from the final-state quarks vanishes in the chiral limit of massless quarks. The contact terms in the QCD light-cone sum rule evaluation of the weak annihilation amplitudes are also discussed.

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1 Introduction

Radiative decays of B-mesons, such as $B \to K^*\gamma$ or $B \to \rho\gamma$ provide important tests of the Standard Model and of new physics scenarios. However, besides the dominant pointlike $b \to s\gamma$ or $b \to d\gamma$ transitions generated by loops of heavy particles, there are ordinary weak decay mechanisms. In particular the decay $B \to \rho\gamma$ can proceed via the usual four-Fermi weak transition accompanied by a photon radiated from the quarks inside the initial or final mesons. Clearly, this "weak annihilation" mechanism depicted in Fig. 1 must be under theoretical control in order to predict the decay rate with a reasonable accuracy.

In recent years, several calculations of the weak annihilation contribution to $B \to V\gamma$ were reported [1−9]. Various techniques, from quark models and effective Lagrangians to dispersion relations and QCD sum rules were used.

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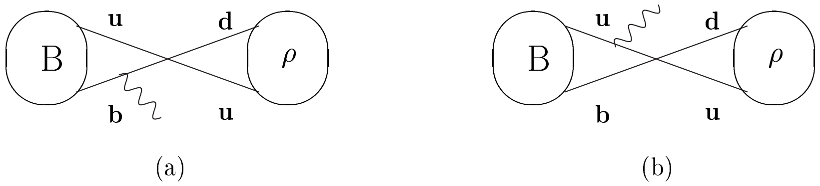

Figure 1. Weak annihilation mechanism for the decay $B^- \to \rho^- \gamma$ with photon emission from the initial state quarks.

In all these calculational schemes perturbative photon emission is involved which gives rise to gauge non-invariant terms in the photon field, the so called contact terms. Clearly these have to vanish in the final answer.

Going through the papers including our own [5] we have to admit that the way of handling the contact terms often looks arbitrary or even mysterious, and may cause dissatisfaction among careful readers. In the most recent work [9], the contact terms seem to combine into a gauge-invariant combination and contribute to the physical decay amplitude.

In this paper we would like to investigate this issue in more details concentrating on the emergence and cancellation of contact terms in the weak annihilation amplitudes in $B \to V\gamma$. The elements of our analysis such as Ward identities for the divergences of conserved currents or decomposition in invariant amplitudes are familiar and were used in some of the papers [7,9]. However we are not aware of a complete and comprehensive analysis which puts all dots over i's.

2 Radiative leptonic decay $B \to l\nu\gamma$

In order to make things as clear as possible, we start our discussion with the better known decay $B \to l\nu\gamma$ [10−14] whose discussion parallels the well known process $\pi \to e\nu\gamma$. It is customary to divide the amplitude of the latter decay into "internal bremsstrahlung" (IB) and "structure dependent"(SD) contributions (for a review of $\pi \to e\nu\gamma$ see e.g. [15]). The IB contribution collects the photon radiation from the charged lepton (Fig.2 a) and from the meson (Fig. 2b) and shows the same helicity (mass) suppression as the leading decay $\pi \to l\nu$. However, a simple calculation of the emission from the lepton

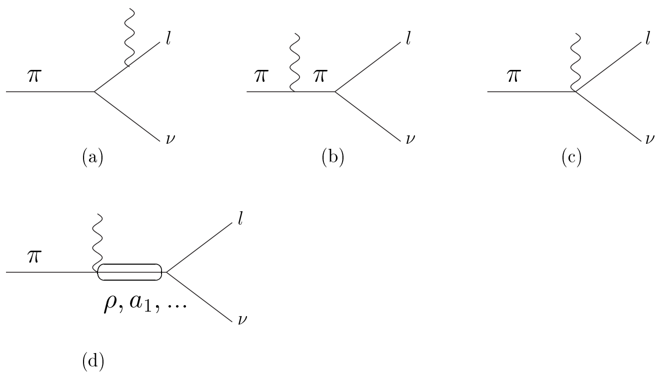

Figure 2. Diagrams for $\pi \to l\nu\gamma$

yields a helicity unsupressed amplitude,

$$A(\pi \to l\nu\gamma)_{IB} = -ie\frac{G_F}{\sqrt{2}}\bar{u}_l\Gamma_\alpha v_\nu \epsilon^\alpha f_\pi \,, \tag{1}$$

even if $m_l = 0$. Here, $e = \sqrt{4\pi\alpha_{em}}$ is the basic electric charge, G_F the Fermi constant, $\Gamma_\alpha = \gamma_\alpha(1-\gamma_5)$, u_l, v_ν are the lepton spinors, ϵ is the photon polarization vector and f_π the pion decay constant. Besides having no helicity suppression, this result is also not gauge-invariant in the photon field. But it is well known that Eq. (1) is not the complete answer for the physical decay amplitude. To recover the latter, one has to add the contribution of an additional diagram (Fig. 2c) corresponding to an effective four-particle vertex of the form $\pi A_\mu \bar{l}\Gamma^\mu \nu_l$ where A_μ is the photon field. In the framework of an effective meson theory with a point-like pion this vertex can be incorporated into the usual coupling by replacing the derivative of the pion field by the covariant one; the effective interaction is then $D_\mu \pi \bar{l}\Gamma^\mu \nu_l$, where $D_\mu = \partial_\mu - ieA_\mu$. Note that the extra interaction is a *contact term* in the sense that the weak and the electromagnetic interactions (currents) originate in same space-time point. Fig. 2c yields

$$A(\pi \to l\nu\gamma)_{CT} = ie\frac{G_F}{\sqrt{2}}\bar{u}_l\Gamma_\alpha v_\nu \epsilon^\alpha f_\pi \,, \tag{2}$$

exactly canceling the amplitude (1): The emission of the photon from the final state leptons is indeed absent in the "chiral limit" of massless leptons. The only surviving contribution comes from the photon emission in the initial state which is "structure dependent" and corresponds to the diagrams of Fig. 2d. The corresponding amplitude can be parametrized in a gauge-invariant form

$$A(\pi \to l\nu\gamma)_{SD} = e\frac{G_F}{\sqrt{2}}\,(\bar{u}_l\Gamma_\alpha v_\nu)\,\Big(iF_A^{(\pi)}(p^2)(\epsilon^\alpha(p\cdot q)-(\epsilon\cdot p)q^\alpha) \\ +F_V^{(\pi)}(p^2)\epsilon^{\alpha\mu\lambda\rho}\epsilon_\mu p_\lambda q_\rho\Big). \qquad (3)$$

Here, p is the sum of the lepton momenta and q is the momentum of the real photon, $q^2=0$. The SD amplitude is determined by the axial and vector form factors $F_{A,V}^{(\pi)}$.

The SD contributions come from intermediate quark-antiquark states with $J^P = 1^-$ and 1^+ and are therefore very difficult to calculate without a reliable method to handle long-distance quark-gluon interactions. A way out is to represent the form factors $F_{A,V}^{(\pi)}$ in terms of dispersion relations in the variable p^2 with a set of intermediate hadronic states with the quantum numbers of vector and axial-vector mesons, respectively.

We now consider the radiative leptonic decays in the framework of QCD where mesons are composite particles. The amplitude for $B\to l\nu\gamma$ can be written as

$$A(B^-\to l\bar{\nu}_l\gamma) = \frac{G_F}{\sqrt{2}}V_{ub}\langle\, l\bar{\nu}(p)\;\gamma(q)\mid\big(\bar{l}\Gamma_\rho\nu\big)\,(\bar{u}\Gamma^\rho b)\mid B^-(p+q)\,\rangle\,. \qquad (4)$$

To first order in the e.m. interactions the matrix element above can be rewritten as a sum of two physically distinct contributions:

$$\langle\, l\bar{\nu}(p)\;\gamma(q)\mid\big(\bar{l}\Gamma_\rho\nu\big)\,(\bar{u}\Gamma^\rho b)\mid B^-(p+q)\,\rangle \\ = ie\epsilon^\mu\Big[(\bar{u}_l\Gamma_\rho v_\nu)\int d^4x\,e^{iqx}\langle 0\mid T\{j_\mu^{em}(x)\,\bar{u}\Gamma^\rho b(0)\}\mid B^-(p+q)\rangle \\ -\int d^4x\,e^{iqx}\langle l\bar{\nu}(p)\mid T\{j_\mu^{em}(x)\bar{l}\Gamma_\rho\nu(0)\}\mid 0\rangle if_B(p+q)^\rho\Big]\,, \qquad (5)$$

where $j_\mu^{em} = -\bar{l}\gamma_\mu l + \sum_{q=u,d,s,c,b} e_q\bar{q}\gamma_\mu q$ is the e.m. current and e_q the quark e.m. charges in the units of e. The first term on the r.h.s. of Eq. (5) corresponds to the photon emission from the initial B meson state and the leptonic part is trivially factorized out. In the second term the photon is emitted from the final charged lepton and the hadronic matrix element is factorized using the standard definition of the B-meson decay constant : $\langle 0\mid\bar{u}\gamma^\rho\gamma_5 b\mid B(p+q)\rangle = if_B(p+q)^\rho$. The remaining lepton-photon matrix element in this term can be simply calculated using Feynman rules of QED.

As expected, at $m_l=0$ the result is very similar to Eq. (1):

$$e\epsilon^\mu\int d^4x\, e^{iqx}\langle l\bar{\nu}(p)\mid T\{j_\mu^{em}(x)\,\bar{l}\Gamma_\rho\nu(0)\}\mid 0\rangle f_B(p+q)^\rho = -ie\bar{u}_l\Gamma_\mu v_\nu\epsilon^\mu f_B\,, \quad (6)$$

and again has a typical structure of a contact, gauge non-invariant term. In analogy to the pion case we expect that photon emission from the initial B meson contains a contact term which cancels Eq. (6).

To see this explicitly we use a generic covariant decomposition of the hadronic matrix element

$$T_{\mu\rho}^{(B)}(p,q) = i\int d^4x\, e^{iqx}\langle 0\mid T\{j_\mu^{em}(x)\,\bar{u}\Gamma_\rho b(0)\}\mid B^-(p+q)\rangle \quad (7)$$

in two independent 4-momenta p and q:

$$T_{\mu\rho}^{(B)}(p,q) = g_{\mu\rho}\,a + p_\mu q_\rho\,b + q_\mu p_\rho\,c + p_\mu p_\rho\,d + q_\mu q_\rho\,e + \epsilon_{\rho\mu\lambda\sigma}p^\lambda q^\sigma F_V^{(B)}\,. \quad (8)$$

where a,b,c,d,e and $F_V^{(B)}$ are invariant amplitudes. We apply the standard electromagnetic Ward identity to the matrix element (7) using the conservation of the e.m. current. In momentum space it corresponds to a multiplication by q_μ. The well-known additional contribution due to differentiation of the θ-function in the T product yields a contact term:

$$q^\mu T_{\mu\rho}^{(B)} = i(p+q)_\rho f_B\,. \quad (9)$$

Applied to the decomposition (8), the same operation yields

$$q^\mu T_{\mu\rho}^{(B)} = q_\rho\,a + (p\cdot q)q_\rho\,b + (p\cdot q)p_\rho\,d\,. \quad (10)$$

Comparing the coefficients in two above equations at independent 4-momenta one gets the relations

$$a + (p\cdot q)\,b = if_B, \quad (11)$$

and

$$(p\cdot q)\,d = if_B, \quad (12)$$

The first of these relations connects the unknown amplitudes a via b whereas the second fixes the amplitude d. As a result we can rewrite $T_{\mu\rho}^{(B)}$ in the following general form

$$\begin{aligned} T_{\mu\rho}^{(B)}(p,q) &= (g_{\mu\rho}(p\cdot q) - p_\mu q_\rho)\,iF_A^{(B)} + g_{\mu\rho}(p\cdot q)\,\alpha + p_\mu q_\rho\,\beta \\ &+q_\mu p_\rho\,c + i\frac{p_\mu p_\rho}{(p\cdot q)}f_B + q_\mu q_\rho\,e + \epsilon_{\rho\mu\lambda\sigma}p^\lambda q^\sigma\,F_V^{(B)}\,, \end{aligned} \quad (13)$$

introducing new invariant amplitudes $F_A^{(B)}$, α and β where the latter must satisfy the condition (Ward identity)

$$\alpha+\beta = i\frac{f_B}{(p\cdot q)}. \tag{14}$$

The values of α and the corresponding β themselves are arbitrary and not fixed by the electromagnetic Ward identity. In Eq. (13) the terms proportional to $F_A^{(B)}$, c and $F_V^{(B)}$ are gauge-invariant (they vanish after being multiplied by q_μ) whereas the term proportional to f_B disappears in the chiral limit after being multiplied by the lepton current. The remaining contact-term part of $T_{\mu\rho}^{(B)}$ containing α and β is gauge noninvariant. Different choices of α and β simply reflect different choices of $F_A^{(B)}$ and allow us to rewrite $T_{\mu\rho}^{(B)}$ in many ways.

Let us set $\beta=0$; this gives

$$\begin{aligned} T_{\mu\rho}^{(B)}(p,q) &= (g_{\mu\rho}(p\cdot q)-p_\mu q_\rho)\, iF_A^{(B)} + ig_{\mu\rho}f_B \\ +q_\mu p_\rho\, c + i\frac{p_\mu p_\rho}{(p\cdot q)}f_B &+ q_\mu q_\rho\, e + \epsilon_{\rho\mu\lambda\sigma}p^\lambda q^\sigma\, F_V^{(B)}. \end{aligned} \tag{15}$$

Substituting Eq. (15) together with Eq. (6) in Eq. (5) one finally obtains for the decay amplitude

$$\begin{aligned} A(B^-\to l\nu\gamma) = \; & e\frac{G_F}{\sqrt{2}}V_{ub}\Big\{ (\bar{u}_l\Gamma^\rho v_\nu)\Big[(\epsilon_\rho(p\cdot q)-(\epsilon\cdot p)\,q_\rho)\; iF_A^{(B)} + i\epsilon_\rho\, f_B \\ & +\epsilon_{\rho\mu\lambda\sigma}\epsilon^\mu p^\lambda q^\sigma\, F_V^{(B)}\Big] - i\,(\bar{u}_l\Gamma^\rho v_\nu)\,\epsilon_\rho\, f_B\Big\}, \end{aligned} \tag{16}$$

where the terms in the brackets correspond to the initial state photon and the remaining term is the only effect of the final state emission, the contact term (6). We see that the two contact terms cancel in the r.h.s. of this expression as expected. The remaining "structure dependent" amplitude is a combination of two gauge-invariant form factors.

The story is however not yet finished. What about choosing another set of values for α and β? Instead of a contact term proportional to $f_B\epsilon_\rho$ in Eqs. (15) and (16) we will recover the combination $\alpha(p\cdot q)\epsilon_\rho+\beta(p\cdot\epsilon)q^\rho$ which looks completely different. But when the "final-state" contact term Eq. (6) is added and the relation (14) is used, a gauge invariant result is obtained, with a form factor $iF_A^{(B)}\to iF_A^{(B)}-\beta$. The choice $\beta\neq 0$ looks less attractive, because it leaves the impression that the helicity unsuppressed contact term of the leptonic photon emission is part of the answer. But it is not necessarily wrong. Everything depends on how the 'gauge invariant' form factor $F_A^{(B)}$ is calculated in a given framework. If we calculate the coefficient of the kinematical structure $g_{\mu\rho}$ in $T^{(B)}_{\mu\rho}$, then we have to add the additional contact term because the diagrams used to calculate this invariant amplitude implicitly contain such a term.[a] Below we will see this in the particular example of the correlation function used in light-cone sum rules. We prefer the scheme which corresponds to the $\beta = 0$ choice where the contact terms vanish and which is in accordance with the physical picture. Since the structure of the contact term in the final state radiation is fixed (it is the $g_{\mu\rho}$ structure), one should look for a formulation of the initial state radiation which yields the **other** kinematical structure (in this case $p_\mu q_\rho$). This statement is clearly independent of the choice of β.

Thus, we have once more convinced ourselves that there is no photon emission from charged leptons in the massless lepton limit. It is clear that the same statement will be valid for final massless quarks if the strong interactions are neglected, that is for the weak annihilation contribution to the $B \to X_d\gamma$ inclusive width calculated at the partonic level. But since the inclusive width is a sum of positive exclusive widths, it is not possible that *any* of the exclusive channels gets chirally unsuppressed final state radiation. Nevertheless, let us demonstrate that explicitly using the same technique of symmetry relations and Ward identities.

3 $B \to \rho\gamma$

We now turn to the case of interest, the decay $B^- \to \rho^-\gamma$. In order to trace the contact terms we choose the charged B mode and consider only the weak annihilation decay mechanism; furthermore we employ the factorization approximation. The decay amplitude is similar to Eq. (4) if the lepton pair is replaced by the light-quark pair and the final state correspondingly by a ρ meson:

$$A(B^- \to \rho^-\gamma)_{WA} = \frac{G_F}{\sqrt{2}} V_{ub}V^*_{ud} \langle \rho^-(p)\,\gamma(q) \,|\, \left(\bar{d}\Gamma_\nu u\right)\left(\bar{u}\Gamma^\nu b\right) | \,B^-(p+q)\,\rangle. \quad (17)$$

In the above, we omit the combination of Wilson coefficients $a_1 = c_1 + c_2/3$ which is numerically close to 1 and irrelevant for our discussion. Again, the matrix element can be rewritten as a sum of two contributions:

$$\begin{aligned}\langle\, \rho(p)\;\gamma(q)\;|\;\left(\bar{d}\Gamma_\nu u\right)\left(\bar{u}\Gamma^\nu b\right)\;|\;B^-(p+q)\,\rangle &= e\epsilon^\mu\epsilon^{(\rho)\nu} f_\rho m_\rho T^{(B)}_{\mu\nu} \\ &\quad - ie\epsilon^\mu (p+q)^\nu f_B\, T^{(\rho)}_{\mu\nu}\,, \qquad (18)\end{aligned}$$

[a] one of us (A.K.) is grateful to D. Melikhov for a discussion on this point.

where $\epsilon^{(\rho)}$ is the polarization vector, $\epsilon^{(\rho)} \cdot p = 0$ and f_ρ is the decay constant of ρ, defined as $\langle \rho(p) \mid \bar{d}\gamma_\nu u \mid 0 \rangle = m_\rho f_\rho \epsilon_\nu^{(\rho)}$. The first term in Eq. (18) contains the matrix element of the photon emission from the B meson already analysed in the previous section. The second represents the final-state emission and includes the hadronic matrix element

$$T_{\mu\nu}^{(\rho)} = i \int d^4x\, e^{iqx} \langle\, \rho(p) \mid T\{j_\mu^{em}(x)\, \bar{d}\Gamma_\nu u(0)\} \mid 0 \rangle \tag{19}$$

which is multiplied by $p+q$. To analyse this object we again apply the Ward identity for the e.m. current:

$$q^\mu T_{\mu\nu}^{(\rho)} = f_\rho m_\rho \epsilon_\nu^{(\rho)}\,, \tag{20}$$

or equivalently

$$q^\mu (p+q)^\nu T_{\mu\nu}^{(\rho)} = f_\rho m_\rho (\epsilon^{(\rho)} \cdot q)\,. \tag{21}$$

The general decomposition of the product $(p+q)^\nu T_{\mu\nu}^{(\rho)}$ reads

$$\begin{aligned} (p+q)^\nu T_{\mu\nu}^{(\rho)} &= \epsilon_\mu^{(\rho)}\, a^{(\rho)} + (\epsilon^{(\rho)} \cdot q) p_\mu\, b^{(\rho)} \\ &+ (\epsilon^{(\rho)} \cdot q) q_\mu\, c^{(\rho)} + \epsilon_{\nu\mu\lambda\sigma} \epsilon^{(\rho)\nu} p^\lambda q^\sigma\, F_V^{(\rho)}. \end{aligned} \tag{22}$$

Multiplying both parts of this equation by q_μ one obtains:

$$q^\mu (p+q)^\nu T_{\mu\nu}^{(\rho)} = (\epsilon^{(\rho)} \cdot q)\, a^{(\rho)} + (\epsilon^{(\rho)} \cdot q)(p \cdot q)\, b^{(\rho)}. \tag{23}$$

The Ward identity (21) and Eq. (23) yield then a relation between the invariant amplitudes $a^{(\rho)}$ and $b^{(\rho)}$:

$$a^{(\rho)} + (p \cdot q)\, b^{(\rho)} = f_\rho m_\rho\,. \tag{24}$$

It might seem again that there is an arbitrariness (as in Eq. (13)) in writing the amplitude. However, at this point it is important to notice that the final-state weak current is also conserved in the chiral limit. Therefore there is an additional Ward identity for the hadronic matrix element $T_{\mu\nu}^{(\rho)}$:

$$(p+q)^\nu T_{\mu\nu}^{(\rho)} = f_\rho m_\rho \epsilon_\mu^{(\rho)}\,. \tag{25}$$

The situation is simplified even more by the fact that the product $(p+q)^\nu T_{\mu\nu}^{(\rho)}$ entering the decay amplitude is by itself the l.h.s. of the Ward identity. The result of this consideration are the constraints

$$a^{(\rho)} = f_\rho m_\rho, \quad b^{(\rho)} = c^{(\rho)} = F_V^{(\rho)} = 0\,, \tag{26}$$

consistent with the e.m. Ward identity (24). Most importantly, Eq. (26) fixes the final state emission uniquely as was the case in the leptonic decay. The final expression for the $B \to \rho\gamma$ weak annihilation amplitude is then

$$A(B^- \to \rho^- \gamma)_{WA} = e\frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* f_\rho m_\rho \Big\{ \Big[\Big((\epsilon \cdot \epsilon^{(\rho)})(p \cdot q) - (\epsilon \cdot p)(\epsilon^{(\rho)} \cdot q) \Big) \, iF_A^{(B)} + if_B(\epsilon \cdot \epsilon^{(\rho)}) + \epsilon_{\nu\mu\lambda\sigma} \epsilon^{(\rho)\nu} \epsilon^\mu p^\lambda q^\sigma \, F_V^{(B)} \Big] - if_B(\epsilon \cdot \epsilon^{(\rho)}) \Big\} \,, \quad (27)$$

where the part proportional to $T_{\mu\nu}^{(B)}$ is indicated by brackets. The contact terms again cancel each other, if, as explained in the previous section, the structure proportional to $p_\mu q_\nu$ has been chosen to calculate the form factor $F_A^{(B)}$. Thus in the chiral limit there is no photon emission from the final ρ in $B \to \rho\gamma$. We also note that $F_V^{(\rho)} = 0$ prohibits the emission also in the vector part of the amplitude .[b] The weak annihilation amplitude again has the form of a sum of two "structure dependent" terms corresponding to the photon emission from the initial state.

4 $B \to D^* \gamma$

We now come to the physically different case where the final state contains a c quark, e.g. the $B \to D^* \gamma$ decay which is in fact dominated by the weak annihilation mechanism. The point is that the mass of the charm quark can not be neglected. While a more complete analysis of this decay using QCD light-cone sum rules will be presented elsewhere [16], we limit ourselves to the issue of contact terms. The decay amplitude is similar to Eq. (17) with obvious replacements in the quark current and final state:

$$A(B^- \to D^{*-}\gamma) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^* \langle \, D^{*-}(p) \; \gamma(q) \mid \big(\bar{d}\Gamma_\nu c\big) \, (\bar{u}\Gamma^\nu b) \mid B^-(p+q) \, \rangle \,. \quad (28)$$

In the factorization approximation:

$$\begin{aligned} &\langle \, D^*(p) \; \gamma(q) \mid \big(\bar{d}\Gamma_\nu c\big) \, (\bar{u}\Gamma^\nu b) \mid B^-(p+q) \, \rangle \\ &= e\epsilon^\mu \epsilon^{(D^*)\nu} f_{D^*} m_{D^*} T_{\mu\nu}^{(B)} - ie\epsilon^\mu (p+q)^\nu f_B T_{\mu\nu}^{(D^*)} \,, \end{aligned} \quad (29)$$

where $\epsilon^{(D^*)}$ and f_{D^*} are the polarization vector and the decay constant of D^*. The hadronic matrix element $T_{\mu\nu}^{(B)}$ responsible for the initial-state photon

[b] Sometimes the axial and vector form factors for $B \to V\gamma$ are called parity-violating and parity-conserving, respectively.

emission is exactly the same as in $B \to l\nu\gamma$ or $B \to \rho\gamma$, whereas the matrix element determining the photon emission from the final D^* is defined as

$$T_{\mu\nu}^{(D^*)} = i\int d^4x\, e^{iqx} \langle D^*(p) \mid T\{j_\mu^{em}(x)\, \bar{d}\Gamma_\nu c(0)\} \mid 0\rangle\,, \tag{30}$$

With the most general decomposition (13) we obtain for the first term on r.h.s. of Eq. (29):

$$e\epsilon^\mu \epsilon^{(D^*)\nu} f_D^* m_{D^*} T_{\mu\nu}^{(B)} = e f_D^* m_{D^*} \Big\{ \Big[(\epsilon\cdot\epsilon^{(D^*)})(p\cdot q) - (\epsilon\cdot p)(\epsilon^{(D^*)}\cdot q) \Big]\, iF_A^{(B)}$$
$$+(\epsilon\cdot\epsilon^{(D^*)})(p\cdot q)\,\alpha + (p\cdot\epsilon)(q\cdot\epsilon^{(D^*)})\,\beta + \epsilon_{\nu\mu\lambda\sigma}\epsilon^{(D^*)\nu}\epsilon^\mu p^\lambda q^\sigma\, F_V^{(B)} \Big\}\,, \tag{31}$$

with α and β related by Eq. (14). The product $(p+q)^\nu T_{\mu\nu}^{(D^*)}$ for the final state emission can again be constrained by the Ward identity of the weak current. In this case the $\bar{d}\Gamma_\nu c$ current is **not** conserved: $\partial_\nu(\bar{d}\Gamma_\nu c) = m_c \bar{d}(1-\gamma_5)c$ and therefore

$$(p+q)^\nu T_{\mu\nu}^{(D^*)} = f_D^* m_D^* \epsilon_\mu^{(D^*)}$$
$$+ i m_c \int d^4x e^{ipx} \langle D^*(p) \mid T\{j_\mu^{em}(x)\bar{d}(1-\gamma_5)c\} \mid 0\rangle\,. \tag{32}$$

Matching this expression with the general decomposition analogous to Eq. (22)

$$(p+q)^\nu T_{\mu\nu}^{(D^*)} = \epsilon_\mu^{(D^*)}\, a^{(D^*)} + (\epsilon^{(D^*)}\cdot q)p_\mu\, b^{(D^*)}$$
$$+(\epsilon^{(D^*)}\cdot q)q_\mu\, c^{(D^*)} + \epsilon_{\nu\mu\lambda\sigma}\epsilon^{(D^*)\nu} p^\lambda q^\sigma\, F_V^{(D^*)}\,, \tag{33}$$

we conclude that

$$a^{(D^*)} = f_{D^*} m_{D^*} + O(m_c), \quad b^{(D^*)}, c^{(D^*)}, F_V^{(D^*)} \sim O(m_c)\,, \tag{34}$$

that is the photon emission from the final state has an nonvanishing amplitude proportional to m_c. Finally, the electromagnetic Ward identity yields a relation similar to (24):

$$a^{(D^*)} + (p\cdot q)\, b^{(D^*)} = f_{D^*} m_{D^*}\,. \tag{35}$$

As in the case of $T^{(B)}$ analysed above there is a certain freedom in using this constraint. In particular, it is possible to rewrite

$$(p+q)^\nu T_{\mu\nu}^{(D^*)} = \Big(\epsilon_\mu^{(D^*)}(p\cdot q) - p_\mu(\epsilon^{(D^*)}\cdot q)\Big)\, iF_A^{(D^*)} + \epsilon_\mu^{(D^*)}(p\cdot q)\,\alpha^{(D^*)}$$
$$+(\epsilon^{(D^*)}\cdot q)p_\mu \beta^{(D^*)} + (\epsilon^{(D^*)}\cdot q)q_\mu\, c^{(D^*)} + \epsilon_{\nu\mu\lambda\sigma}\epsilon^{(D^*)\nu} p^\lambda q^\sigma\, F_V^{(D^*)}\,, \tag{36}$$

introducing new amplitudes $F_A^{(D^*)}$, $\alpha^{(D^*)}$ and $\beta^{(D^*)}$ with the condition

$$\alpha^{(D^*)} + \beta^{D^*} = \frac{m_{D^*} f_{D^*}}{(p\cdot q)}\,. \tag{37}$$

Multiplying Eq. (36) by $-ie\epsilon^\mu f_B$ we obtain the general form of the final-state emission amplitude which has to be added to the initial-state one given in Eq. (31). It seems that we have now a problem in adjusting the arbitrary coefficients in both the initial and the final state emission. However, it is easy to see that the constraints (14) and (37) allow to arrange the gauge-invariant combinations of form factors for both terms $T^{(B)}$ and $T^{(D^*)}$ *separately* so that the remaining contact terms proportional to $f_B f_{D^*}$ cancel. Like for the initial emission where the choice $\beta = 0$ was preferable, we also suggest using $\beta^{D^*} = 0$ for $T^{(D^*)}$. In this case the final expression for the decay amplitude

$$\begin{aligned} A(B^- \to D^{*-}\gamma) \;&= e\frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^* \Bigg\{ \Big[\Big((\epsilon \cdot \epsilon^{(D^*)})(p \cdot q) - (\epsilon \cdot p)(\epsilon^{(D^*)} \cdot q) \Big)\; iF_A^{(B)} \\ &+ i f_B (\epsilon \cdot \epsilon^{(D^*)}) + \epsilon_{\nu\mu\lambda\sigma} \epsilon^{(D^*)\nu} \epsilon^\mu p^\lambda q^\sigma \; F_V^{(B)} \Big] m_{D^*} f_{D^*} \\ &- i f_B \Big[\Big((\epsilon \cdot \epsilon^{(D^*)})(p \cdot q) - (\epsilon \cdot p)(\epsilon^{(D^*)} \cdot q) \Big)\; iF_A^{(D^*)} \\ &+ f_{D^*} m_{D^*} (\epsilon \cdot \epsilon^{(D^*)}) + i\epsilon_{\nu\mu\lambda\sigma} \epsilon^{(D^*)\nu} \epsilon^\mu p^\lambda q^\sigma \; F_V^{(D^*)} \Big] \Bigg\}, \end{aligned} \tag{38}$$

contains, in addition to the form factors $F_{A,V}^{(B)}$ of the massless case, also two new form factors $F_{A,V}^{(D^*)}$ corresponding to the photon emission from the D^*. The respective contact terms cancel as desired: None of the spurious contact terms appear in the gauge-invariant amplitudes. As before, the relevant structure to calculate is $(\epsilon \cdot p)(q \cdot \epsilon^{(D^*)})$. This concludes our general considerations.

5 Applying light-cone sum rules to $B \to V\gamma$

As pointed out before, the way the structure-dependent contributions are calculated is important for identifying the right terms and discarding the ones which cancel. Let us illustrate this for the QCD light-cone sum rule (LCSR) approach used to calculate the weak annihilation amplitude for $B \to \rho\gamma$ [5,6] and $B \to l\nu\gamma$ [5] previously. In these papers the object $T_{\mu\nu}^{(B)}$ defined in Eq. (7) was calculated. The method is to introduce a correlation function

$$\Pi_\nu(p,q) = i \int d^4x \, e^{ipx} \langle 0 \mid T\{\bar{u}\Gamma_\nu b(x), m_b \bar{b} i\gamma_5 u(0)\} \mid 0 \rangle_{F(q)} \tag{39}$$

of two heavy-light currents in the external photon field with momentum q. The B meson is interpolated by the quark current. In first order in the e.m.

interaction $\Pi_\nu = e\epsilon^\mu \Pi_{\mu\nu}$ where

$$\Pi_{\mu\nu}(p,q)=-\int d^4x\, d^4y\, e^{ipx+iqy}\langle 0 \mid T\{j_\mu^{em}(y), \bar{u}\Gamma_\nu b(x), m_b\bar{b}i\gamma_5 u(0)\} \mid 0\rangle. \quad (40)$$

The Ward identity with respect to the e.m. current for this three-point correlator can be easily derived with the following nontrivial result:

$$q^\mu \Pi_{\mu\nu} = p_\nu \Pi^{CT}(p^2) - (p+q)_\nu\, \Pi^{CT}((p+q)^2)\,, \quad (41)$$

where Π^{CT} is the invariant amplitude determining the two-point correlator

$$\Pi_\nu^{CT}(r) = i\int d^4x\, e^{irx}\langle 0 \mid T\{\bar{u}\Gamma_\nu b(x), m_b\bar{b}i\gamma_5 u(0)\} \ \mid 0\rangle = r_\nu \Pi^{CT}(r^2)\,. \quad (42)$$

Before continuing, let us make the following remark. In the LCSR approach the dominant contribution to the correlation function comes from the long-distance photon emission parametrized by the light-cone distribution amplitudes of the photon. This part is explicitly gauge-invariant in e.m. field and cannot give rise to contact terms. Our concern here is the short-distance part of Eq. (39) which corresponds to the perturbative emission of the photon from quark lines.

Returning to the counterterms, we see that a combination of the correlators in Eq. (41) now plays the role of the contact term for the three-point correlation function. Importantly, one of them is a function of p only, whereas the other one depends on $p+q$. Diagrammatically, the first term in Eq. (41) corresponds to the emission of the photon in the point of the B meson current whereas the second one in the point of the weak current. The correlation function (39) itself is calculated using the operator-product expansion (OPE); the result can be expressed as a general decomposition

$$\Pi_{\mu\nu} = g_{\mu\nu}\Pi_a + p_\mu q_\nu \Pi_b + q_\mu p_\nu \Pi_c + p_\mu p_\nu \Pi_d + q_\mu q_\nu \Pi_e + i\epsilon_{\mu\nu\lambda\sigma}p^\lambda q^\sigma \Pi_V\,, \quad (43)$$

in terms of independent invariant amplitudes Π_a, Π_b, etc. From the Ward identity (41) we see that the result is not explicitly gauge-invariant: The amplitudes in Eq. (43) do not combine to form a gauge-invariant expression. If we multiply both parts of Eq. (43) by q_μ and compare the result with Eq. (41) we obtain

$$\Pi_a + (p\cdot q)\Pi_b = -\Pi^{CT}((p+q)^2), \quad (p\cdot q)\Pi_d = \Pi^{CT}(p^2) - \Pi^{CT}((p+q)^2). \quad (44)$$

These constraints must be obeyed by the OPE result, or any other calculation of the correlation function. Now let us assume that we have calculated only the invariant amplitude for the $p_\mu q_\nu$ structure in Eq. (43). Without knowing the result for Π_a and Π_d we can simply use the relations (44) and rewrite the initial correlator as

$$\Pi_\nu = e\Big\{ \Big((\epsilon\cdot p)q_\nu - \epsilon_\nu(p\cdot q)\Big)\,\Pi_b - \epsilon_\nu \Pi^{CT}((p+q)^2)$$
$$+\frac{(\epsilon\cdot p)}{(p\cdot q)}p_\nu\Big(\Pi^{CT}(p^2) - \Pi^{CT}((p+q)^2)\Big) + i\epsilon_{\mu\nu\lambda\sigma}\epsilon^\mu p^\lambda q^\sigma \Pi_V\Big\}\,. \qquad (45)$$

We emphasize that the amplitudes in this expression are fixed as a result of a direct calculation. But we note that the relation (44) can be used in a more general way (compare Eqs. (11) and (13)), to include contact terms proportional to ϵ_ν as well as to q_ν. In analogy to the previous considerations we have anticipated here the choice corresponding to $\beta = 0$, that is the form (45) without a contact term proportional to q_ν.

The next step in the sum rule derivation is in writing down the dispersion relation by inserting in Eq. (39) a complete set of hadronic states with the B meson quantum numbers,

$$\Pi_\nu(p,q) = \frac{\langle 0 \mid \bar u \Gamma_\nu b \mid B\rangle_{F(q)}\langle B \mid m_b \bar b i\gamma_5 u\} \mid 0\rangle}{m_B^2 - (p+q)^2} + \dots\,.$$
$$= \frac{e\epsilon^\mu T^{(B)}_{\mu\nu} f_B m_B^2}{m_B^2-(p+q)^2} + \dots\,. \qquad (46)$$

We will also need the dispersion relation for the correlator Π^{CT} which reads:

$$\Pi^{CT}_\nu(p+q) = \frac{\langle 0 \mid \bar u \Gamma_\nu b \mid B\rangle\langle B \mid m_b \bar b i\gamma_5 u\} \mid 0\rangle}{m_B^2 - (p+q)^2} + \dots\,.$$
$$= \frac{-f_B^2 m_B^2}{m_B^2-(p+q)^2}(p+q)_\nu + \dots\,. \qquad (47)$$

In the above we retained the ground-state B meson terms denoting the contribution of excited and continuum states by ellipses. The latter states are not important for our analysis because their contributions one by one obey the same symmetry properties as the B meson term. The matrix element of the photon emission from B meson entering Eq. (46) can now be calculated matching the dispersion relation (46) with the result of OPE for the correlation function Π_ν. Simultaneously, the dispersion relation (47) for Π^{CT} yields the usual two-point QCD sum rule result for f_B. We skip important points of the sum-rule procedure (use of quark-hadron duality, continuum subtraction and Borel transformation) which are explained in detail in the literature (see e.g. [17]). In particular, after the Borel transformation in the relevant variable $(p+q)^2$ the term proportional to $\Pi^{CT}(p^2)$ in Eq. (45) vanishes. The most important circumstance for our analysis is the fact that each invariant amplitude in the expansion (45) yields, via the dispersion relation, a separate sum-rule relation for the corresponding invariant amplitude in the matrix element. Thus, as a result of sum rule calculation and taking into account Eq. (47) one obtains

$$\begin{aligned}\epsilon^{\mu}T^{(B)}_{\mu\nu} &= (\epsilon_\nu (p\cdot q) - (\epsilon\cdot p) q_\nu)\, i\bar{F}^{(B)}_A + i\bar{f}_B\epsilon_\nu \\ &+ i\frac{(\epsilon\cdot p)p_\nu}{(p\cdot q)}\bar{f}_B + \epsilon_{\nu\mu\lambda\sigma}\epsilon^{\mu}p^{\lambda}q^{\sigma}\bar{F}^{(B)}_V\,, \end{aligned}\tag{48}$$

where $\bar{F}^{(B)}_A$, $\bar{F}^{(B)}_V$ and $\bar{f}_B$ are the hadronic amplitudes calculated from the QCD sum rules for Π_b , Π_V and Π^{CT} respectively. The above equation exactly reproduces the structure of Eq. (15) together with the contact term.

Thus, we have shown that the calculational procedure which starts from the structure $p_\mu q_\nu$ in the correlator yields the proper gauge-invariant term and a contact term with structure $g_{\mu\nu}$. This latter is an inseparable part of the matrix element and cancels the (contact) term generated by the final state emission.

Finally, let us illustrate the above analysis by taking one of the short-distance contributions to the correlation function, namely the quark-condensate term. This contribution is not very important numerically in the final sum rule but is simply calculated and therefore convenient as a study case. Physically, it corresponds to the photon emission at short distances from the virtual b quark line whereas light u quarks are soft and approximated by a local quark condensate. The expression for this part of the correlation function is easily obtained by taking in Eq. (40) $j_\mu = e_b\bar{b}\gamma_\mu b$, contracting the b quark fields into free-quark propagators, and replacing the u quark fields by the vacuum condensate $\langle\bar{u}u\rangle$ (further details see in [5]). The result reads:

$$\begin{aligned}\Pi^{\langle\bar{u}u\rangle}_{\mu\nu} &= ie_b\frac{\langle\bar{u}u\rangle m_b}{(p^2-m_b^2)((p+q)^2-m_b^2)} \\ &\times\left[g_{\mu\nu}(m_b^2-p^2-(p\cdot q)) + p_\mu q_\nu + q_\mu p_\nu + 2p_\mu p_\nu + i\epsilon_{\mu\nu\alpha\beta}p^{\alpha}q_{\beta}\right]\end{aligned}\tag{49}$$

It is easy to check that the Ward identity is indeed valid and that the contact term is equal to the quark condensate contribution to the two-point correlator:

$$\Pi^{CT\langle\bar{u}u\rangle}(r^2) = ie_b\frac{\langle\bar{u}u\rangle m_b}{r^2-m_b^2}\,.\tag{50}$$

Using the latter expression we can rewrite Eq. (49) in the form of Eq. (45) where

$$\Pi^{\langle\bar{u}u\rangle}_b = \Pi^{\langle\bar{u}u\rangle}_V = ie_b\frac{\langle\bar{u}u\rangle m_b}{(p^2-m_b^2)((p+q)^2-m_b^2}\,,\tag{51}$$

which agrees with the expression given in [5]. Thus we can uniquely identify the relevant form factor; again it is the coefficient of the $(\epsilon \cdot p)q_\nu$ structure. We have checked that the more complicated short-distance contributions of perturbative diagrams to the sum rule calculated in [5] are also in accordance with this procedure.

To summarize: In this paper we have identified a procedure based on simple Ward identities to extract the correct form factors in the calculation of the weak annihilation contribution to the radiative decays of the form $B \to V\gamma$.

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Note added: After this paper was finished we became aware of the recent work [18] where Ward identities and symmetry relations are applied but where vector-dominance is used. This is a special model and allows only limited statements, unlike the general QCD picture employed here. The conclusion reached in [18] seems to be in disagreement with the general result we present.

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