New predictions for inclusive heavy-quarkonium P-wave decays

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We show that some NRQCD colour-octet matrix elements can be written in terms of (derivatives of) wave functions at the origin and non-perturbative universal constants once the factorization between the soft and ultrasoft scale is achieved by using an effective field theory where only ultrasoft degrees of freedom are kept as dynamical entities. This allows us to derive a new set of relations between inclusive heavy-quarkonium P-wave decays into light hadrons with different principal quantum number and with different heavy flavour. In particular, we can estimate the branching ratios of bottomonium P-wave states by using charmonium data.

Inclusive P-wave decays to light hadrons have proved to be an optimal testing ground of our understanding of heavy quarkonia. The use of NRQCD [1,2] allowed a description of these decays in terms of expectation values of some 4-heavy-quark operators at a quantum-field level in a systematic way. Besides the so-called colour-singlet operators, for which their expectation values could be related to wave functions in an intuitive way, there were also colour-octet operators. The latter were decisive in solving the infrared sensitivity of earlier calculations [3]. It has been thought so far that these colour-octet expectation values could not be related with a Schrödinger-like formulation in any way.

We show in this letter that it is not so. For certain states, the expectation values of colour-octet operators can also be written in terms of wave functions and additional bound-state-independent non-perturbative parameters. We shall focus on the operators relevant to P-wave decays into light hadrons, but it should become apparent that this is a general feature.

The line of developments that has led us to this result is the following. It was pointed out in Ref. [4] that NRQCD still contains dynamical scales, which are not relevant to the kinematical situation of the lower-lying states in heavy quarkonium (energy scales larger than the ultrasoft scale: mv^2, v being the relative velocity of the heavy quark and m its mass). Hence, further simplifications occur if we integrate them out. We call pNRQCD the resulting effective field theory (as in [5]; note that in [6], in the situation $\Lambda_{\rm QCD} \gg mv^2$, the EFT was called pNRQCD$'$). When the typical scale of non-perturbative physics, say $\Lambda_{\rm QCD}$, is smaller than the soft scale mv, and larger than the ultrasoft scale mv^2, the soft scale can be integrated out perturbatively. This leads to an intermediate EFT that contains, besides the singlet, also octet fields and ultrasoft gluons as dynamical degrees of freedom [4,6]. These are eventually integrated out by the (non-perturbative) matching to pNRQCD [6]. When $\Lambda_{\rm QCD}$ is of the order of the soft scale, the (non-perturbative) matching to pNRQCD has to be done in one single step. This framework has been developed in a systematic way in Ref. [5].

In this letter we will compute the inclusive P-wave decay widths into light hadrons at leading order for $\Lambda_{\rm QCD} \gg mv^2$ by using pNRQCD. In this situation the singlet is the only dynamical field in pNRQCD (Goldstone bosons are also present, but they play a negligible role in the present analysis and will be ignored), if hybrids and other degrees of freedom associated with heavy–light meson pair threshold production develop a mass gap of $\mathcal{O}(\Lambda_{\rm QCD})$, as we will assume in what follows [6,5], or if they play a minor role in the heavy-quarkonium dynamics. Therefore, the pNRQCD Lagrangian reads [6,5]

$$\mathcal{L}_{\rm pNRQCD} = {\rm Tr}\left\{ {\rm S}^\dagger \left(i\partial_0 - h \right) {\rm S} \right\}, \qquad (1)$$

where h is the pNRQCD Hamiltonian, to be determined by matching the EFT to NRQCD. The total decay width of the singlet heavy quarkonium state is then given by

$$\Gamma = -2\,{\rm Im}\,\langle n,L,S,J|h|n,L,S,J\rangle, \qquad (2)$$

where $|n,L,S,J\rangle$ are the eigenstates of the Hamiltonian h. The imaginary parts are inherited from the 4-heavy-fermion NRQCD Wilson coefficients and, for P-wave decays, first appear as local (delta-like)

$\mathcal{O}(1/m^4)$ potentials in the pNRQCD Lagrangian. The relevant structure reads (we shall concentrate on potentials, which inherit imaginary parts from the NRQCD operators and which contribute to P-wave states at first order in quantum-mechanical perturbation theory (QMPT)):

$$-2\,\mathrm{Im}\, h\Big|_{\mathrm{P-wave}} = F_{SJ}\mathcal{T}^{ij}_{SJ}\frac{\boldsymbol{\nabla}^i_{\mathbf{r}}\delta^{(3)}(\mathbf{r})\boldsymbol{\nabla}^j_{\mathbf{r}}}{m^4}\,, \qquad (3)$$

where $\mathcal{T}^{ij}_{SJ}$ corresponds to the spin and total angular momentum wave-function projectors. What is now left is to compute F_{SJ}, i.e. to perform the matching between NRQCD and pNRQCD. For the situation A): when $mv \gg \Lambda_{\rm QCD} \gg mv^2$, by taking the results of [6], and for the more general situation B): when $\Lambda_{\rm QCD} \lesssim mv$, by using the formalism of Refs. [5]. In both situations we get:

$$F_{SJ} = -2N_c\,\mathrm{Im}\, f_1(^{2S+1}\mathrm{P}_J) - \frac{4T_F}{9N_c}\,\mathcal{E}\,\mathrm{Im}\, f_8(^{2S+1}\mathrm{S}_S), \qquad (4)$$

where $f_1(^{2S+1}\mathrm{L}_J)$ and $f_8(^{2S+1}\mathrm{L}_J)$ are the short-distance Wilson coefficients of NRQCD as defined in Ref. [2] and

$$\mathcal{E} = T_F\int_0^\infty d\tau\,\tau^3\,\langle g\mathbf{E}^a(\tau,\mathbf{0})\Phi_{ab}(\tau,0;\mathbf{0})g\mathbf{E}^b(\tau,\mathbf{0})\rangle\,. \qquad (5)$$

A) P-wave potentials for $mv \gg \Lambda_{\rm QCD} \gg mv^2$. In this case the matching from NRQCD to pNRQCD at the scale $\Lambda_{\rm QCD}$ can be done in two steps. In the first step, which can be done perturbatively, we integrate out the scale mv and end up with an EFT, which contains singlet (S) and octet (O) fields as dynamical degrees of freedom. At the next-to-leading order in the multipole expansion the Lagrangian reads [4,6]:

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}\left\{\mathrm{S}^\dagger\,(i\partial_0 - h_s)\,\mathrm{S} + +\mathrm{O}^\dagger\,(iD_0 - h_o)\,\mathrm{O}\right\}\\ &+\mathrm{Tr}\left\{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{S} + \mathrm{h.c.} + \frac{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{O}}{2} + \frac{\mathrm{O}^\dagger\mathrm{O}\mathbf{r}\cdot g\mathbf{E}}{2}\right\}\\ &-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a}\,,\end{aligned} \qquad (6)$$

where h_s and h_o have to be determined by matching to NRQCD. They read as follows

$$\begin{aligned} h_s &= -\frac{\boldsymbol{\nabla}^2_{\mathbf{r}}}{m} + V_s(r) + \ldots\\ &+ N_c\, f_1(^{2S+1}\mathrm{P}_J)\mathcal{T}^{ij}_{SJ}\frac{\boldsymbol{\nabla}^i_{\mathbf{r}}\delta^{(3)}(\mathbf{r})\boldsymbol{\nabla}^j_{\mathbf{r}}}{m^4} + \ldots\\ h_o &= -\frac{\boldsymbol{\nabla}^2_{\mathbf{r}}}{m} + V_o(r) + \ldots\\ &+ T_F\, f_8(^{2S+1}\mathrm{S}_S)\mathcal{T}_S\frac{\delta^{(3)}(\mathbf{r})}{m^2} + \ldots\,,\end{aligned} \qquad (7)$$

neglecting centre-of-mass recoil terms; $\mathcal{T}_S$ corresponds to the total spin projector. Beyond $\mathcal{O}(1/m)^0$ we have only displayed the terms that are relevant to our calculation. In the second step we integrate out (non-perturbatively) the gluons and the octet field ending up with the pNRQCD Lagrangian (1). The Hamiltonian h has to be determined by matching the two effective field theories. It reads $h = h_s + \delta h_s$, with (at leading non-vanishing order in the multipole expansion)

$$\begin{aligned}\delta h_s &= -i\frac{T_F}{N_c}\int_0^\infty d\tau\, e^{ih_s\frac{\tau}{2}}\,\langle \mathbf{r}\cdot g\mathbf{E}^a(\tau,\mathbf{0})\\ &e^{-ih_o\tau}\,\Phi_{ab}(\tau,0;\mathbf{0})\,g\mathbf{E}^b(0,\mathbf{0})\cdot\mathbf{r}\rangle\, e^{ih_s\frac{\tau}{2}}\,,\end{aligned} \qquad (8)$$

where consistency with $\Lambda_{\rm QCD} \gg mv^2$ requires an expansion of the exponentials of h_o and h_s. Taking into account that we are interested in P-wave states, only the perturbation that puts one h_o to each side of the $\mathcal{O}(1/m^2)$ S-wave potential survives at leading order. The final result reads:

$$\mathrm{Im}\,\delta h_s\Big|_{\mathrm{P-wave}} = \frac{2T_F}{9N_c}\mathcal{E}\,\frac{\boldsymbol{\nabla}_{\mathbf{r}}\delta^{(3)}(\mathbf{r})\boldsymbol{\nabla}_{\mathbf{r}}}{m^4}\mathcal{T}_S\,\mathrm{Im} f_8(^{2S+1}\mathrm{S}_S) \qquad (9)$$

which plugged into Eq. (3) gives Eq. (4). This shows how a colour-octet operator in NRQCD becomes a colour-octet potential in the EFT of Eq. (6) and, eventually, contributes to a colour-singlet potential in pNRQCD, which is one of our main points.

B) P-wave potentials for $\Lambda_{\rm QCD} \lesssim mv$. In the case $\Lambda_{\rm QCD} \lesssim mv$ the matching from NRQCD to pNRQCD at the scale $\Lambda_{\rm QCD}$ has to be done directly, since no other relevant scales are supposed to lie between m and mv. The only dynamical degree of freedom of pNRQCD is the heavy-quarkonium singlet field S. The Lagrangian has been written in (1). The Hamiltonian h is obtained by matching (non-perturbatively) to NRQCD, order by order in $1/m$, within a Hamiltonian formalism [5]. In this letter we only sketch the main steps of the derivation. In short, we can formally expand the NRQCD Hamiltonian in $1/m$:

$$H_{\rm NRQCD} = H^{(0)}_{\rm NRQCD} + \frac{1}{m}H^{(1)}_{\rm NRQCD} + \cdots\,. \qquad (10)$$

The eigenstates of the heavy-quark-antiquark sector can be labelled as

$$|\underline{g};\mathbf{x}_1,\mathbf{x}_2\rangle = |\underline{g};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} + \frac{1}{m}|\underline{g};\mathbf{x}_1,\mathbf{x}_2\rangle^{(1)} + \cdots,$$

where $\underline{g}$ labels the colour-related degrees of freedom (we do not explicitly display spin labels for simplicity). Assuming a mass gap of $\mathcal{O}(\Lambda_{\rm QCD})$ much larger than mv^2, all the excitations ($\underline{g} \neq \underline{0}$) decouple and the ground state ($\underline{g} = \underline{0}$) corresponds to the singlet state. Therefore, the matching condition reads

$$\begin{aligned}&\langle \underline{0}; \mathbf{x}_1, \mathbf{x}_2 | H | \underline{0}; \mathbf{x}_1', \mathbf{x}_2' \rangle \qquad (11)\\ &= h(\mathbf{x}_1, \mathbf{x}_2, \boldsymbol{\nabla}_{\mathbf{x}_1}, \boldsymbol{\nabla}_{\mathbf{x}_2}) \delta^{(3)}(\mathbf{x}_1 - \mathbf{x}_1') \delta^{(3)}(\mathbf{x}_2 - \mathbf{x}_2').\end{aligned}$$

Up to $\mathcal{O}(1/m^4)$ the imaginary contributions are only carried by the Wilson coefficients of the dimension 6 and 8 4-heavy-fermion operators in NRQCD. Since we are only interested in Eq. (3) a huge simplification occurs and only two contributions survive. From the dimension 8 operators we obtain

$$\begin{aligned}&\mathrm{Im}\, \delta h \, \delta^{(3)}(\mathbf{x}_1 - \mathbf{x}_1') \delta^{(3)}(\mathbf{x}_2 - \mathbf{x}_2') \qquad (12)\\ &= \frac{1}{m^4} \mathrm{Im}\, {}^{(0)}\langle \underline{0}; \mathbf{x}_1, \mathbf{x}_2 | H^{(4)}_{\mathrm{NRQCD}} | \underline{0}; \mathbf{x}_1', \mathbf{x}_2' \rangle^{(0)} \Big|_{\mathrm{P-wave}} \\ &= N_c \, \mathcal{T}^{ij}_{SJ} \, \mathrm{Im}\, f_1({}^{2S+1}\mathrm{P}_J) \frac{\boldsymbol{\nabla}^i_{\mathbf{r}} \delta^{(3)}(\mathbf{r}) \boldsymbol{\nabla}^j_{\mathbf{r}}}{m^4} \\ &\quad \times \delta^{(3)}(\mathbf{x}_1 - \mathbf{x}_1') \delta^{(3)}(\mathbf{x}_2 - \mathbf{x}_2').\end{aligned}$$

On the other hand, we also have contributions from the iteration of lower-order $1/m$ corrections to the NRQCD Hamiltonian with the dimension 6 4-heavy-fermion operators. The only term that contributes to Eq. (3) is

$$\begin{aligned}&\mathrm{Im}\, \delta h \, \delta^{(3)}(\mathbf{x}_1 - \mathbf{x}_1') \delta^{(3)}(\mathbf{x}_2 - \mathbf{x}_2') \qquad (13)\\ &= \frac{1}{m^4} \mathrm{Im}\, {}^{(1)}\langle \underline{0}; \mathbf{x}_1, \mathbf{x}_2 | H^{(2)}_{\mathrm{NRQCD}} | \underline{0}; \mathbf{x}_1', \mathbf{x}_2' \rangle^{(1)} \Big|_{\mathrm{P-wave}}.\end{aligned}$$

The explicit computation of the right-hand side of Eq. (13) gives (as far as the P-wave contribution is concerned) Eq. (9). Therefore, the sum of the contributions from Eqs. (12) and (13) coincides with Eq. (3), after the replacements (4) and (5).

We can now obtain the decay widths by using Eq. (3). At first order in QMPT, we obtain :

$$\begin{aligned}&\Gamma(\chi^S_{QJ}(n\mathrm{P}) \to \mathrm{LH}) = \Big[\frac{3N_c}{\pi} \mathrm{Im}\, f_1({}^{2S+1}\mathrm{P}_J) + \frac{2T_F}{3\pi N_c} \\ &\mathrm{Im}\, f_8({}^{2S+1}\mathrm{S}_S)\, \mathcal{E} \Big] \frac{|R'_{Qn1}(0)|^2}{m^4}, \qquad (14)\end{aligned}$$

where $\chi^1_{QJ}(n\mathrm{P}) := \chi_{QJ}(n\mathrm{P})$, $\chi^0_{QJ}(n\mathrm{P}) := h_Q(n\mathrm{P})$ ($Q = b, c$), n is the principal quantum number, and $R_{Qn1}(r)$ the radial wave function at leading order. Comparing with the standard NRQCD formula, where spin symmetry has already been used, we have:

$$\langle h_Q(n\mathrm{P}) | O_8({}^1\mathrm{S}_0) | h_Q(n\mathrm{P}) \rangle(\mu) = \frac{|R'_{Qn1}(0)|^2}{3\pi N_c m^2} T_F \mathcal{E}(\mu). \qquad (15)$$

The information gained with this formula is that all non-perturbative flavour and principal quantum number dependence is encoded in the wave function, as in the colour singlet operators. The additional non-perturbative parameter $\mathcal{E}(\mu)$ is universal: it only depends on the light degrees of freedom of QCD. This implies that the following relation between decay widths is also universal:

$$\begin{aligned}&\mathcal{E}(\mu) = -\frac{9N_c^2}{2T_F} \times \qquad (16)\\ &\frac{\mathrm{Im}\, f_1({}^{2S+1}\mathrm{P}_J) - \mathrm{Im}\, f_1({}^{2S'+1}\mathrm{P}_{J'}) \frac{\Gamma(\chi^S_{QJ}(n\mathrm{P}) \to \mathrm{LH})}{\Gamma(\chi^{S'}_{QJ'}(n\mathrm{P}) \to \mathrm{LH})}}{\mathrm{Im}\, f_8({}^{2S+1}\mathrm{S}_S) - \mathrm{Im}\, f_8({}^{2S'+1}\mathrm{S}_{S'}) \frac{\Gamma(\chi^S_{QJ}(n\mathrm{P}) \to \mathrm{LH})}{\Gamma(\chi^{S'}_{QJ'}(n\mathrm{P}) \to \mathrm{LH})}}.\end{aligned}$$

It is interesting to notice that the UV behaviour of $\mathcal{E}$ has the logarithmic divergence:

$$\mathcal{E}(\mu) \simeq 12\, N_c C_F \frac{\alpha_s}{\pi} \ln \mu, \qquad (17)$$

which matches exactly the IR log of the $\mathcal{O}(\alpha_s)$ correction of $\mathrm{Im}\, f_1({}^{2S+1}\mathrm{P}_J)$, and hence the cancellation originally observed in [1] is fulfilled. Then one could consider the LL RG improvement of $\mathcal{E}$ by using the results of Ref. [2] for the running of the octet-matrix element. One obtains ($\beta_0 = 11C_A/3 - 4n_f T_F/3$):

$$\mathcal{E}(\mu) = \mathcal{E}(\mu') + \frac{24 N_c C_F}{\beta_0} \ln \frac{\alpha_s(\mu')}{\alpha_s(\mu)}. \qquad (18)$$

Let us apply the above results to actual quarkonium. For the P-wave states observed in nature, the formalism is safer for $\chi_b(1P)$ than for $\chi_b(2P)$ and $\chi_c(1P)$, since the latter are closer to threshold. Nevertheless, we will assume that our framework provides a reasonable description of all these states.

The numerical extraction of $\mathcal{E}$ is a delicate task, since several of the Wilson coefficients (see Ref. [7] for a full list of them) have large next-to-leading order contributions, which may spoil the convergence of the perturbative series. This problem is not specific of our formalism, but belongs to the standard formulation of NRQCD. Here, in order to give an estimate, we only use those data that provide more stable results in going from the leading to the next-to-leading order, more precisely the average of Eq. (16) for $(J, S) = (1, 1)$, $(J', S') = (0, 1)$, and $(J, S) = (1, 1)$, $(J', S') = (2, 1)$. The experimental data have been taken from [8] and updated accordingly to [9,10]. Our final value reads

$$\mathcal{E}(1\,\mathrm{GeV}) = 5.3^{+3.5}_{-2.2}(\mathrm{exp}), \qquad (19)$$

where we have just added the known NLO term to the LL result. The errors only refer to the experimental uncertainties on the decay widths. Theoretical uncertainties mainly come from subleading operators in the power counting ($\mathcal{O}(v)$ suppressed) and subleading terms in the perturbative expansion of the Wilson coefficients ($\mathcal{O}(\alpha_{\rm s})$ suppressed) whose bad convergence may affect considerably the figure of Eq. (19). We feel, therefore, that further studies, maybe along the lines of Refs. [11], are needed before a complete numerical analysis, including theoretical uncertainties, can be done. In any case, the above figure is compatible with the values that are usually assigned to the NRQCD octet and singlet matrix elements (e.g. from the fit of [12] one gets $\mathcal{E}(1\,{\rm GeV}) = 3.6^{+3.6}_{-2.9}({\rm exp})$). The above figure is also compatible with the charmonium (quenched) lattice data of [13], whereas, if the running (18) is taken into account, bottomonium lattice data, quenched [13] and unquenched [14], appear to give a lower value. Note that, in the language of Refs. [13,14], Eq. (15) reads $\mathcal{E}(\mu) = 81\, m_{b,c}^2 \mathcal{H}_8(\mu)/\mathcal{H}_1|_{b,c}$, which implies $\mathcal{H}_8(\mu)/\mathcal{H}_1|_b \times \mathcal{H}_1/\mathcal{H}_8(\mu)|_c = m_c^2/m_b^2$. This equality must be fulfilled by lattice results for any number of light fermions and for any values of the heavy-quark masses such that our assumptions hold.

By using the estimate (19), we can also predict the branching ratios for the $n = 1, 2$ P-wave bottomonium states. We obtain

$$\frac{\Gamma(\chi^1_{b1}(1{\rm P}))}{\Gamma(\chi^1_{b2}(1{\rm P}))} = \frac{\Gamma(\chi^1_{b1}(2{\rm P}))}{\Gamma(\chi^1_{b2}(2{\rm P}))} = 0.50^{+0.06}_{-0.04}, \qquad (20)$$

where only the errors inherited from Eq. (19) have been included. For what concerns theoretical uncertainties, the comments after Eq. (19) apply also here (with a better behaviour of the perturbative series). Note that the first equality holds independently from Eq. (19) and from the use of charmonium data and, hence, provides a more robust prediction. The remaining branching ratios can be obtained using spin symmetry. Notice also that, although no model-independent predictions can be made for the decay widths (they depend on the wave function at the origin, which is flavour and state dependent), our results allow any model that gives a definite value to $R'_{Qn1}(0)$ to make definite predictions.

In conclusion, we have exploited the fact that NRQCD still contains irrelevant degrees of freedom for certain heavy quarkonium states, which can be integrated out in order to constrain the form of the matrix elements of colour-octet operators. We have focused on the operators relevant to P-wave decays, which allowed us to produce concrete, new, rigorous results. However, it should be clear from the structure of the pNRQCD Lagrangian itself, that similar results can be obtained for matrix elements of any colour-octet operator.

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