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Non-perturbative effects in semi-leptonic B_c decays

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Abstract

We discuss the impact of the soft degrees of freedom inside the B_c meson on its rate in the semi-leptonic decay $B_c \to X\ell\bar{\nu}_\ell$ where X denotes light hadrons below the D^0 threshold. In particular we identify contributions involving soft hadrons which are non-vanishing in the limit of massless leptons. These contributions become relevant for a measurement of the purely leptonic B_c decay rate, which due to helicity suppression involves a factor m_ℓ^2 and thus is much smaller than the contributions involving soft hadrons.

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1 Introduction

Within the standard model of elementary particle physics the B_c is the only meson which contains a heavy quark-antiquark ($Q\overline{Q}$) pair of different flavour. After more then two decades of purely theoretical investigations its observation by the CDF collaboration in 1998 [1,2] pushed open the gate for experimental exploration. The newest and forthcoming generation of experiments (Tevatron and LHC) will be able to determine the B_c properties that are hardly known at present [3].

Theoretically, the fact that the B_c is a bound state of heavy quarks offers an interesting possibility for an efficient way of calculating its properties. While the bound state is dominated by soft scales the heavy quark mass m_Q which is the typical scale for production and decay processes of such mesons is much larger. Due to this hierarchy of scales it is obvious to use an effective theory approach. One could treat the B_c within the Heavy Quark Effective Theory (HQET) at intermediate scales (i.e. where the charm (anti)quark is considered to be light) but this does not take into account that m_c can still be considered a perturbative scale. Assuming that the c quark is also heavy (i.e. using that both $m_b, m_c \gg \Lambda_{\mathrm{QCD}}$) we may work with approaches developed for quarkonia-like systems. In this case the B_c stands somehow in between of charmonia and bottomonia which usually are investigated within the framework of Non-relativistic Quantumchromodynamics (NRQCD) [4].

In this article we concentrate on the semi-leptonic B_c decay $B_c \to X\ell\bar{\nu}_\ell$ where X denotes only *light* hadrons which means hadrons below the charm threshold. In this kinematic region the B_c has to decay through electroweak annihilation. The light hadrons thus originate from gluons or other light degrees of freedom inside the B_c meson which makes this decay mode particularly interesting, since it tests the light degrees of freedom in a quarkonia-like system. But also for practical purposes it is of relevance: We will show that the semi-leptonic decay rate exceeds the leptonic one (at least for light leptons) even in the limit of extremely soft light hadrons which could complicate substantially the measurement of the latter.

The total rate of the purely leptonic mode is given by the expression

$$\Gamma[B_c \to \ell\bar{\nu}_\ell] = \frac{G_F^2}{8\pi}|V_{cb}|^2 f_{B_c}^2 M^3 \left[\frac{m_\ell^2}{M^2}\left(1-\frac{m_\ell^2}{M^2}\right)^2\right] \tag{1}$$

where G_F is the Fermi constant, V_{cb} the CKM matrix element corresponding to the underlying partonic decay process $b\bar{c} \to \ell\bar{\nu}_\ell$, and M is the mass of the B_c. The B_c decay constant f_{B_c} is defined via

$$i f_{B_c} p^\mu := \langle 0|\,\bar{b}\gamma^\mu\gamma^5 c\,|B_c(p)\rangle\ . \tag{2}$$

Obviously the total rate (1) is suppressed by the ratio m_ℓ^2/M^2 of the lepton and the B_c mass squared. This is caused by the helicity flip which is necessary in a back-to-back decay of a spinless particle in the ground state into a left-handed lepton (right-handed antilepton) and its right-handed antineutrino (left-handed neutrino).

Contrary to the quark model where the $(b\bar{c})$ systems has to be in a 0^- state, NRQCD and related effective approaches allow for higher Fock states, i.e. the $(b\bar{c})$ pair also can exist with other quantum numbers than $J^P = 0^-$ inside the B_c. Although these states are suppressed by non-perturbative factors as $\bar{v}^2$ in NRQCD[1] their contribution becomes measurable if the m_ℓ^2 suppression of the leading Fock state is stronger than the $\bar{v}^2$ suppression of a subleading Fock state which does not suffer from helicity suppression. However, these contributions are not purely leptonic anymore since in such a case the soft degrees of freedom inside the higher Fock states should give rise to some soft hadronic signal in the final state.

Thus we expect, for instance, semi-leptonic contributions related to the NRQCD matrix elements $\langle \mathcal{O}_1(^3S_1)\rangle$ and $\langle \mathcal{O}_1(^1P_1)\rangle$ where the short distance coefficients arise without any m_ℓ^2/M^2 suppression since it is not necessary to flip any helicity if the decaying $(b\bar{c})$ pair has spin one or is in a P wave state. Additionally there also should be unsuppressed contributions from higher Fock states with the $(b\bar{c})$ pair in a $^1S_0$ state since the soft degrees of freedom inside the B_c can carry away some angular momentum or spin to prevent the helicity suppression in the leptonic sector. Thus the corresponding short distance coefficients should (at least for light leptons) overcome the enhancement of the leading order matrix element $\langle \mathcal{O}_1(^1S_0)\rangle$ against the higher orders in the non-relativistic expansion. They also should exceed the short distance coefficients from radiative leptonic B_c decays where the prize of avoiding helicity suppression by radiating off an additional photon is paid for by a factor $\alpha/(4\pi)$.

In order to calculate the lepton energy spectrum in the decay $B_c \to X\ell\bar{\nu}_\ell$ we apply an effective approach almost equivalent to standard NRQCD. Even though we use a different notation namely a covariant one borrowed from HQET our effective Lagrangian matches the NRQCD one. However, we enlarge the operator basis at dimension-8 by operators related with the centre-of-mass (cms) momentum of the $Q\overline{Q}$ pair inside the bound state since we expect contributions that are not helicity suppressed just from these operators. According to the standard NRQCD velocity rules the matrix elements of such operators are suppressed with respect to matrix elements of dimension-8 operators that merely depend on the relative momentum of quark and antiquark. On the other hand, it is known that these power counting rules are only applicable for systems which obey the scale hierarchy $m_Q \gg m_Q\bar{v} \gg m_Q\bar{v}^2 \sim \Lambda_{\rm QCD}$ [5].

Fleming et al. [6] proposed to keep the standard NRQCD velocity rules only for bottomonia while they suggest to switch to a dimensional power counting in the charmonium sector. The gist of this analysis is a new weighting of a spin-flip transition that is compatible with data [7] and possibly could explain some of the NRQCD drawbacks concerning the

[1]We have chosen $\bar{v}$ instead of the commonly used v to denote the typical (average) non-relativistic (anti)quark velocity inside the quarkonia, i.e. more precisely the absolute value of its three vector, to avoid confusion with the covariant four vector v_μ we need to define the quarkonia velocity.

J/ψ polarization at high transverse momentum and the endpoint spectrum in radiative quarkonia decays.

Here we do not only discuss the relative weight of a dipole and a spin-flip transition. Rather the semi-leptonic decay rate may also help to clarify the power counting of the cms derivatives on the $Q\overline{Q}$ pair. Our procedure follows the strategy of Heavy Quarkonia Effective Theory (HQ$\overline{\text{Q}}$ET) [8, 9]. We just eliminate some redundancies and rotate the operator basis into the familiar one of NRQCD.

In the following we will start with setting up the basics of the effective approach. We sketch how the Lagrangian is constructed and how B_c annihilation processes are implemented. Furthermore we provide the basis of four fermion operators up to dimension-8 and discuss the power counting rules. After applying this framework on the lepton energy spectrum in the decay $B_c \to X\ell\bar{\nu}_\ell$ we address the structure of its endpoint region. We also relate our result to the shape function improved one of NRQCD. Finally we discuss the total semi-leptonic width of the B_c and implied consequences on measuring its purely leptonic width.

2 Effective field theory for the B_c

In this section we shall briefly discuss the effective field theory set up as it is appropriate for the case of a B_c decay. In general, quarkonia are more complicated than mesons containing only one heavy (anti)quark due to the fact that several scales are involved, some of which are dynamically generated.

2.1 Lagrangian and fields

In the first step one uses the fact that the heavy quark mass is large compared to any other scale in the problem. Thus one performs a $1/m_Q$ expansion of the full QCD Lagrangian, which can be done either by integrating out the heavy degrees of freedom, which are the small components of the heavy quark spinor [10], or by performing a sequence of Foldy-Wouthuysen transformations [11]. In both cases one obtains a $1/m_Q$ expansion of the heavy quark field as well as of the Lagrangian. Although these expansions look slightly different, the results for the matrix elements are the same. Up to $1/m_Q^2$ one obtains

$$\mathcal{L}_v = \bar{h}_v^{(+)}(ivD)h_v^{(+)} + \frac{1}{2m_Q}(K_v + M_v) + \mathcal{O}\left(\frac{1}{m_Q^2}\right) \tag{3}$$

with

$$K_v = \bar{h}_v^{(+)} iD^\mu (g_{\mu\nu} - v_\mu v_\nu) iD^\nu h_v^{(+)}\,, \tag{4a}$$

$$M_v = \frac{ig_s}{2}\bar{h}_v^{(+)}(-i\sigma^{\mu\nu})G_{\mu\nu}h_v^{(+)} \tag{4b}$$

where $v_\mu = P_\mu/M$ is the four velocity of the heavy meson and $h_v^{(+)}$ the field operator of a heavy quark in the limit $m_Q \to \infty$. The terms K_v and M_v correspond to the kinetic energy and chromomagnetic moment, respectively, where the gluonic field strength tensor $G_{\mu\nu}$ is defined by the relation

$$ig_s G_{\mu\nu} := [iD_\mu, iD_\nu] \,. \tag{5}$$

Likewise, we obtain the corresponding relations for an antiquark in the infinite mass limit by reversing the sign of the velocity $v \to -v$.

In order to describe a quarkonium-like system one wants to treat both the quark and the antiquark in the infinite mass limit, assuming that both are sitting in a bound state moving with a velocity v and that both heavy constituents have small residual momentum. Thus one would start with the Lagrangian

$$\mathcal{L}_{\text{stat.}} = \bar{h}_v^{(+)}(ivD)h_v^{(+)} - \bar{h}_{v'}^{(-)}(iv'D)h_{v'}^{(-)} \tag{6}$$

and add the corresponding pieces of the Lagrangian for the light degrees of freedom.

However, it is known that this simple ansatz misses an important piece of physics, which is that the dynamics responsible for the binding generates additional scales. These scales are the size and the binding energy of the bound system and are related to the relative (three-)velocity v_{rel} which can be written as

$$v_{\text{rel}} = \sqrt{(vv')^2 - 1} \,. \tag{7}$$

Clearly in a quarkonium we have $v_{\text{rel}} \ll 1$ and in the limit $v \to v'$ the Lagrangian (6) exhibits certain pathologies indicating that bound states cannot be described in the static limit. As a example one may consider the matrix element $\langle H_{\text{stat.}}|\, \bar{h}_v^{(+)}\Gamma h_{v'}^{(-)}\,|0\rangle$ in the static limit; computation of the anomalous dimension reveals an imaginary part in the anomalous dimension of the respective current $\bar{h}_v^{(+)}\Gamma h_{v'}^{(-)}$ as $v' \to v$ [12]. The divergence in the limit $v' \to v$ leads to a phase factor in the solution of the renormalization group equation which is related to the coulombic part of the one-gluon exchange [13,14]. This reflects the possibility of heavy bound states.

This problem can be fixed by including the kinetic term into the leading order Lagrangian, i.e. one switches to a non-relativistic description. The Lagrangian then reads

$$\mathcal{L}_0 = \bar{h}_v^{(+)}\left(ivD - \frac{(iD_\perp)^2}{2m_Q}\right)h_v^{(+)} - \bar{h}_v^{(-)}\left(ivD + \frac{(iD_\perp)^2}{2m_Q}\right)h_v^{(-)} \tag{8}$$

where the component $iD_\perp^\mu$ orthogonal the v_μ is given by

$$iD_\perp^\mu := (g^{\mu\nu} - v^\mu v^\nu)iD_\nu \,. \tag{9}$$

The form of the leading order term evidently shows that is impossible to operate with a purely dimensional power counting in this case, since the equation of motion is not homogeneous in $1/m_Q$:

$$(ivD)\, h_v^{(\pm)} = \pm \frac{(iD_\perp)^2}{2m_Q} h_v^{(\pm)} \,. \tag{10}$$

Usually it is argued that this corresponds to an expansion in the relative velocity $\bar{v}$ which is assumed to be small. In fact, the non-relativistic approximation (8) can generate bound states of (inverse) size $m_Q \bar{v}$ and of binding energy $m_Q \bar{v}^2$ which introduce two more, dynamically generated scales into the problem. This fact makes the construction of an effective field theory approach more complicated.

We also note that the chromomagnetic moment M_v is taken to be higher order compared to the kinetic energy K_v although it is of the same mass dimension. This means that the antisymmetric combination of two covariant derivatives acting on the field $h_v^{(\pm)}$ is suppressed with respect to the symmetric combination:

$$[iD^\mu, iD^\nu]\, h_v^{(\pm)} \quad \ll \quad \{iD^\mu, iD^\nu\}\, h_v^{(\pm)} \,. \tag{11}$$

The standard rules for power counting in non-relativistic system can be found in [15], but we shall return to this question at the end of this section.

The static as well as the non-relativistic limit of QCD has (compared to full QCD) an additional spin symmetry which is evident from the Lagrangian (8). In the case of a quarkonium-like system this symmetry predicts quadruplets which are degenerate up to terms of order $\bar{v}^2$. For the ground state quarkonia this quadruplet consists of the 0^- states and the three polarizations of the 1^- state. For excited quarkonia this quadruplet structure is more complicated and involves a possible orbital angular momentum.

Finally one may formulate NRQCD in the rest frame of the quarkonium in which $v = (1, \mathbf{0})$ which transforms (8) into the usual NRQCD Lagrangian. Rewriting the (relativistic) four-dimensional fields $h_v^{(\pm)}$ in terms of the (non-relativistic) two-dimensional spinors ψ and χ for quark and antiquark, respectively,

$$h_v^{(+)} = \begin{pmatrix} \psi \\ 0 \end{pmatrix} , \qquad h_v^{(-)} = \begin{pmatrix} 0 \\ \chi \end{pmatrix} \tag{12}$$

one actually gets the NRQCD Lagrangian

$$\mathcal{L}_0 = \psi^\dagger \left(iD_t + \frac{(i\boldsymbol{D})^2}{2m_Q} \right) \psi + \chi^\dagger \left(iD_t - \frac{(i\boldsymbol{D})^2}{2m_Q} \right) \chi \,. \tag{13}$$

2.2 Operator Product Expansion for semileptonic B_c decays

The main contribution to semileptonic B_c decays comes from the part of the effective Hamiltonian mediating $b \to c$ transitions

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} (\bar{b}_L \gamma_\mu c_L)(\bar{\nu}_{\ell,L} \gamma^\mu \ell_L) \tag{14}$$

where the subscript L denotes the left-handed component of the the corresponding fermion.

In order to consider the inclusive case we write the rate as

$$d\Gamma = \frac{8G_F^2 |V_{cb}|^2}{M} \int \frac{d^4q}{(2\pi)^4} \int d^4x \, e^{-iqx} \langle B_c | \left(\bar{b}_L(x) \gamma_\mu c_L(x) \right) \left(\bar{c}_L(0) \gamma_\nu b_L(0) \right) | B_c \rangle \, I^{\mu\nu}(q) \tag{15}$$

where $I^{\mu\nu}(q)$ denotes the leptonic part.

We perform an operator product expansion (OPE) for the product of the hadronic currents making use of the fact that both m_c and m_b are large scales, which can be removed from the matrix element by redefining the phase of the heavy fields. We consider the decay of a B_c^- which consists of a anti-c and a b quark; therefore we define

$$b(x) =: e^{-im_b vx} B_v(x) \quad \textit{and} \quad c(x) =: e^{im_c vx} C_v(x) \tag{16}$$

where B_v and C_v eventually become the nonrelativistic fields considered in the last paragraph, and v is the four velocity of the B_c meson. In terms of these fields we have for the hadronic tensor

$$H_{\mu\nu} = \int d^4x \, \exp\{-i(q - M_{b\bar{c}} v)x\} \langle B_c(v) | \left(\bar{B}_{v,L}(x) \gamma_\mu C_{v,L}(x) \right) \left(\bar{C}_{v,L}(0) \gamma_\nu B_{v,L}(0) \right) | B_c(v) \rangle \tag{17}$$

where

$$M_{b\bar{c}} = m_b + m_c \tag{18}$$

is the sum of the quark masses.

We perform an OPE assuming that $(q - M_{b\bar{c}} v)^2$ and $qv - M_{b\bar{c}}$ are of order $M_{b\bar{c}}^2$ and $M_{b\bar{c}}$, respectively. The differential rate takes the form

$$d\Gamma = \sum_n \mathcal{C}_n(\mu) \, \langle \mathcal{O}[n] \rangle \Big|_\mu \tag{19}$$

where $\mathcal{C}_n$ are short distance coefficients which are calculated in perturbation theory and $\langle \mathcal{O}[n] \rangle$ are forward matrix elements of local operators $\mathcal{O}[n]$ with non-relativistic B_c states. The parameter μ is the renormalization scale; the dependence of the coefficients on μ is canceled by the corresponding dependence of the matrix elements, such that the total rate is independent of μ. The sum runs over an infinite set of operators which are of increasing mass dimension; this dimension is compensated by the coefficients $\mathcal{C}_n$ which contain only the perturbative and large mass scale $M_{b\bar{c}}$, while the matrix elements $\langle\mathcal{O}[n]\rangle$ do not depend on this mass scales any more; they depend only on a non-perturbative small mass scale $\bar{\Lambda}$ corresponding e.g. to the binding energy of a coulombic system. Thus higher dimensional operators are expected to be suppressed by powers of $\bar{\Lambda}/M_{b\bar{c}}$ and thus the series in (19) may be truncated and we shall consider only the first nontrivial terms.

In leading order we have two operators of dimension-6:

$$\mathcal{O}_{(1\times1)}[{}^1S_0^{(C)}] := -\left(\bar{h}_v^{(+)}\,\gamma_5 C\, h_v^{(-)}\right)\left(\bar{h}_v^{(-)}\,\gamma_5 C\, h_v^{(+)}\right)\,, \tag{20a}$$
$$\mathcal{O}_{(1\times1)}[{}^3S_1^{(C)}] := -\left(\bar{h}_v^{(+)}\,\gamma_\mu^\perp C\, h_v^{(-)}\right)\left(\bar{h}_v^{(-)}\,\gamma_\perp^\mu C\, h_v^{(+)}\right)\,. \tag{20b}$$

Here we have introduced the spectral notation $n = {}^{2S+1}L_J^{(C)}$ where the upper index $C = 1, 8$ denote the the colour states that are indicated by $C = 1$ and $C = T^a$ for singlet and octet states, respectively, on the right hand side of equations (20). Using (12) we recognize that the leading order four fermion operators again coincide with the one of NRQCD [4]:

$$\mathcal{O}_{(1\times1)}[{}^1S_0^{(C)}] \equiv \mathcal{O}_C({}^1S_0)\,, \qquad \mathcal{O}_{(1\times1)}[{}^3S_1^{(C)}] \equiv \mathcal{O}_C({}^3S_1)\,. \tag{21}$$

In subleading order our set of operators is more general than the standard NRQCD basis, since we also include contributions related to the motion of the $Q\overline{Q}$ pair, more precisely its centre of mass, inside the quarkonium. It is known that these contributions are important in certain kinematical regions, e.g. in the endpoint of photon spectra in radiative quarkonia decays [16, 17], even though they are missed by naive use of NRQCD. To disentangle the relative motion of the quark and the antiquark from the motion of its centre of mass we decompose their residual momentum inside the meson according to

- Residual cms momentum (RCM):

$$iD_\mu\left(\bar{Q}_v^{(\pm)}\,\Gamma C\, Q_v^{(\mp)}\right) := \bar{Q}_v^{(\pm)}\,\Gamma C(iD_\mu)\,Q_v^{(\mp)} + \bar{Q}_v^{(\pm)}\,(i\overleftarrow{D}_\mu)\Gamma C\, Q_v^{(\mp)} \tag{22}$$

- Residual relative momentum (RRM):

$$\bar{Q}_v^{(\pm)}\,\Gamma\left(\tfrac{i}{2}\overleftrightarrow{D}_\mu\right)C\, Q_v^{(\mp)} := \frac{1}{2}\left[\bar{Q}_v^{(\pm)}\,\Gamma C(iD_\mu)\,Q_v^{(\mp)} - \bar{Q}_v^{(\pm)}\,(i\overleftarrow{D}_\mu)\Gamma C\, Q_v^{(\mp)}\right] \tag{23}$$

Besides we introduce $T^{(\mu\nu)} := (T^{\mu\nu} + T^{\nu\mu})/2 - g^{\mu\nu}T_\lambda{}^\lambda/4$ to denote the symmetric traceless part of a tensor. In this way we can distinguish three types of dimension-8 operators:
RCM × RCM:

$$\mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[{}^1S_0^{(C)}] := -\left[iD_\mu^\perp\left(\bar{h}_v^{(+)}\,\gamma_5 C\, h_v^{(-)}\right)\right]\left[iD_\perp^\mu\left(\bar{h}_v^{(-)}\,\gamma_5 C\, h_v^{(+)}\right)\right] \tag{24a}$$
$$\mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[{}^3S_1^{(C)}] := -\left[iD_\mu^\perp\left(\bar{h}_v^{(+)}\,\gamma_\nu^\perp C\, h_v^{(-)}\right)\right]\left[iD_\perp^\mu\left(\bar{h}_v^{(-)}\,\gamma_\perp^\nu C\, h_v^{(+)}\right)\right] \tag{24b}$$
$$\mathcal{P}_{(\mathrm{C}\times\mathrm{C})}[{}^3S_1^{(C)}] := -\left[iD_{(\mu}^\perp\left(\bar{h}_v^{(+)}\,\gamma_\perp^\mu C\, h_v^{(-)}\right)\right]\left[iD_{\nu)}^\perp\left(\bar{h}_v^{(-)}\,\gamma_\perp^\nu C\, h_v^{(+)}\right)\right] \tag{24c}$$

RCM $\times$ RRM:

$$\mathcal{O}_{(\mathrm{C}\times\mathrm{R})}[{}^1S_0^{(C)}] := -\frac{1}{2}\left\{\left[iD_\mu^\perp\left(\bar{h}_v^{(+)}\gamma_5 C\,h_v^{(-)}\right)\right]\left[\bar{h}_v^{(-)}\gamma_5\left(-\tfrac{i}{2}\overleftrightarrow{D}_\perp^\mu\right)C\,h_v^{(+)}\right]+\text{h.c.}\right\} \tag{25a}$$

$$\mathcal{O}_{(\mathrm{C}\times\mathrm{R})}[{}^3S_1^{(C)}] := -\frac{1}{2}\left\{\left[iD_\mu^\perp\left(\bar{h}_v^{(+)}\gamma_\nu^\perp C\,h_v^{(-)}\right)\right]\left[\bar{h}_v^{(-)}\gamma_\perp^\nu\left(-\tfrac{i}{2}\overleftrightarrow{D}_\perp^\mu\right)C\,h_v^{(+)}\right]+\text{h.c.}\right\} \tag{25b}$$

$$\mathcal{P}_{(\mathrm{C}\times\mathrm{R})}[{}^3S_1^{(C)}] := -\frac{1}{2}\left\{\left[iD_{(\mu}^\perp\left(\bar{h}_v^{(+)}\gamma_\perp^\mu C\,h_v^{(-)}\right)\right]\left[\bar{h}_v^{(-)}\gamma_\perp^\nu\left(-\tfrac{i}{2}\overleftrightarrow{D}_{\nu)}^\perp\right)C\,h_v^{(+)}\right]+\text{h.c.}\right\} \tag{25c}$$

RRM $\times$ RRM:

$$\mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^1P_1^{(C)}] := \left[\bar{h}_v^{(+)}\gamma_5\left(-\tfrac{i}{2}\overleftrightarrow{D}_\mu^\perp\right)C\,h_v^{(-)}\right]\left[\bar{h}_v^{(-)}\gamma_5\left(-\tfrac{i}{2}\overleftrightarrow{D}_\perp^\mu\right)C\,h_v^{(+)}\right] \tag{26a}$$

$$\mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_0^{(C)}] := \frac{1}{3}\left[\bar{h}_v^{(+)}\left(-\tfrac{i}{2}\overleftrightarrow{\not{D}}^\perp\right)C\,h_v^{(-)}\right]\left[\bar{h}_v^{(-)}\left(-\tfrac{i}{2}\overleftrightarrow{\not{D}}^\perp\right)C\,h_v^{(+)}\right] \tag{26b}$$

$$\mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_1^{(C)}] := \frac{1}{2}\left[i\epsilon_\mu{}^{\alpha\beta\gamma}v_\gamma\,\bar{h}_v^{(+)}\gamma_\alpha^\perp\left(-\tfrac{i}{2}\overleftrightarrow{D}_\beta^\perp\right)C\,h_v^{(-)}\right]\left[i\epsilon^{\mu\alpha'\beta'\gamma'}v_{\gamma'}\,\bar{h}_v^{(-)}\gamma_{\alpha'}^\perp\left(-\tfrac{i}{2}\overleftrightarrow{D}_{\beta'}^\perp\right)C\,h_v^{(+)}\right] \tag{26c}$$

$$\mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_2^{(C)}] := \left[\bar{h}_v^{(+)}\gamma_{(\mu}^\perp\left(-\tfrac{i}{2}\overleftrightarrow{D}_{\nu)}^\perp\right)C\,h_v^{(-)}\right]\left[\bar{h}_v^{(-)}\gamma_\perp^{(\mu}\left(-\tfrac{i}{2}\overleftrightarrow{D}_\perp^{\nu)}\right)C\,h_v^{(+)}\right] \tag{26d}$$

$$\mathcal{P}_{(\mathrm{R}\times\mathrm{R})}[{}^1S_0^{(C)}] := \frac{1}{2}\left\{\left[\bar{h}_v^{(+)}\gamma_5 C\,h_v^{(-)}\right]\left[\bar{h}_v^{(-)}\gamma_5\left(-\tfrac{i}{2}\overleftrightarrow{D}^\perp\right)^2C\,h_v^{(+)}\right]+\text{h.c.}\right\} \tag{27a}$$

$$\mathcal{P}_{(\mathrm{R}\times\mathrm{R})}[{}^3S_1^{(C)}] := \frac{1}{2}\left\{\left[\bar{h}_v^{(+)}\gamma_\mu^\perp C\,h_v^{(-)}\right]\left[\bar{h}_v^{(-)}\gamma_\perp^\mu\left(-\tfrac{i}{2}\overleftrightarrow{D}^\perp\right)^2C\,h_v^{(+)}\right]+\text{h.c.}\right\} \tag{27b}$$

$$\mathcal{P}_{(\mathrm{R}\times\mathrm{R})}[{}^3S_1^{(C)},{}^3D_1^{(C)}] := \frac{1}{2}\left\{\left[\bar{h}_v^{(+)}\gamma_\mu^\perp C\,h_v^{(-)}\right]\left[\bar{h}_v^{(-)}\gamma_\nu^\perp\left(-\tfrac{i}{2}\overleftrightarrow{D}_\perp^{(\mu}\right)\left(-\tfrac{i}{2}\overleftrightarrow{D}_\perp^{\nu)}\right)C\,h_v^{(+)}\right]+\text{h.c.}\right\} \tag{27c}$$

The main difference to the operator basis of NRQCD is the inclusion of operators of the type RCM $\times$ RCM and RCM $\times$ RRM. Dealing with the cms momentum of the $Q\overline{Q}$ pair inside the quarkonium they were neglected in the original work [4] since they do not yield contributions from the dominant Fock state $|Q\overline{Q}\rangle$. Nevertheless it was shown that they play an important role in quarkonia decay [17] and production [18] processes as long as the observable is not inclusive enough. More precisely these operators are responsible for the shift from the partonic to the hadronic endpoint in energy spectra caused by the mass difference $M-2m_Q\sim m_Q\bar{v}^2$. After resumming the leading cms operators to so-called shape functions the NRQCD prediction is extendible up to higher energy values. Numerical analyses of this effect using a model consistent with the shape function formalism indicate on the one hand a natural solution of the HERA problem in the J/ψ photoproduction channel [19, 20] but generate on the other hand some new obscurities in the radiative Υ decay [21].

The operators or the RRM $\times$ RRM sector common with NRQCD are related to its dimension-8 operators in the following way (cf. eq. (12)):

$$\mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^{2S+1}L_J^{(C)}] = \mathcal{O}_C({}^{2S+1}L_J)\,,\qquad \mathcal{P}_{(\mathrm{R}\times\mathrm{R})}[{}^{2S+1}L_J^{(C)}] = \mathcal{P}_C({}^{2S+1}L_J)\,. \tag{28}$$

2.3 Power counting

Instead of applying strictly the standard NRQCD velocity scaling rules we propose to use a slightly generalized power counting scheme. In particular we want to direct the readers attention on contributions related to the cms movement of the $Q\overline{Q}$ pair inside the meson. These contributions potentially exceed the prediction by standard NRQCD power counting, which we will critically recapitulate first.

To determine the suppression factor of NRQCD matrix elements of four-fermion operators we have to consider both the scaling of the operator itself and additional orders of $\bar{v}$ coming from the Fock decomposition of the hadronic state:

$$|H\rangle = \sum_n \left(c_0^H[n] \, |Q\overline{Q}[n]\rangle + c_1^H[n] \, |Q\overline{Q}[n]g\rangle + c_2^H[n] \, |Q\overline{Q}[n]gg\rangle + \dots \right). \tag{29}$$

While the standard velocity rules for the operators are straightforward the assignment of suppression factors to the coefficients $c_i[n]$ in (29) is quite complicated since it depends on the non-perturbative dynamics inside the bound state. In the original NRQCD paper [4] quarkonia states are assumed to obey the scale hierarchy $m_Q \gg m_Q\bar{v} \gg m_Q\bar{v}^2 \sim \Lambda_{\mathrm{QCD}}$. In other words their binding is assumed to be coulombic which enables the use of NRQED velocity rules of [15]. In particular this implies the validity of multipole expansion for calculating the coefficients, i.e. a spin-flip transition (M1) is suppressed by $\bar{v}^2$ in contrast to a dipole transition (E1) which only accounts for $\bar{v}$.

Beneke mentioned that this is no longer true if the scale hierarchy is $m_Q \gg m_Q\bar{v} \sim \Lambda_{\mathrm{QCD}} \gg m_Q\bar{v}^2$ [5]. He proposed to consider M1 transitions as $\lambda\bar{v}$ where $\lambda = \bar{v}$ for coulombic systems while $\lambda \sim \mathcal{O}(1)$ for $m_Q\bar{v} \sim \Lambda_{\mathrm{QCD}}$. A recent analysis [6] elaborates this feature to claim that the standard NRQCD velocity rules are only applicable in the bottomonium sector while it suggests a HQET-like power counting scheme in $\Lambda_{\mathrm{QCD}}/m_Q$ for charmonia.

We take a further step towards generalizing the counting rules questioning the scaling of a cms derivative inside a NRQCD operator. According to

$$|\boldsymbol{p}_{Q\overline{Q}}| = |\boldsymbol{p}_{\mathrm{soft}}| = E_{\mathrm{soft}} = \frac{M^2 - 4m_Q^2}{2M} = \frac{M + 2m_Q}{2M}(M - 2m_Q) \tag{30}$$

the total momentum $\boldsymbol{p}_{Q\overline{Q}}$ of the $Q\overline{Q}$ pair inside a quarkonium is connected with its binding energy $M - 2m_Q$. Therefore in the standard formalism the cms derivative on a quark-antiquark bilinear accounts as $\bar{v}^2$. However, if the binding of the quarkonia-like system is not coulombic anymore the binding energy no longer should be purely coulombic either. Rather there also should be non-negligible contributions of order $\Lambda_{\mathrm{QCD}}/m_Q \gg \bar{v}^2$. Hence we assume a cms derivative to be of order $\kappa\bar{v}$ keeping in mind that the standard NRQCD power counting is restored by $\kappa = \bar{v}$.

With this set of rules the B_c decay matrix elements of the operators up to dimension-8 scale as indicated in Fig. 1. Note that colour selection rules forbid the existence of solely one gluon in the higher Fock state if we restrict the perturbative part to leading order α_s. To estimate the leading contributions according to the non-relativistic expansion we have to know if the quarkonia-like B_c behaves rather as bottomonia or as charmonia. In the first case ($\kappa, \lambda \to \bar{v}$) the first five configurations of Fig. 1 would contribute up to order $\mathcal{O}(\bar{v}^8)$ while for $\kappa, \lambda \to 1$ the main matrix elements are the ones of the first row and the first column in Fig. 1.

3 Lepton energy spectrum

Using these assumptions we shall now calculate the lepton energy spectrum $d\Gamma/dz$ in the semi-leptonic decay $B_c \to X\ell\bar{\nu}_\ell$ where X denote only light hadrons.

$$z := \frac{2E_\ell}{M_{b\bar{c}}} \tag{31}$$

is the lepton energy E_ℓ in the B_c rest frame normalized on half of the sum of the heavy quark masses.

Note that $M_{b\bar{c}}$ is slightly smaller than the mass M of the B_c. The mass difference $M - M_{b\bar{c}}$ just count for the binding energy of the $(b\bar{c})$ system inside the B_c meson which is of the order $\bar{v}^2$ within usual NRQCD. However, this has to be taken into account to get the correct endpoint of the energy spectrum [17]. We will return to this point later again.

The underlying partonic process is the electroweak decay $b + \bar{c} \to \ell\bar{\nu}_\ell$. Thus we get

$$\frac{d\Gamma}{dz} = \frac{1}{M}\left(\frac{4G_F}{\sqrt{2}}\right)^2 |V_{cb}|^2 \int d^4x\, \frac{d^4q}{(2\pi)^4}\, e^{-ix[q-M_{b\bar{c}}v]}\, I_{\mu\nu}(q)\, \langle B_c|\, J_v^\mu(x)\bar{J}_v^\nu(0)\, |B_c\rangle\ . \tag{32}$$

The B_c rest frame is characterized by $v_\mu = (1, \mathbf{0})$. There the tensor $I_{\mu\nu}$ solely depends on the momentum q that flow through the effective vertex

$$I_{\mu\nu}(q) = \int \widetilde{dp_\ell}\widetilde{dp_{\bar{\nu}_\ell}}\, (2\pi)^4\delta^4(q - p_\ell - p_{\bar{\nu}_\ell})\, L_{\mu\nu}\, \delta(z - \frac{2p_\ell^0}{M_{b\bar{c}}}) \tag{33}$$

which means simultaneously that only the symmetric terms of the leptonic tensor

$$L^{\mu\nu} = 2\left(p_\ell^\mu p_{\bar{\nu}_\ell}^\nu + p_\ell^\nu p_{\bar{\nu}_\ell}^\mu + g^{\mu\nu}(p_\ell \cdot p_{\bar{\nu}_\ell}) - i\epsilon^{\mu\nu}{}_{\rho\sigma} p_\ell^\rho p_{\bar{\nu}_\ell}^\sigma\right) \tag{34}$$

survive. Let us turn to the hadronic matrix element now. Since we have factorized off their main phase space dependence the fields $B_v(x)$ and $C_v(x)$ in the left-handed current

$$J_v^\mu(x) = \bar{B}_v(x)\frac{\gamma^\mu(1-\gamma_5)}{2}C_v(x) \tag{35}$$

only depend on residual momenta. Hence we can expand the hadronic matrix element in the coordinate space around $x = 0$ after we have ensured hermiticity by symmetrizing the matrix element previously:

$$\langle B_c | \, J_v^\mu(x) \bar{J}_v^\nu(0) \, | B_c \rangle \to \langle B_c | \, \tfrac{1}{2} [J_v^\mu(x) \bar{J}_v^\nu(0) + J_v^\mu(0) \bar{J}_v^\nu(-x)] \, | B_c \rangle =: R_v^{\mu\nu} \, . \tag{36}$$

Then the expansion in x correspond to an expansion in the residual cms momentum of the $(b\bar{c})$ pair inside the B_c. Up to second order we get

$$R_v^{\mu\nu}(x) = R_v^{\mu\nu}(0) + i\partial^\alpha R_v^{\mu\nu}(x)\big|_{x=0} (-ix_\alpha) + \frac{1}{2} \, i\partial^\alpha i\partial^\beta R_v^{\mu\nu}(x)\big|_{x=0} (-ix_\alpha)(-ix_\beta) + \ldots \tag{37}$$

where the currents on their parts are also expansions parameterizing the residual relative momentum of the $(b\bar{c})$ system. It is given in eq. (66) of the Appendix.

The x dependence in the phase space integrals of the form

$$\int d^4x \, \frac{d^4q}{(2\pi)^4} \, \{1, (-ix_\alpha), (-ix_\alpha)(-ix_\beta)\} \, e^{-ix[q - M_{b\bar{c}} v]} \, I_{\mu\nu}(q) \tag{38}$$

is regarded by expressing the factor $-ix_\alpha$ as derivative $\partial/\partial q^\alpha$ on the exponential and subsequent integration by parts. At this stage arise higher derivative of delta functions, and the result for the spectrum is of the structure

$$\frac{d\Gamma}{dz} = \Gamma_0 \left\{ \mathcal{A}\delta(1+\varepsilon-z) + \frac{1}{(2m)^2} \left[\mathcal{B}_0 \delta(1+\varepsilon-z) + \mathcal{B}_1 \delta'(1+\varepsilon-z) + \mathcal{B}_2 \delta''(1+\varepsilon-z) \right] \right\} . \tag{39}$$

The fact that the differential decay rate is proportional to localized distributions (i.e. δ functions and its derivatives) reflects the two body structure of the decay at (partonic) tree level; the kinematics fix the lepton energy to be $z = 1 + \varepsilon$ where

$$\varepsilon := \frac{m_\ell^2}{M_{b\bar{c}}^2} \tag{40}$$

is the rescaled lepton mass squared. Clearly ε is a small quantity for the electron and the muon case, in which we can use only the leading term. However, for the τ the parameter ε is sizable and we thus keep the full dependence on ε.

The coefficient $\mathcal{A}$ contains contributions from both the spin singlet and the spin triplet dimension-6 operator:

$$\mathcal{A} = \mathcal{A}_{(1\times1)}[^1S_0^{(1)}] \, \langle B_c | \, \mathcal{O}_{(1\times1)}[^1S_0^{(1)}] \, | B_c \rangle + \mathcal{A}_{(1\times1)}[^3S_1^{(1)}] \, \langle B_c | \, \mathcal{O}_{(1\times1)}[^3S_1^{(1)}] \, | B_c \rangle \, . \tag{41}$$

We expect that the leading term represented by $\mathcal{A}_{(1\times1)}[^1S_0^{(1)}]$ is suffering from helicity suppression just like the purely leptonic decay. In the inclusive case this is due to spin symmetry making its contribution to $R_{\mu\nu}$ entirely proportional to $v_\mu v_\nu$. If vacuum insertion is applied, this contribution becomes exactly equal to the one from the purely leptonic case. In fact $\Gamma_0 \mathcal{A}_{(1\times1)}[{}^1S_0^{(1)}]$ reproduces the total rate (1) of the purely leptonic decay: Defining

$$\Gamma_0 := \frac{G_F^2}{8\pi M} |V_{cb}|^2 M_{b\bar{c}}^2 \tag{42}$$

the short distance coefficient reads

$$\mathcal{A}_{(1\times1)}[{}^1S_0^{(1)}] = \varepsilon(1-\varepsilon)^2 \,. \tag{43}$$

While the Wilson coefficient of the spin singlet operator explicitly shows a factor ε we do not expect any helicity suppression in the spin triplet case: We obtain for the corresponding short distance coefficient

$$\mathcal{A}_{(1\times1)}[{}^3S_1^{(1)}] = \frac{(1-\varepsilon)^2(2+\varepsilon)}{3} \tag{44}$$

which does not vanish as $\varepsilon \to 0$. From this observation we deduce that the spin triplet operator could become important even though its NRQCD matrix element is suppressed by $\lambda^2\bar{v}^6$ with respect to the leading matrix element of the operator $\mathcal{O}_{(1\times1)}[{}^1S_0^{(1)}]$. If we compare the magnitude of the suppression factors

$$\frac{\mathcal{A}_{(1\times1)}[{}^1S_0^{(1)}]}{\mathcal{A}_{(1\times1)}[{}^3S_1^{(1)}]} \approx \frac{3}{2}\left(\frac{m_\ell}{M_{b\bar{c}}}\right)^2 \quad \leftrightarrow \quad \lambda^2\bar{v}^6 \approx \frac{\langle B_c|\,\mathcal{O}_{(1\times1)}[{}^3S_1^{(1)}]\,|B_c\rangle}{\langle B_c|\,\mathcal{O}_{(1\times1)}[{}^1S_0^{(1)}]\,|B_c\rangle} \tag{45}$$

we realize that the spin triplet contribution overcomes the spin singlet contribution if the lepton mass is small enough. For sure this condition is fulfilled in the electron case. For muons both contributions are comparable although it depends on the value of λ and $\bar{v}$ for B_c. In the case $\ell = \tau$ we can neglect the spin triplet part.

The coefficients $\mathcal{B}_i$ arise with a factor $1/(2m)^2$ where m is the effective (= twice the reduced) mass of the b quark and the c antiquark. Furthermore we have introduced the dimensionless variable η to parameterize their mass difference:

$$m := \frac{2m_b m_c}{m_b + m_c}\,, \qquad \eta := \frac{m_b - m_c}{m_b + m_c}\,. \tag{46}$$

Generally operators of higher powers in the inverse mass m are accounted to be of higher order. However, there is a big *caveat* in the endpoint region of the energy spectrum. Because the expansion is an expansion in terms of $1/(M_{b\bar{c}} - 2E_\ell)$ rather than in terms of $1/(2m)$ where

$$2m = (1-\eta^2) M_{b\bar{c}} \tag{47}$$

it breaks down in the endpoint region. In the framework of NRQCD it was shown that the availability of the theoretical prediction can be extended to higher energy values if the contributions of the order $[m_Q \bar{v}^2/(m_Q - E_\ell)]^n$ are summed up to so-called shape functions [17]. Thereby the endpoint also shifts from the partonic value characterized by mass of the $Q\overline{Q}$ pair to the hadronic value which is a function of the mass of the decaying particle. In our case higher order derivatives of the leading delta function $\delta(1+\varepsilon-z)$ represent contributions to this shape function. We will elaborate this in Subsection 3.1 more detailed.

First we present the result for the coefficients $\mathcal{B}_i$ in (39). Due to heavy quark spin symmetry the spin triplet operators get an additional suppression factor compared to the spin singlet operators when they are inserted into the B_c matrix element. On the other hand they should not suffer from helicity suppression as we have seen from $\mathcal{O}_{(1\times 1)}[{}^3S_1^{(1)}]$. Therefore we will not only keep the spin singlet contributions to $\mathcal{B}_0$ but also the most relevant triplet operator $\mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_J^{(1)}]$ according to Fig. 1:

$$\begin{aligned}\mathcal{B}_0 &= \mathcal{B}^{(0)}_{(\mathrm{C}\times\mathrm{C})}[{}^1S_0^{(1)}]\,\langle B_c|\,\mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[{}^1S_0^{(1)}]\,|B_c\rangle + \mathcal{B}^{(0)}_{(\mathrm{C}\times\mathrm{R})}[{}^1S_0^{(1)}]\,\langle B_c|\,\mathcal{O}_{(\mathrm{C}\times\mathrm{R})}[{}^1S_0^{(1)}]\,|B_c\rangle \\ &+ \mathcal{C}^{(0)}_{(\mathrm{R}\times\mathrm{R})}[{}^1S_0^{(1)}]\,\langle B_c|\,\mathcal{P}_{(\mathrm{R}\times\mathrm{R})}[{}^1S_0^{(1)}]\,|B_c\rangle + \mathcal{B}^{(0)}_{(\mathrm{R}\times\mathrm{R})}[{}^1P_1^{(1)}]\,\langle B_c|\,\mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^1P_1^{(1)}]\,|B_c\rangle \\ &+ \mathcal{B}^{(0)}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_0^{(1)}]\,\langle B_c|\,\mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_0^{(1)}]\,|B_c\rangle\ .\end{aligned} \tag{48}$$

For light leptons the pieces which do no suffer from helicity suppression can be as sizable as the dimension-6 contribution $\mathcal{A}$. If we consider the limit $\varepsilon\to 0$ approximately fulfilled by electrons we should find non-vanishing contributions from $\mathcal{B}^{(0)}_{(\mathrm{R}\times\mathrm{R})}[{}^1P_1^{(1)}]$ and $\mathcal{B}^{(0)}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_0^{(1)}]$ where the quark-antiquark pair is in a P-wave state. Additionally we expect the coefficients $\mathcal{B}^{(0)}_{(\mathrm{C}\times\mathrm{C})}[{}^1S_0^{(1)}]$ and $\mathcal{B}^{(0)}_{(\mathrm{C}\times\mathrm{R})}[{}^1S_0^{(1)}]$ to survive in this limit. There the soft degrees of freedom moving relative to the quark-antiquark pair carry away some spin or angular momentum. Indeed $\mathcal{C}^{(0)}_{(\mathrm{R}\times\mathrm{R})}[{}^1S_0^{(1)}]$ is the only piece of $\mathcal{B}_0$ that is proportional to ε:

$$\mathcal{B}^{(0)}_{(\mathrm{C}\times\mathrm{C})}[{}^1S_0^{(1)}] = \frac{2(6-8\eta^2+3\eta^4)}{3} - \varepsilon\left[1+\eta^2+2(3-6\eta^2+2\eta^4)\varepsilon - \frac{3-5\eta^2}{3}\varepsilon^2\right], \tag{49a}$$

$$\mathcal{B}^{(0)}_{(\mathrm{C}\times\mathrm{R})}[{}^1S_0^{(1)}] = \frac{8\eta(2-\eta^2)}{3} - 8\eta\varepsilon\left[1-\varepsilon+\frac{2-\eta^2}{3}\varepsilon^2\right], \tag{49b}$$

$$\mathcal{C}^{(0)}_{(\mathrm{R}\times\mathrm{R})}[{}^1S_0^{(1)}] = -4\varepsilon\left[1-2(2-\eta^2)\varepsilon+(3-2\eta^2)\varepsilon^2\right], \tag{49c}$$

$$\mathcal{B}^{(0)}_{(\mathrm{R}\times\mathrm{R})}[{}^1P_1^{(1)}] = \frac{8\eta^2}{3} - \frac{4\eta^2}{3}\varepsilon\left[3-\varepsilon^2\right], \tag{49d}$$

$$\mathcal{B}^{(0)}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_0^{(1)}] = 8 - 4\varepsilon\left[3(1-\eta^2)+6\eta^2\varepsilon-(1+3\eta^2)\varepsilon^2\right]. \tag{49e}$$

Note that the terms odd in η changes their sign if we consider $B_c^+ \to X\ell^+\nu_\ell$ instead of $B_c^- \to X\ell\bar{\nu}_\ell$. However, this does not mean that the result violates CP invariance, since the sign of the operator $\mathcal{O}_{(\mathrm{C}\times\mathrm{R})}[{}^1S_0^{(C)}]$ also changes. Furthermore we have used heavy quark spin symmetry to summarize the P wave channels ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$:

$$\langle B_c|\,\mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_J^{(1)}]\,|B_c\rangle = (2J+1)\,\langle B_c|\,\mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_0^{(1)}]\,|B_c\rangle\ . \tag{50}$$

Next we deal with the contributions $\mathcal{B}_1$ and $\mathcal{B}_2$. As mentioned above they become relevant close to the endpoint since they are more singular than the leading contribution. Once the δ function and its derivatives are resummed we will find a shape function enhancement close to the endpoint. For reasons which will become clear in a moment we also keep the $\mathcal{B}_1$ pieces stemming from $\mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}]$ and $\mathcal{P}_{(\mathrm{C}\times\mathrm{R})}[^3S_1^{(1)}]$:

$$\begin{aligned}\mathcal{B}_1 = {} & \mathcal{B}^{(1)}_{(\mathrm{C}\times\mathrm{C})}[^1S_0^{(1)}]\,\langle B_c|\,\mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[^1S_0^{(1)}]\,|B_c\rangle + \mathcal{B}^{(1)}_{(\mathrm{C}\times\mathrm{R})}[^1S_0^{(1)}]\,\langle B_c|\,\mathcal{O}_{(\mathrm{C}\times\mathrm{R})}[^1S_0^{(1)}]\,|B_c\rangle \\ & + \mathcal{C}^{(1)}_{(\mathrm{R}\times\mathrm{R})}[^1S_0^{(1)}]\,\langle B_c|\,\mathcal{P}_{(\mathrm{R}\times\mathrm{R})}[^1S_0^{(1)}]\,|B_c\rangle + \mathcal{B}^{(1)}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}]\,\langle B_c|\,\mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}]\,|B_c\rangle \\ & + \mathcal{B}^{(1)}_{(\mathrm{C}\times\mathrm{R})}[^3S_1^{(1)}]\,\langle B_c|\,\mathcal{O}_{(\mathrm{C}\times\mathrm{R})}[^3S_1^{(1)}]\,|B_c\rangle + \mathcal{C}^{(1)}_{(\mathrm{R}\times\mathrm{R})}[^3S_1^{(1)}]\,\langle B_c|\,\mathcal{P}_{(\mathrm{R}\times\mathrm{R})}[^3S_1^{(1)}]\,|B_c\rangle\ .\end{aligned} \tag{51}$$

While the spin singlet contributions are still helicity suppressed the coefficients corresponding to a quark-antiquark pair in a spin triplet state do not vanish for $\varepsilon \to 0$:

$$\mathcal{B}^{(1)}_{(\mathrm{C}\times\mathrm{C})}[^1S_0^{(1)}] = \frac{1-\eta^2}{6}\varepsilon\left[13 - 12\eta^2 - 3(7-6\eta^2)\varepsilon + 3\varepsilon^2 - (11-10\eta^2)\varepsilon^3\right], \tag{52a}$$

$$\mathcal{B}^{(1)}_{(\mathrm{C}\times\mathrm{R})}[^1S_0^{(1)}] = -\frac{2\eta(1-\eta^2)}{3}\varepsilon\left[1-\varepsilon\right]^3, \tag{52b}$$

$$\mathcal{C}^{(1)}_{(\mathrm{R}\times\mathrm{R})}[^1S_0^{(1)}] = -2(1-\eta^2)\varepsilon\left[1-\varepsilon\right]^3, \tag{52c}$$

$$\begin{aligned}\mathcal{B}^{(1)}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}] = {} & -\frac{1-\eta^2}{3} + \frac{1-\eta^2}{30}\varepsilon\big[29 + \eta^2 - 15(1+\eta^2)\varepsilon \\ & \qquad - (17-27\eta^2)\varepsilon^2 - 15(1-\eta^2)\varepsilon^3\big],\end{aligned} \tag{52d}$$

$$\mathcal{B}^{(1)}_{(\mathrm{C}\times\mathrm{R})}[^3S_1^{(1)}] = -\frac{4\eta(1-\eta^2)}{3} + \frac{\eta(1-\eta^2)}{3}\varepsilon\left[11 - 9\varepsilon + \varepsilon^2 + \varepsilon^3\right], \tag{52e}$$

$$\mathcal{C}^{(1)}_{(\mathrm{R}\times\mathrm{R})}[^3S_1^{(1)}] = -\frac{4(1-\eta^2)}{3} + \frac{2(1-\eta^2)}{3}\varepsilon\left[5 - 3\varepsilon - \varepsilon^2 + \varepsilon^3\right]. \tag{52f}$$

Finally we quote the results for the coefficient $\mathcal{B}^{(2)}$. In that case we only find contributions to RCM $\times$ RCM operators:

$$\begin{aligned}\mathcal{B}_2 = {} & \mathcal{B}^{(2)}_{(\mathrm{C}\times\mathrm{C})}[^1S_0^{(1)}]\,\langle B_c|\,\mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[^1S_0^{(1)}]\,|B_c\rangle + \mathcal{B}^{(2)}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}]\,\langle B_c|\,\mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}]\,|B_c\rangle \\ & + \mathcal{C}^{(2)}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}]\,\langle B_c|\,\mathcal{P}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}]\,|B_c\rangle\end{aligned} \tag{53}$$

where

$$\mathcal{B}^{(2)}_{(\mathrm{C}\times\mathrm{C})}[^1S_0^{(1)}] = \frac{(1-\eta^2)^2}{6}\varepsilon\left[1-\varepsilon^2\right]^2, \tag{54a}$$

$$\mathcal{B}^{(2)}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}] = \frac{7(1-\eta^2)^2}{60} - \frac{(1-\eta^2)^2}{60}\varepsilon\left[25 - 44\varepsilon + 38\varepsilon^2 - 9\varepsilon^3 - 3\varepsilon^4\right], \tag{54b}$$

$$\mathcal{C}^{(2)}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}] = -\frac{(1-\eta^2)^2}{15} + \frac{(1-\eta^2)^2}{15}\varepsilon\left[5 - 12\varepsilon + 14\varepsilon^2 - 7\varepsilon^3 + \varepsilon^4\right]. \tag{54c}$$

Again only the spin singlet pieces show helicity suppression. However, this feature of the coefficients of higher order derivatives of the delta function does not surprise since it is given by the phase space structure of the process as we will explain in the following subsection.

3.1 Shape function formalism

The spectrum within the operator product expansion is given as a series of δ functions and its derivatives and thus is not physical. However, it can be interpreted within the shape function formalism. As mentioned before the NRQCD expansion breaks down for lepton energies to close to the endpoint. The physical reason for this breakdown comes from the lack of phase space for radiating off soft gluons if the lepton energy reaches it maximum value. Hence in the endpoint region the non-perturbative subprocess $B_c \to (\bar{b}c) +$ *soft degrees of freedom* has to be parametrized by so-called shape functions reflecting the phase space dependence rather than just by numbers like the NRQCD matrix elements.

The formal construction of the shape functions in B_c decays proceeds analogously to the one in quarkonia decays [17]. The phase space of the short distance subprocess is given by

$$d\mathrm{PS} = \widetilde{dp_\ell}\widetilde{dp_{\bar{\nu}_\ell}}(2\pi)^4\delta^4(P + l - p_\ell - p_{\bar{\nu}_\ell}) \tag{55}$$

where $P = M_{b\bar{c}}v$ and l are the main and the residual cms momenta of the heavy quark-antiquark system in the meson rest frame, respectively. If we integrate over $d^4p_{\bar{\nu}_\ell}$ and expand the lepton energy spectrum in the small residual cms momentum we recognize that only the light cone component $l_+ = \hat{n}\cdot l$ is relevant for the soft gluon dynamics. Here the light cone vector is given by $\hat{n} = 2(P - p_\ell)/M_{b\bar{c}}$. Then the lepton energy spectrum reads[2]

$$\frac{d\Gamma}{dE_\ell} = \sum_n \mathcal{C}[n] \int dl_+\, f_{B_c}[n](l_+)\, \delta(M_{b\bar{c}}/2 - E_\ell + l_+/2) \tag{56}$$

where the shape functions are defined via

$$f_{B_c}[n](l_+) = \langle B_c|\left[\psi^\dagger\mathcal{K}_n\chi\right]\delta(l_+ - iD_+)\left[\chi^\dagger\mathcal{K}'_n\psi\right]|B_c\rangle\ . \tag{57}$$

As for the matrix elements there is one shape function per each $(b\bar{c})$ configuration indicated by the kernels $\mathcal{K}_n$ and $\mathcal{K}'_n$ in (57). In case of spin singlet configurations vacuum saturation is applicable and the covariant derivative iD_+ reduces to the time component which picks up the binding energy $M - M_{b\bar{c}}$ of the B_c. Thus the shape function shifts the endpoint of the lepton energy spectrum from the unphysical partonic value $M_{b\bar{c}}/2$ to the physical $M/2$ expressed by the hadronic B_c mass.

Since the shape function resums purely kinematical effects originating from the phase space the short distance coefficients $\mathcal{C}[n]$ in eq. (56) are not affected at all. This means particularly that the coefficient (43) of the shape function $f_{B_c}[^1S_0^{(1)}]$ is still helicity suppressed while the coefficient (44) of the spin triplet shape function $f_{B_c}[^3S_1^{(1)}]$ do not vanish for $\varepsilon \to 0$.

After changing to our four-dimensional notation and expanding the shape functions in the residual momentum $iD_+/M_{b\bar{c}}$ we reobtain the spectrum in terms of delta functions

[2] We ignore the lepton mass m_ℓ in this discussion.

and their derivatives like eq. (39). Acting on the spin triplet structure of the dimension-6 operator $\mathcal{O}_{(1\times1)}[^3S_1^{(1)}]$ the zeroth component of the covariant derivative generates the following dimension-8 operators (cf. eq. (47)):

$$
\begin{aligned}
&\frac{1}{2}\left\{\left[\bar{h}_v^{(+)}\,\gamma_\mu^\perp\,h_v^{(-)}\right]\left[\frac{(ivD)}{M_{b\bar{c}}}\big(\bar{h}_v^{(-)}\,\gamma_\perp^\mu\,h_v^{(+)}\big)\right]+\text{h.c.}\right\} \\
&\qquad=\frac{1}{4m^2}\left(-\frac{1-\eta^2}{2}\right)\left\{\mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[^3S_1^{(1)}]+4\eta\mathcal{O}_{(\mathrm{C}\times\mathrm{R})}[^3S_1^{(1)}]+4\mathcal{P}_{(\mathrm{R}\times\mathrm{R})}[^3S_1^{(1)}]\right\}
\end{aligned}
\tag{58}
$$

We re-observe the same structure as we have seen in the pieces of the coefficients (52d-52f) that remain in the limit $\varepsilon \to 0$. Actually these pieces are completely determined by the coefficient (44) of $\mathcal{O}_{(1\times1)}[^3S_1^{(1)}]$ and the expansion of the corresponding shape function. Similarly one can derive the non-vanishing pieces in (54b) and (54c) by applying $iD_\perp/M_{b\bar{c}}$ two times on $\mathcal{O}_{(1\times1)}[^3S_1^{(1)}]$.

Note that the shape function do not determine the complete coefficients $\mathcal{B}_1$ and $\mathcal{B}_2$. The pieces proportional to ε contain terms which cannot be directly derived from the shape function. Hence the structure of the spin singlet coefficients in $\mathcal{B}_1$ and $\mathcal{B}_2$ is more complicated. Furthermore we want to stress that the origin of the contributions to $\mathcal{B}_0$ which do not vanish in the limit of massless leptons is not kinematical. To explain these pieces it is not sufficient to introduce shape functions. Rather one has to consider terms that are generated dynamically by the non-perturbative physics inside the B_c taken into account in the bilinear expansion (66).

3.2 Moments of the spectrum

Another way to understand the unphysical series of delta functions and their derivatives is by the means of moments which implies some smearing [22].

The two-body structure of the decay at tree level yields a spectrum in terms of an expansion in singular functions

$$\frac{d\Gamma}{dz}=\sum_{k=0}^{\infty}\frac{M_k}{k!}\,\delta^{(k)}(1+\varepsilon-z) \tag{59}$$

where the coefficients of the expansion are given by the moments

$$M_k=\int_0^{1+\varepsilon}dz\,\frac{d\Gamma}{dz}(1+\varepsilon-z)^k\,. \tag{60}$$

A comparison of the result (39) with data can thus be performed by sampling the data into moments, for which we get a theoretical prediction. The zeroth moment M_0 is simply the total rate for which we get

$$\Gamma = \Gamma_0 \left[\mathcal{A} + \frac{1}{(2m)^2}\, \mathcal{B}_0 \right] \tag{61}$$

while the first an the second moments are at least of order $1/m^2$

$$M_1 = \frac{\Gamma_0}{(2m)^2}\, \mathcal{B}_1\,, \qquad M_2 = 2\frac{\Gamma_0}{(2m)^2}\, \mathcal{B}_2\,. \tag{62}$$

Hence measuring the second moments of the lepton energy spectra in the semi-leptonic B_c decay provides direct access on the order of magnitude of the RCM $\times$ RCM sector. The fact that the values for the matrix elements do not depend on the type of the lepton in the final state may help to improve the accuracy of this determination. Likewise measurements of the first and zeroth moments may provide some information for the matrix elements of the other operators.

3.3 Total rate in the limit $m_\ell \to 0$

Finally we discuss the limit of vanishing lepton mass. In this limit only subleading terms in $\bar{v}$ survive since the leading term has to vanish due to helicity arguments. On the other hand, this limit is certainly a good approximation in the case of the electron spectrum.

For $\varepsilon \to 0$ non-vanishing contribution to the lepton energy spectrum (39) contains a piece which is proportional to $\delta(1-z)$:

$$\begin{aligned} \frac{d\Gamma}{dz} = \Gamma_0 \Bigg\{ \Bigg(& \frac{2}{3} \langle B_c |\, \mathcal{O}_{(1\times 1)}[{}^3S_1^{(1)}]\, | B_c \rangle + \frac{1}{6m^2} \Big[(6 - 8\eta^2 + 3\eta^4) \, \langle \mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[{}^1S_0^{(1)}] \rangle \\ & + 4\eta(2-\eta^2)\, \langle \mathcal{O}_{(\mathrm{C}\times\mathrm{R})}[{}^1S_0^{(1)}] \rangle + 4\eta^2 \, \langle \mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^1P_1^{(1)}] \rangle + 12\, \langle \mathcal{O}_{(\mathrm{R}\times\mathrm{R})}[{}^3P_0^{(1)}] \rangle \Big] \Bigg) \delta(1-z) \\ & - \frac{1-\eta^2}{12m^2} \Big[\langle \mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[{}^3S_1^{(1)}] \rangle + 4\eta\, \langle \mathcal{O}_{(\mathrm{C}\times\mathrm{R})}[{}^3S_1^{(1)}] \rangle + 4\, \langle \mathcal{P}_{(\mathrm{R}\times\mathrm{R})}[{}^3S_1^{(1)}] \rangle \Big] \, \delta'(1-z) \\ & + \frac{(1-\eta^2)^2}{60m^2} \Big[7\, \langle \mathcal{O}_{(\mathrm{C}\times\mathrm{C})}[{}^3S_1^{(1)}] \rangle - 4\, \langle \mathcal{P}_{(\mathrm{C}\times\mathrm{C})}[{}^3S_1^{(1)}] \rangle \Big] \, \delta''(1-z) \Bigg\} \end{aligned} \tag{63}$$

This piece will contribute to the total rate of the semi-leptonic decay $B_c \to X \ell \bar{\nu}_\ell$ even if the energy and momentum of the light hadrons in the final state are extremely soft. Independent of the exact values of the counting parameters κ, λ, and $\bar{v}$ this piece should be sizable whereas the purely leptonic B_c decay rate (1) vanishes for $m_\ell \to 0$. Consequently we conclude that the semi-leptonic rate exceeds the leptonic one by orders of magnitude for $\ell = e$ while it at least reaches it in the muon case (cf. eq. 45). Since we expect the light hadrons in the final state to be rather soft the experimental search for the exclusive channel $B_c \to \ell \bar{\nu}_\ell$ become very difficult unless the matrix elements in (63) are unnaturally small.

4 Conclusions

We have investigated the semi-leptonic decay $B_c \to X \ell \bar{\nu}_\ell$ where X denotes light hadrons below the D^0 threshold. This decay provides a unique testing ground in quarkonia physics, since the two heavy quarks have to annihilate through weak interaction and hence one may investigate in this way the light degrees of freedom in a quarkonium. The total semi-leptonic rate into light hadrons does not show any helicity suppression at subleading order and thus can exceed the purely leptonic one for light leptons, i.e. for electrons and possibly also for muons. Thus the measurement of the leptonic B_c decay rate could become a quite complicated maybe even infeasible experimental task.

The B_c is an intermediate case between the Υ and the J/ψ. While NRQCD is believed to be a reasonable approximation at least for the low-lying states of the Υ, the non-relativistic approximation becomes questionable for the J/ψ. This point is of relevance for power counting, since the usual NRQCD power counting may not be valid for the J/ψ. This will also affect the contributions related to the centre-of-mass movement of the heavy quark-antiquark pair inside a quarkonia-like bound state. For this reason we propose a slightly generalized power counting scheme with respect to standard NRQCD velocity rules taking care of possible contributions from the cms-motion.

We have shown that the leptonic energy spectrum provides a testing ground for estimating the relevance of such contributions. Measurements of the moments of the spectrum may also help to clarify the power counting rules for spin-flip transitions which are a recent subject of investigation in charmonia physics.

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A Bilinears

To expand the currents $\bar{Q}_v^{(\pm)}\,\Gamma\,Q_v^{(\mp)}$ we use the field expansion that can be read off if the effective Lagrangian (3) is compared with the full QCD Lagrangian. Additionally considering eq. (11) and the equations of motion (10) this yields

$$Q_v^{(\pm)}(x) = \left(1 + \frac{1}{2m_\pm}\, i\not{D}_\perp + \frac{1}{8m_\pm^2}\,(iD_\perp)^2\right) h_v^{(\pm)}(x) + \mathcal{O}\left(\frac{1}{m_\pm^3}\right). \tag{64}$$

Instead of using the masses of the quark (m_+) and the antiquark (m_-) we present the expansion in terms of the effective mass m and the dimensionless parameter η parameterizing the mass difference:

$$m := \frac{2m_+m_-}{m_+ + m_-} \qquad \text{and} \qquad \eta := \frac{m_+ - m_-}{m_+ + m_-} \tag{65}$$

Then the bilinear expansion up to $\mathcal{O}(1/m^2)$ reads:

$$\begin{aligned}
\bar{Q}_v^{(+)}\,\Gamma\,Q_v^{(-)} = {} & \bar{h}_v^{(+)}\,\Gamma\,h_v^{(-)} \\
& + \frac{1}{4m}\,\Big\{\, i\partial_\perp^\mu\left(\bar{h}_v^{(+)}\,[\Gamma,\gamma_\mu]\,h_v^{(-)}\right) + \bar{h}_v^{(+)}\,\{\Gamma, i\overleftrightarrow{\not{D}}_\perp\}\,h_v^{(-)} \\
& \qquad + \eta\left[i\partial_\perp^\mu\left(\bar{h}_v^{(+)}\,\{\Gamma,\gamma_\mu\}\,h_v^{(-)}\right) + \bar{h}_v^{(+)}\,[\Gamma, i\overleftrightarrow{\not{D}}_\perp]\,h_v^{(-)}\right]\Big\} \\
& + \left(\frac{1}{4m}\right)^2\Big\{(1+\eta^2)\left[(i\partial_\perp)^2\left(\bar{h}_v^{(+)}\,\Gamma\,h_v^{(-)}\right) + \bar{h}_v^{(+)}\,\Gamma(i\overleftrightarrow{D}_\perp)^2\,h_v^{(-)}\right] \\
& \qquad\qquad - (1-\eta^2)\left[i\partial_\perp^\mu i\partial_\perp^\nu\left(\bar{h}_v^{(+)}\,\frac{1}{2}\{\gamma_\mu\Gamma\gamma_\nu + \gamma_\nu\Gamma\gamma_\mu\}\,h_v^{(-)}\right)\right. \\
& \qquad\qquad\qquad \left. - \bar{h}_v^{(+)}\frac{1}{2}\{\gamma_\mu\Gamma\gamma_\nu + \gamma_\nu\Gamma\gamma_\mu\} i\overleftrightarrow{D}_\perp^\mu i\overleftrightarrow{D}_\perp^\nu\,h_v^{(-)}\right] \\
& \qquad\qquad + 4\,\eta\; i\partial_\perp^\mu\left(\bar{h}_v^{(+)}\,\Gamma i\overleftrightarrow{D}_\mu^\perp\,h_v^{(-)}\right)\Big\} + \mathcal{O}\left(\frac{1}{m^3}\right).
\end{aligned} \tag{66}$$

In the case of colour octet case ($\bar{Q}_v^{(+)}\,\Gamma\,T^a\,Q_v^{(-)}$):

- insert T^a into the currents $\bar{h}_v^{(+)} \ldots h_v^{(-)}$
- replace $i\partial_\perp^\mu$ by the covariant derivative $iD_\perp^\mu$

Finally the corresponding formula for $\bar{Q}_v^{(-)}\,\Gamma C\,Q_v^{(+)}$ is obtained by the replacement $\eta \to -\eta$.

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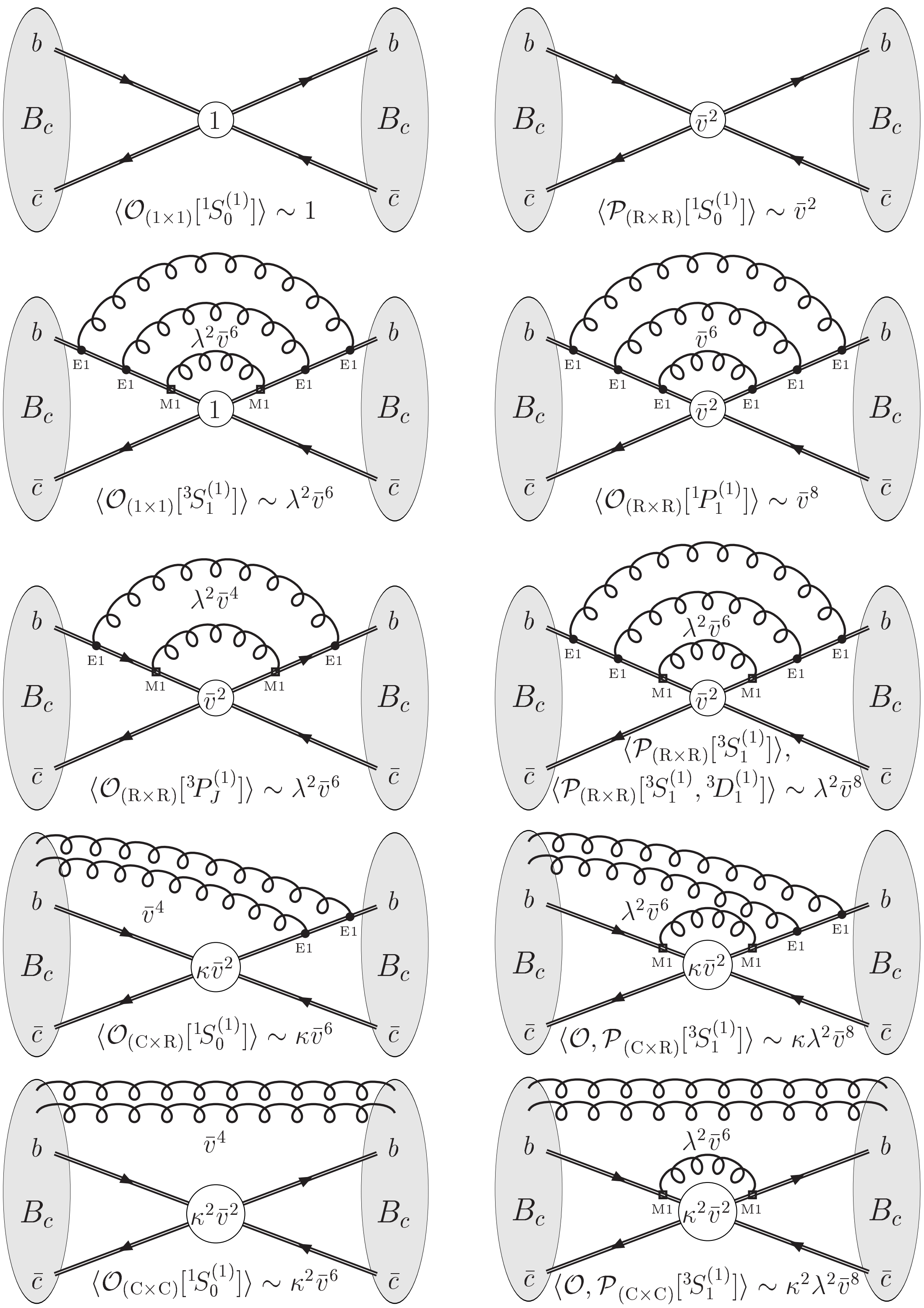

Figure 1: Scaling rules for B_c matrix elements of operators up to dimension-8. The operator power counting is denoted inside the central circle, the Fock state suppression above.