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Developments in Deep-inelastic Structure Function Calculations

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ABSTRACT: We review recent developments in the calculation of deep-inelastic structure functions to next-to-next-to leading order in perturbative QCD. We discuss the impact of these corrections on the determination of the strong coupling α_s and the parton distributions.

KEYWORDS: QCD; Perturbative; Deep-inelastic Scattering; Structure functions.

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STRUCTURE functions in inclusive deep-inelastic scattering offer the possibility for extremely precise determinations of the strong coupling α_s and the parton distribution functions. The high statistical accuracy of the present experimental measurements, and the data expected from the electron-proton collider HERA after the luminosity upgrade, demand analyses in perturbative QCD beyond the standard next-to leading order (NLO) corrections. In order to match the experimental precision, it is therefore necessary to calculate higher order perturbative QCD corrections for the structure functions F_2, F_3 and F_L, in particular the complete next-to-next-to leading order (NNLO) corrections. This information is not fully available yet. Some time ago, the two-loop coefficient functions of F_2, F_3 and F_L have been calculated [1], and more recently, they have been completely checked [2]. However, for the three-loop anomalous dimensions $\gamma_{\mathrm{pp}}^{(2)}$, only partial results are available thus far. These include a finite number of fixed Mellin moments [3, 4, 5, 6], both for F_2 and F_3, the large n_f-limit [7] of $\gamma_{\mathrm{qq}}^{(2)}$ and $\gamma_{\mathrm{gg}}^{(2)}$, in the latter case only the coefficient of the colour factor $n_f^2 C_A$, and several terms relevant in the small-x limit [8].

1. Prospects

To explore the prospects of a complete NNLO analysis in this situation, it has been a fruitful strategy to combine all theoretical information presently available together with reasonable assumptions about the parton distributions or the functional form of the three-loop splitting functions $P_{\mathrm{pp}}^{(2)}(x)$. These investigations resulted in approximate expressions [9] for $P_{\mathrm{pp}}^{(2)}(x)$ with negligible residual uncertainties for $x \gtrsim 10^{-2}$. Subsequent studies of the NNLO evolution of parton distributions revealed a much reduced scale dependence in comparison to standard NLO analyses. Together with recent developments to account for correlated experimental errors [10] in analyses and a rigorous statistical treament [11] of parton distributions, these efforts seem to allow for quantitative estimates of the parton distribution uncertainties. Let us emphasize, that this task is of basic importance for precise luminosity measurements via W- and Z-boson production in upcoming experiments at hadron colliders, and therefore essential for future searches of the Higgs particle or effects of new physics.

The precision determination of the running coupling α_s based on the available partial information about NNLO QCD corrections has also been performed. The CCFR data for xF_3 from νN-scattering [12] has been analyzed [13] using the fixed Mellin moments [3, 4, 5, 6] for F_3. The SLAC and HERA data from eP-scattering [14] have been averaged with Bernstein polynomials to extract α_s in a direct fit of Mellin moments to experimental data [15]. The analyses agree within their errors and, more importantly, indicate that an absolute error for the strong coupling $\Delta\alpha_s \lesssim 1\%$ is possible. In addition, the effect of higher twist contributions, i.e. terms suppressed as $1/Q^2$, have been studied [13] and the NNLO evolution of parton distributions has even been used to put bounds on squark and gluino masses [15]. Finally, it is worth mentioning that the knowledge of a number of fixed Mellin moments of the three-loop coefficient functions [3, 4, 5, 6] enables analyses even beyond NNLO and allows for estimates of the effect of the next-to-NNLO corrections.

In summary, with the complete NNLO QCD analyses for the structure functions F_2, F_3 and F_L a new level of precision is reached in comparing theoretical predictions to experimental data. Moreover, the chance arises to address new questions which could not be studied before.

2. Progress

Progress towards the calculation of the three-loop anomalous dimensions and the coefficient functions crucially relies on the ability to perform all necessary loop integrations. A promising approach [16, 17] which allows to calculate the integrals in Mellin space analytically as a general function of N, is based on the optical theorem and the operator product expansion (OPE). The parameters of the OPE are directly related to the Mellin moments of the structure functions. For F_2 we can write

$$\begin{aligned} F_2^N(Q^2) &= \int_0^1 dx\, x^{N-2} F_2(x,Q^2) \\ &= \sum_{j=\alpha,\mathrm{q},\mathrm{g}} C_{2,j}^N\left(\frac{Q^2}{\mu^2},\alpha_s\right) A_{\mathrm{P},N}^j\left(\mu^2\right)\,, \end{aligned} \tag{2.1}$$

and similar relations define F_3^N and F_L^N. Here, $C_{2,j}^N$ denote the coefficient functions and $A_{\mathrm{P},N}^j$ the spin averaged hadronic matrix elements of singlet operators O^{q}, O^{g} and non-singlet operators O^α, $\alpha = 1,2,\ldots,(n_f^2-1)$, of leading twist. The coefficient functions and the renormalized operator matrix elements in eq.(2.1) both satisfy renormalization group equations. Due to current conservation they are governed by the same anomalous dimensions γ_{jk}, which determine the scale evolution of deep-inelastic structure functions,

$$\begin{aligned} \sum_{k=\alpha,\mathrm{q},\mathrm{g}} \Big[\Big\{\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_s(\mu^2))\frac{\partial}{\partial\alpha_s(\mu^2)}\Big\}\,\delta_{jk} & \\ +\gamma_{jk}(\alpha_s(\mu^2))\Big] A_{\mathrm{P},N}^j\left(\mu^2\right) = 0\,, & \end{aligned} \tag{2.2}$$

$$\begin{aligned} \sum_{k=\alpha,\mathrm{q},\mathrm{g}} \Big[\Big\{\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha_s(\mu^2))\frac{\partial}{\partial\alpha_s(\mu^2)}\Big\}\,\delta_{jk} & \\ -\gamma_{jk}(\alpha_s(\mu^2))\Big] C_{2,k}^N\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right) = 0\,, & \end{aligned} \tag{2.3}$$

where $j = \alpha, \mathrm{q}, \mathrm{g}$ and β represents the β-function of QCD.

The task is the calculation of the coefficient functions $C_{2,j}^N$ and the anomalous dimensions γ_{jk} in dimensionally regulated perturbation theory. In practice, at a given order in perturbation theory, this amounts to calculating the N-th moment of all contributing four-point diagrams with external partons of momentum $p, p^2 = 0$ and photons of momentum q, which is precisely the coefficient of $(p \cdot q/q^2)^N$.

In order to do so, it is useful to set up a hierarchy among all diagrams depending on the number of p-dependent propagators. We define basic building blocks (BBB) as diagrams in which the parton momentum p flows only through a single line in the diagram. Composite building blocks (CBB) are all diagrams with more than one p-dependent propagator. At the three-loop level, there are 10 BBB's, and 32 CBB's respectivley, which correspond to genuine three-loop topologies of the ladder, benz or non-planar type. In addition, there exist numerous BBB's and CBB's at three loops with an effective two-loop topology and a self-energy insertion in one line.

For the BBB's, the single p-dependent propagator, say $1/(p-l)\cdot(p-l)$ in a loop with momentum l, can be expanded into a geometrical sum using $p^2 = 0$. Then, scaling arguments require the final answer for the N-th moment to be proportional to the coefficient $(2\,p \cdot l/l^2)^N$. Thus, one is left with two-point functions with symbolic powers of scalar products in the numerator and denominator. For these objects, one sets up a reduction scheme, that relates the BBB under consideration to simpler diagrams, where certain lines are eliminated, such that the topology simplifies. For the CBB, a straightforward expansion of the p-dependent propagators leads to multiple nested sums, which in general are very difficult to evaluate. Hence, one has to seek a reduction scheme, that maps a given CBB onto BBB's. This is achieved, if one can remove a p-dependent propagator. If one can get rid of a p-independent propagator, usually the topology simplifies.

For the BBB's and the CBB's alike the reduction schemes are determined with the help of integration-by-parts identities [18, 19] and scaling identities [2]. The reduction identities often involve explicitly the parameter N of the Mellin moment and sometimes one has to set up difference equations in N for the N-th moment $F(N)$ of a diagram ,

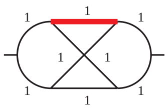
$$\begin{aligned} &a_0(N)F(N) + a_1(N)F(N-1) + \\ &\quad \ldots + a_n(N)F(N-n) + G(N) = 0\,, \end{aligned} \tag{2.4}$$

where $G(N)$ denotes the N-th Mellin moment of simpler diagrams. In the reduction schemes for three-loop diagrams, we have encountered difference equations up to the third order. First order difference equations can be solved at the cost of one sum over Γ-functions in dimensional regularization, $D = 4 - 2\epsilon$. The Γ-functions can be expanded in ϵ and the sum can be solved to any order in ϵ in terms of harmonic sums [20, 21]. Higher order difference equations could be solved constructively. In general, the approach to calculate Mellin moments of structure functions relies on particular mathematical concepts [22], such as harmonic sums [20, 21] and related sums [23]. Subsequently, the inverse Mellin transformation to x-space requires harmonic polylogarithms [24], or more general, multiple polylogarithms [25]. Difference equations for the evaluation of Feynman diagrams, although not in the context of Mellin moments, have recently also been studied by other authors [26, 27].

Let us present an example of a diagram of the non-planar type, that gives rise to a third order difference equation,

$$\int d^D l_1 \, d^D l_2 \, d^D l_3 \, \frac{1}{l_1^2 \, (p + l_2)^2 \, l_3^2 \, \ldots l_8^2} \,, \tag{2.5}$$

where $l_4 = l_3 - q$, $l_5 = l_1 - l_2 + l_3 - q$, $l_6 = l_1 - q$, $l_7 = l_2 - l_1$ and $l_8 = l_2 - l_3$, see ref. [28] for the conventions of the momentum flow. The result for the N-th Mellin moment of this diagram is given by the coefficient c_N,



$$= c_N \, \left(\frac{p \cdot q}{q^2} \right)^N \,. \tag{2.6}$$

The solution for c_N is expressed in terms of harmonic sums of weight six, as it is expected for the finite terms of a three-loop diagram. We obtain for c_N,

$$\begin{aligned} c_N = {} & \\ & \frac{(-1)^N}{N+1} \Big(8S_{-3,-2}(N+1) + 8S_2(N+1)\zeta_3 \\ & +4S_{2,-3}(N+1) - 4S_{2,3}(N+1) \\ & +4S_{3,-2}(N+1) + 4S_{3,2}(N+1) + 10\zeta_5 \Big) \\ & + \frac{1}{N+1} \Big(4S_{-3,-2}(N+1) + 4S_{-3,2}(N+1) \\ & +8S_{-2}(N+1)\zeta_3 + 4S_{-2,-3}(N+1) \\ & -4S_{-2,3}(N+1) + 8S_{3,-2}(N+1) + 10\zeta_5 \Big) \,. \end{aligned} \tag{2.7}$$

This illustrates nicely that our method will not only provide the anomalous dimensions, which are proportional to the single pole in ϵ in dimensional regularization, but also the coefficient functions which are determined by the finite terms at three loops.

In a systematic study we could set up, solve and program the complete reduction identities for all three-loop BBB's and all genuine three-loop CBB's of the ladder, benz and non-planar type. We have used FORM [29] for this task and performed checks at all stages of the calculation with the standard MINCER routine [28] by evaluating the expressions for a number of fixed values of the Mellin moment N. Optimization of the program requires the tabulation of several thousand two- and three-loop integrals in order to evaluate a given Feynman diagram in reasonable time. Thus far, the creation of these tables has already used a large amount of computer time. The complete database of Feynman diagrams for the structure functions F_2, F_3 and F_L has been generated with QGRAF [30] and contains roughly 11000 diagrams up to three loops. At present, the implementation of the reduction scheme is not complete yet. We expect that finishing the program along with the testing and finally the actual calculation of all Feynman diagrams still requires a lot of work.

3. Conclusions

The complete NNLO perturbative QCD corrections for deep-inelastic structure functions significantly reduce the theoretical uncertainties in the determination of the strong coupling α_s and the parton distributions. A precise knowledge of these quantities, including a quantitative error estimate, will be particularly important for new hadron collider experiments.

The present approach based on the OPE, to calculate the Mellin moments of the three-loop structure functions seems to allow a successful completion. The approach relies on the ability to solve all nested sums as functions of N in terms of harmonic sums, to set up and solve difference equations in N and, finally, to reconstruct the complete analytical expressions of the results in x-space by means of an inverse Mellin transformation.

As far as the theoretical developments are concerned, we believe that there is a realistic chance for very high precision measurements in deep-inelastic scattering.

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