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Duality in Semi-Exclusive Processes

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Semi-exclusive processes like $\gamma p \to \pi^+ Y$ are closely analogous to DIS, $ep \to eX$, in the limit where the momentum transfer $|t|$ to the pion and the mass of the inclusive system Y are large but still much smaller than the total CM energy. We apply Bloom-Gilman duality to this semi-exclusive process. The energy dependence of the $\gamma p \to \pi^+ n$ cross section given by semi-local duality agrees with data for moderate values of $|t|$, but its normalization is underestimated by about two orders of magnitude. This indicates that rather high momentum transfers are required for the validity of PQCD in the hard subprocess $\gamma u \to \pi^+ d$. In the case of Compton scattering $\gamma p \to \gamma p$ the analogous discrepancy is about one order of magnitude. In electroproduction the virtuality of the incoming photon can be used to directly measure the hardness of the subprocess.

The remarkable relation between Deep Inelastic Scattering (DIS) $eN \to eX$ and exclusive resonance production $eN \to eN^*$ known as Bloom-Gilman duality [1] has recently been confirmed and extended by data from JLab [2]. Empirically,

$$\int_{\delta x} dx F_2^{scaling}(x) \simeq F_2^{N^*}(Q^2) \propto \frac{d\sigma}{dQ^2}(eN \to eN^*) \propto |F_{pN^*}(Q^2)|^2 \quad (1)$$

where $F_2^{N^*}(Q^2)$ denotes the contribution of an N^* resonance to the DIS structure function F_2, and $F_{pN^*}(Q^2)$ is the exclusive $p \to N^*$ electromagnetic form factor. The Bjorken variable x is related to the photon virtuality Q^2 and the invariant mass of the hadronic system W through

$$x = \frac{Q^2}{W^2 + Q^2 - M_N^2} \quad (2)$$

On the lhs of (1), the leading twist structure function $F_2^{scaling}(x)$ is integrated over an interval δx corresponding to a fixed range of W around the resonance mass, $W = M_{N^*}$. This semi-local duality relation is approximately satisfied for each nucleon resonance region including the Born term: $N^* = P_{11}(938)$, $P_{33}(1232)$, $S_{11}(1535)$ and $F_{15}(1680)$ (where the regions are characterized by their main resonance). The magnitude and x-dependence of the scaling structure function is thus related to the magnitude and Q^2-dependence of the N^* electromagnetic form factors via the kinematic relation $x = x(Q^2, W^2 = M_{N^*}^2)$ of Eq. (2). Additional effects due to the logarithmic Q^2-dependence of $F_2^{scaling}$ are discussed in Ref. [3]. The duality relation is approximately satisfied even at low values of Q^2, for which one would not expect perturbative QCD (PQCD) to apply.

The reasons for the validity of semi-local duality are something of a mystery [4,5]. According to PQCD, the inclusive structure function is given by incoherent scattering off single quarks in the target, whereas exclusive form factors depend on a coherent sum over the quark charges. Interference terms on the rhs of (1) due to scattering on different target quarks should systematically cancel only in an average over several resonances [6]. Yet

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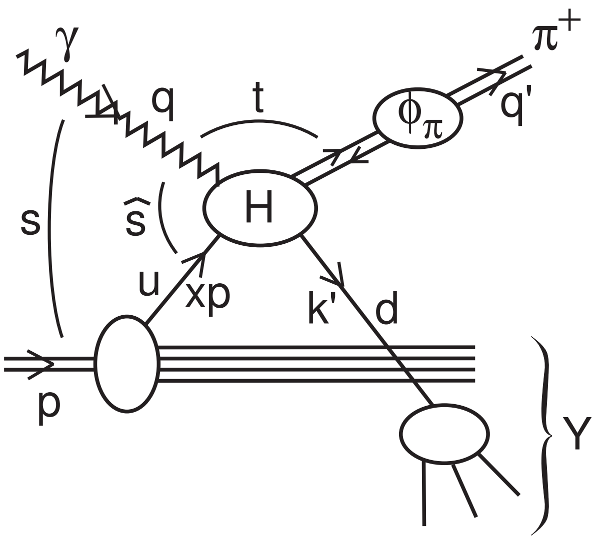

FIG. 1. Semi-exclusive scattering. In the limit $\Lambda_{QCD}^2 \ll |t|, M_Y^2 \ll s$ the cross section factorizes into a hard subprocess cross section $\hat{\sigma}(H)$ times a target parton distribution. The subprocess H acts like a compact probe of target structure analogous to the virtual photon of DIS.

duality appears to be satisfied in a semi-local sense for both proton and deuteron targets [2].

Duality in the $P_{33}(1232)$ region is not satisfied by the Δ resonance alone [7]. The electromagnetic form factor $F_{p\Delta}(Q^2)$ falls faster with increasing Q^2 than predicted by the dimensional scaling laws of PQCD [8]. The 'background' under the $P_{33}(1232)$ resonance becomes relatively more important with Q^2, compensating for the resonance fall-off and maintaining semi-local duality in the $P_{33}(1232)$ region. The corresponding phenomenon is not observed in the $S_{11}(1535)$ and $F_{15}(1680)$ regions, where the resonance contributions do obey dimensional scaling [8]. The spin dependence of semi-local duality is discussed in Refs. [9,10].

In this paper we shall study semi-exclusive processes such as $\gamma p \to \pi^+ Y$ (Fig. 1). In the kinematic limit where the total energy $s = (q+p)^2$ is much larger than the mass of the inclusive system Y, $s \gg M_Y^2 \gg \Lambda_{QCD}^2$, the produced meson π^+ is separated from the hadrons in Y by a rapidity gap. When also the momentum transfer $t = (q-q')^2$ is large the π^+ is produced in a compact configuration in the hard subprocess H and, due to color transparency, does not reinteract in the target. We have then a generalisation of ordinary DIS, with the $eq \to eq$ subprocess replaced by $\gamma q \to \pi^+ q'$ and with the physical cross section given by [11]

$$\frac{d\sigma}{dx\,dt}(\gamma p \to \pi^+ Y) = \sum_{q,q'} q(x) \frac{d\sigma}{dt}(\gamma q \to \pi^+ q') \tag{3}$$

where $q(x)$ is the distribution of the struck quark in the target. From the point of view of the target physics, there is a one-to-one correspondence between the semi-exclusive process and ordinary DIS, with $Q^2 \leftrightarrow -t$ and $W^2 \leftrightarrow M_Y^2$. The momentum fraction of the struck quark is thus in the semi-exclusive process

$$x = \frac{-t}{M_Y^2 - t - M_N^2} \tag{4}$$

The close analogy with DIS makes it natural to assume that semi-exclusive processes obey Bloom-Gilman duality [12] (see also [13]). Such duality links large momentum transfer exclusive cross sections to standard DIS structure functions and, via Eq. (1), to exclusive form factors. The multitude of processes that potentially can be compared in this way may lead to a better understanding of the applicability of PQCD to exclusive processes, and of Bloom-Gilman duality itself.

For the process of Fig. 1 semi-local duality predicts

$$\int_{\delta x} \left[u(x) + \bar{d}(x)\right] \frac{d\sigma}{dt}(\gamma u \to \pi^+ d) \simeq \frac{d\sigma}{dt}(\gamma p \to \pi^+ N^*) \tag{5}$$

where the hard subprocess cross section is given by [14]

$$\frac{d\sigma}{dt}(\gamma u \to \pi^+ d) = \frac{128\pi^2 \alpha \alpha_s^2}{27|t|\hat{s}^2} \left(\frac{e_u}{\hat{s}} + \frac{e_d}{\hat{u}}\right)^2 (\hat{s}^2 + \hat{u}^2) \times \left(\int_0^1 \frac{dz}{z} \phi_\pi(z)\right)^2 \tag{6}$$

Here $\hat{s} = (q+xp)^2 \simeq xs$, $\hat{u} \simeq -\hat{s} - t$ and $\phi(z)$ is the pion distribution amplitude (normalized as $\int dz \phi(z) = f_\pi/\sqrt{12}$ with $f_\pi = 93$ MeV). In the semi-exclusive limit considered in Ref. [11] $|t| \ll s$ and thus $\hat{u} \simeq -\hat{s}$. At large $-t$ and fixed $M_Y^2 = M_{N^*}^2$ we have $x \to 1$ and thus $\hat{s} \simeq s$.

The quark distributions are poorly measured at high x. Furthermore, target mass corrections as well as the choice of δx in (5) are sources of uncertainty particularly for the nucleon Born term [1,10,15]. In the present analysis we may, however, take advantage of the fact that the target physics should be independent of the hard probe H (*cf.* Fig. 1). Hence the target mass corrections and δx should be similar in Eqs. (1) and (5). Noting also that the main contribution to the lhs is in both cases from u-quarks allows us to eliminate the quark distributions in favour of a direct relation between $\sigma(ep \to eN^*)$ and $\sigma(\gamma p \to \pi^+ N^*)$,

$$\frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = 16\pi^2 \frac{\alpha \alpha_s^2 f_\pi^2}{|t| s^2} \frac{G_{Mp}^2(-t)}{(1 + r_{du}/4)} \tag{7}$$

where we took the $s \gg |t|$ limit [11] in (6) and assumed the asymptotic form of the pion distribution amplitude, $\phi_\pi^{as}(z) = \sqrt{3} f_\pi z(1-z)$. The ratio r_{du} is the d-quark distribution integrated over the interval δx, divided by the corresponding quantity for the u-quark (we neglect the antiquark contributions since $x \to 1$ at large $|t|$).

The $1/s^2$ energy dependence of the cross section (7) at fixed $|t|$ is a general consequence of the exchange of two spin $\frac{1}{2}$ quarks in the t-channel. Higher order ladder corrections could modify the power.

Data on $\gamma p \to \pi^+ N^*$ at large momentum transfer exists for $E_\gamma \leq 7.5$ GeV [16]. The related measurements of the semi-exclusive process $\gamma p \to \pi^+ Y$ were apparently never published.

The measured $\gamma p \to \pi^+ n$ cross section is $\propto 1/s^2$ for $|t| \lesssim 2$ GeV^2 [16], in agreement with (7). This indicates the t-range for which the asymptotic energy dependence has set in when $s \lesssim 15$ GeV^2. At $E_\gamma = 7.5$ GeV and $t = -1.95$ GeV^2 the differential cross section is 3.45 ± 0.16 nb/GeV^2. Using $G_{Mp}(Q^2) = 2.79/(1 + Q^2/.71\ \mathrm{GeV}^2)^2$ [8], $\alpha_s(2\ \mathrm{GeV}^2) \simeq 0.4$ [17] and $r_{du} = 0.5$ the rhs. of Eq. (7) is 0.065 nb/GeV^2. Data is thus a factor ~ 50 larger than the duality prediction!

At higher values of $|t|$ the $\gamma p \to \pi^+ n$ cross section falls rapidly with energy, approximately as $1/s^6$ in the $|t| = 3 \ldots 4$ GeV^2 range [16]. Hence the duality relation (7) will be better satisfied at larger s and $|t|$. E.g., for $data/theory \propto 1/s^4$ their ratio approaches unity around $E_\gamma \simeq 20$ GeV.

For DIS the duality relation (1) works down to quite low values of $Q^2 \lesssim 1$ GeV^2 [1,2]. It is perhaps not surprising that the subprocess H in Fig. 1 requires higher values of $|t|$ to become compact – the virtualities of the internal lines in the $\gamma u \to \pi^+ d$ subprocess are generally less than $|t|$. It is possible that duality nevertheless applies to the semi-exclusive process $\gamma p \to \pi^+ Y$ at lower values of $|t|$ in the sense that the shape of the scaling curve measured at large M_Y^2 agrees on average with that of the resonance region. Unfortunately, data on the semi-exclusive cross section is not available.

So far we discussed duality for $|t| \ll s$, a limit which appears to be required for factorization in semi-exclusive processes [11]. The authors of Ref. [12] took $|t| \sim s$ in the semi-exclusive process [14] and were led via duality to a fixed angle limit of the exclusive process. Since $G_{Mp}^2(-t) \propto 1/t^4$, Eq. (7) evaluated at fixed t/s predicts that $d\sigma/dt(\gamma p \to \pi^+ n) \propto 1/s^7$ at fixed angle [12]. This energy dependence agrees with the data which at $\theta_{CM} = 90^\circ$ scales as $s^7 d\sigma/dt \simeq 8.3 \mu\mathrm{b}\ \mathrm{GeV}^{12}$ over practically the whole measured energy range [16].

However, Eq. (7) (with a slight change of normalization due to evaluating (6) at fixed angle) underestimates the measured cross section by more than a factor 200. This disagreement will not improve at higher energies since both data and theory obey the same scaling. We conclude that duality, and more generally the factorization of semi-exclusive processes implied by Fig. 1 and Eq. (3), can be valid only in the $|t| \ll s$ limit [11]. In the fixed angle limit diagrams involving more than one target parton can apparently not be neglected. This was also observed in Ref. [18].

In the case of the process $\gamma p \to \pi^0 p$ we note that the leading order subprocess cross section (6) vanishes for $|t| \ll s$. The cross section measured for this process [16,19] is actually somewhat larger than that for $\gamma p \to \pi^+ n$. This confirms our conclusion above that the expression (6) is not applicable for the present data.

For Compton scattering, $\gamma p \to \gamma p$, the duality prediction analogous to (7) is

$$\frac{d\sigma}{dt}(\gamma p \to \gamma p) = \frac{16\pi\alpha^2}{9s^2} \frac{1 + r_{du}/16}{1 + r_{du}/4} G_{Mp}^2(-t) \tag{8}$$

With the same parameters as above this gives a Compton cross section which is a factor ~ 13 below the data [19] at $E_\gamma = 6$ GeV, $-t = 2.45$ GeV^2. The discrepancy is thus less than for $\gamma p \to \pi^+ n$ but still substantial. The data indicates a $\sim 1/s^3$ energy dependence of the cross section, again suggesting that higher values of both s and $|t|$ are likely to be required for agreement with PQCD.

To conclude, we applied Bloom-Gilman duality to semi-exclusive processes. In a fixed angle limit [12], duality predicts $d\sigma/dt(\gamma p \to \pi^+ n) \propto 1/s^7$ in agreement with data [16]. However, the normalization of the 90° data is more than two orders of magnitude larger than expected from duality. Due to the identical scaling of both data and theory this discrepancy will not decrease with energy. It thus appears that the dynamics of semi-exclusive processes does not factorize into a a hard subprocess and a target parton distribution (*cf.* Fig. 1) in a fixed angle limit.

The energy dependence of the $\gamma p \to \pi^+ n$ cross section is consistent with $1/s^2$ at fixed $|t| \lesssim 2$ GeV^2 as required by duality. However, the duality relation underestimates the normalization by a factor 50 even at $|t| \simeq 2$ GeV^2. At higher values of $|t|$ the data decreases rapidly with energy, indicating that better agreement with the normalization given by the subprocess cross section (6) will be obtained at higher values of s and $|t|$. Such a conclusion is consistent with preliminary data on semi-exclusive ρ photoproduction at HERA, $\gamma p \to \rho Y$, which suggests [20] that large values of $|t| \gtrsim 8$ GeV^2 are required for agreement with PQCD predictions of the dominance of longitudinal ρ polarization.

The empirical success of duality down to $Q^2 \lesssim 1$ GeV^2 in DIS suggests that the shape of the scaling curve may average the resonance contributions at similar values of $|t|$ also in semi-exclusive processes. It would thus be important to measure the $\gamma p \to \pi^+ Y$ cross section at higher M_Y^2 (but consistent with the condition $M_Y^2 \ll s$). This should be feasible at HERMES, COMPASS and HERA.

The effective size of the hard probe H in Fig. 1 can be 'measured' via its dependence on the virtuality Q^2 of the initial photon. The subprocess cross section will be independent of Q^2 as long as the transverse size of H is $\lesssim 1/Q$. Conversely, the probe H can be made more compact by increasing Q^2. This should improve the agreement with the PQCD cross section (6) (modified to account for the photon virtuality).

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