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CP Asymmetries in $b \to (s/d)$ Transitions as a Test of CKM CP Violation

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Abstract

We point out that a simple test of the mechanism of CP violation can be performed by a measurement of the CP asymmetries in exclusive and inclusive radiative rare decays. We show that the rate asymmetries $\Delta\Gamma = \Gamma(B^- \to f\gamma) - \Gamma(B^+ \to \bar{f}\gamma)$ for certain final states f can be predicted in a theoretically clean way. Some implications for $b \to s\ell^+\ell^-$ decays are discussed.

1 Introduction

After the very successful start of the B factories at SLAC and KEK we may expect a large amount of data on decays of B mesons. Rare decays were first observed by the CLEO collaboration [1, 2]; these measurements have been refined [3] and confirmed by other experiments [4, 5]. The theoretical prediction of the Standard Model (SM) up to next-to-leading logarithmic precision for the total decay rate of the $b \to s\gamma$ mode is well in agreement with the experimental data [7]. The $b \to s\gamma$ mode already allows for theoretically clean and rather strong constraints on the parameter space of various extensions of the SM [8, 9].

Also, detailed measurements of CP asymmetries in rare B decays will be possible in the near future. Theoretical predictions for the *normalized* CP asymmetries of the inclusive channels (see [10, 11, 12]) within the Standard Model lead to

$$\alpha_{CP}(b \to s/d\,\gamma) = \frac{\Gamma(\bar{B} \to X_{s/d}\gamma) - \Gamma(B \to X_{\bar{s}/\bar{d}}\gamma)}{\Gamma(\bar{B} \to X_{s/d}\gamma) + \Gamma(B \to X_{\bar{s}/\bar{d}}\gamma)} \tag{1}$$

$$\alpha_{CP}(b \to s\gamma) \approx 0.6\%, \qquad \alpha_{CP}(b \to d\gamma) \approx -16\% \tag{2}$$

when the best-fit values for the CKM parameters [13] are used. An analysis for the leptonic counterparts can be found in [14]. The normalized CP asymmetries may also be calculated for exclusive decays; however, these results are model-dependent. An example of such a calculation may be found in [15].

CLEO has already presented a measurement of the CP asymmetry in inclusive $b \to s\gamma$ decays, yielding [16]

$$\alpha_{CP}(b \to s\gamma) = (-0.079 \pm 0.108 \pm 0.022) \cdot (1.0 \pm 0.030)\,, \tag{3}$$

which indicates that very large effects are already excluded. However, as we point out here, the decays of the form $b \to s\gamma$ and $b \to d\gamma$ as well as their leptonic counterparts provide a stringent test, if the CKM matrix is indeed the only source of CP violation. We shall argue that the exclusive as well as the inclusive decays may be used to perform a clean test of the CKM mechanism of CP violation.

We start with the fact that the CP asymmetry for the sum of the partonic processes $b \to (s+d)\gamma$ vanishes in the limit of $m_d = m_s = 0$. This was first observed by Soares [17] in a partonic calculation of these processes. Later Kagan and Neubert analysed the CP asymmetry in inclusive $b \to s\gamma$ decays and mentioned this as a side remark [11]. This fact is still true when only the weaker condition $m_d = m_s$ holds, which corresponds to the U-spin limit at the quark level.

We shall show in this note that one may use this fact for a stringent test of the CKM mechanism of CP violation. Any CP violation in the standard model has to be proportional to the determinant[1]

$$\begin{aligned} C &= \det\left[\mathcal{M}_U\,,\,\mathcal{M}_D\right] \\ &= i\,J\;\;(m_u - m_c)(m_u - m_t)(m_c - m_t)(m_d - m_s)(m_d - m_b)(m_s - m_b) \end{aligned} \tag{4}$$

where $\mathcal{M}_{U/D}$ are the mass matrices for the up and down quarks and

$$J = \mathrm{Im}[V_{ub} V_{cb}^* V_{cs} V_{us}^*] \tag{5}$$

is the Jarlskog parameter, which is a fourth-order quantity and which is invariant under rephasing of the quarks fields. For a unitary CKM matrix there is exactly one such quantity, and for that reason J (or equivalently C) is the measure of CP violation in the Standard Model. However, comparing different CP-violating processes involves also hadronic matrix elements, which in general are hard to control, making a direct extraction of C or J in general difficult.

One way to gain information about the hadronic matrix elements is to use symmetries. For the case at hand, we shall make use of the U-spin, which is the $SU(2)$ subgroup of flavour $SU(3)$ relating the s and the d quark. In fact U-spin symmetry is a well-known tool and has already been used in the context of non-leptonic decays [18, 19] and general proofs exist for U-Spin relations among arbitrary heavy hadron decays [20]. In the present paper we shall apply this symmetry to relate the decays $b \to s\gamma$ and $b \to d\gamma$.

It is known from hadron spectroscopy that the U-spin symmetry is violated, which at the parton level originates from the different masses of the down and the strange quark. Furthermore, if the down and the strange quark were degenerate, the Standard Model would be CP-conserving, as can be seen from (4).

However, we shall make use of this symmetry only with respect to the strong interactions; although the down and strange quark masses are different, we shall consider (hadronic) final states with masses well above the down and strange quark (current) masses, and thus the U-spin limit is still useful.

In section 2 we discuss the U-spin relations for hadronic matrix elements relevant to the exclusive decays; in section 3 we study inclusive processes. In section 4 we conclude with a short discussion of possible non-standard model scenarios.

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[1]We assume here that the mass matrices for up and down quarks are hermitean.

2 U-Spin Relations: Exclusive Decays

The effective Hamiltonean mediating rare radiative or rare semileptonic transitions from $b \to q\gamma$ ($q = d, s$) can be decomposed into the pieces with different weak phases

$$H_{eff} = \lambda_u^{(q)} A + \lambda_c^{(q)} B + \lambda_t^{(q)} C = \lambda_u^{(q)} (A - C) + \lambda_c^{(q)} (B - C), \qquad (6)$$

where

$$\lambda_u^{(q)} = V_{ub} V_{uq}^* \qquad \lambda_c^{(q)} = V_{cb} V_{cq}^* \qquad \lambda_t^{(q)} = V_{tb} V_{tq}^* \qquad (7)$$

and where we have used the unitarity relation $\lambda_u^{(q)} + \lambda_c^{(q)} + \lambda_t^{(q)} = 0$ to eliminate $\lambda_c^{(q)}$.

As far as the strong interaction U-spin is concerned, the effective Hamiltonian transforms as a doublet under this symmetry, i.e. A, B and C are U-spin doublets. The charged B mesons contain neither the d nor the s flavour and are thus U-spin singlets. The neutral B mesons (B_d, B_s) form a doublet under U-spin, which makes their case more complicated.

The final states for the exclusive processes we can consider are the π, K, ρ and K^* states. Here again the case of charged states is simple: The (π^- K^-) and the (ρ^- K^{*-}) form two U-spin doublets. For the neutral mesons we can form a U-spin singlet and a U-spin triplet. For the vector mesons (assuming that the ϕ is a pure $s\bar{s}$ state) the singlet is a combination of ϕ, ρ and ω: $1/\sqrt{2}[|\phi\rangle + \frac{1}{2}(|\omega\rangle - |\rho\rangle)]$ while for the triplet we have the $|K^0\rangle$ and $|\overline{K}^0\rangle$ as the ± 1 component, and $1/\sqrt{2}[|\phi\rangle - \frac{1}{2}(|\omega\rangle - |\rho\rangle)]$ for the 0 component.

As mentioned above, the U-spin is clearly broken by the different masses m_d, m_s of the down and the strange quark. On the other hand, both m_d and m_s are small with respect to the masses of any hadron, except for the octet of light pseudoscalars, the masses of which vanish in the chiral limit of QCD and thus are presumably more sensitive to the (current quark) masses of the s and d quarks.

To this end, when talking about exclusive final states, we shall mainly consider the vector mesons; these have masses much larger than the (current quark) masses of any of the light quarks. Thus we expect, for the ground state vector mesons, the U-spin symmetry to be quite accurate inspite of the non-degeneracy of m_d and m_s. A measure for the breaking of U-spin symmetry is certainly the relative mass difference between the ρ and the K^*, which is of the order of fifteen percent.

For the decay of neutral mesons such as $B_d \to \rho^0\gamma$ or $B_d \to \pi_0\ell^+\ell^-$, the U-spin relations involve two reduced matrix elements corresponding to the two possibilities to couple the U-spins: the doublet of B mesons can be coupled with the (doublet) Hamltonian either to a singlet or a triplet. The two matrix elements can in principle be disentangled by measuring all the decays of neutral B mesons, including processes like $B_s \to \phi\gamma$. Since this will not be possible in the near future, we shall concentrate on what can be done at the B factories.

Thus the charged modes are more promising since they do not involve any B_s decay. Let us first consider the radiative decay $B^\pm \to V^\pm\gamma$, where $V = \rho$ or K^*. The rate for these decays may be written as

$$\Gamma(B^- \to V^-\gamma) = |\lambda_u M_u + \lambda_c M_c|^2, \tag{8}$$

where we have suppressed the superscript (q) for simplicity, and M_u is the matrix element of $A - C$ and M_c is the one of $B - C$.

The charge-conjugate process is

$$\Gamma(B^+ \to V^+\gamma) = |\lambda_u^* M_u + \lambda_c^* M_c|^2, \tag{9}$$

which yields for the rate difference

$$\begin{aligned} \Delta\Gamma(B^- \to V^-\gamma) &= \Gamma(B^- \to V^-\gamma) - \Gamma(B^+ \to V^+\gamma) \\ &= -4\,\mathrm{Im}(M_u M_c^*)\,\mathrm{Im}(\lambda_u \lambda_c^*). \end{aligned} \tag{10}$$

This is a standard expression showing that CP violation is indeed proportional to J, since $\mathrm{Im}(\lambda_u\lambda_c^*) = \pm J$.

The hadronic matrix elements for $V = \rho$ and $V = K^*$ are related by U-spin. Comparing $B^\pm \to K^{*\pm}\gamma$ with $B^\pm \to \rho^\pm\gamma$, one finds (up to U-spin-breaking effects):

$$M_u^{(K^*)} = M_u^{(\rho)} = M_u \qquad M_c^{(K^*)} = M_c^{(\rho)} = M_c, \tag{11}$$

which means that

$$\Delta\Gamma(B^- \to K^{*-}\gamma) = -4\,\mathrm{Im}\,(M_u M_c^*)\,\mathrm{Im}\,(\lambda_u^{(s)}\lambda_c^{(s)*}) \tag{12}$$

$$\Delta\Gamma(B^- \to \rho^-\gamma) = -4\,\mathrm{Im}\,(M_u M_c^*)\,\mathrm{Im}\,(\lambda_u^{(d)}\lambda_c^{(d)*}). \tag{13}$$

Using unitarity of the CKM matrix one can show easily that

$$J = \mathrm{Im}(\lambda_u^{(s)}\lambda_c^{(s)*}) = -\mathrm{Im}(\lambda_u^{(d)}\lambda_c^{(d)*}) \tag{14}$$

and thus one finds that the rate asymmetries (*not* the CP asymmetries, which are the rate asymmetries normalized to the sum of the rates) satisfy - in the U-spin limit for the hadronic matrix elements - the relation

$$\Delta\Gamma(B^- \to K^{*-}\gamma) = -\Delta\Gamma(B^- \to \rho^-\gamma). \tag{15}$$

This result implicitly may be found in the literature, but to our knowledge nobody has yet considered its implications in detail.

Relation (15) provides us with a relatively clean test of the CKM mechanism of CP violation. The fact that J is the only CP-violating parameter in the Standard Model is deeply related to the unitarity of the CKM matrix and to the fact that there are only three families. Any non-SM scenario has generically more sources of CP violation, which in general would disturb (15), since there will be other weak phases besides the CKM phase.

Clearly the main uncertainty in (15) is U-spin breaking, which means that (15) is only useful with some estimate of this breaking. We shall prceed in a similar way as in [18] and write

$$\begin{aligned} &\Delta\Gamma(B^- \to K^{*-}\gamma) + \Delta\Gamma(B^- \to \rho^-\gamma) \\ &\qquad = -4\, J\, \mathrm{Im}\left(M_u^{(K^*)}(M_c^{(K^*)})^* - M_u^{(\rho)}(M_c^{(\rho)})^*\right) = b_{exc}\Delta_{exc} \end{aligned} \tag{16}$$

where the right hand side is written as a product of a "relative U-spin breaking" b_{exc} and a "typical size" Δ_{exc} of the CP violating rate difference. Explicitely we have

$$b_{exc} = \frac{\mathrm{Im}\left(M_u^{(K^*)}(M_c^{(K^*)})^* - M_u^{(\rho)}(M_c^{(\rho)})^*\right)}{\frac{1}{2}\mathrm{Im}\left(M_u^{(K^*)}(M_c^{(K^*)})^* + M_u^{(\rho)}(M_c^{(\rho)})^*\right)} \tag{17}$$

and

$$\Delta_{exc} = -2\, J\, \mathrm{Im}\left(M_u^{(K^*)}(M_c^{(K^*)})^* + M_u^{(\rho)}(M_c^{(\rho)})^*\right) \tag{18}$$

which is half of the difference of the two rate asymmetries.

The advantage to write the right hand side as $b_{exc}\Delta_{exc}$ is that, although we know neither b_{exc} nor Δ_{exc} precisely, we still can estimate it. Δ_{exc} can only be computed in a model, but the relative breaking b_{exc} of U-spin can be estimated e.g. from spectroscopy. This leads us to

$$|b_{exc}| = \frac{M_{K^*} - m_\rho}{\frac{1}{2}(M_{K^*} + m_\rho)} = 14\% \tag{19}$$

which takes into account our ignorance about the sign of b_{exc}. Certainly also other estimates are possible, such as a comparison of f_ρ and f_{K^*}[2]; both quantities can be determined from τ decays and one finds in this case a very small U-spin breaking. We shall use the more conservative value for b_{exc}, which is also compatible with sum rule calculations of form factors (see [21]).

[2] There are various definitions for these quantities in the literature; we choose to define them as $\langle V(p,\epsilon)|j_\mu|0\rangle = f_V m_V \epsilon_\mu$.

For Δ_{exc} we use the model result from [15] and get

$$\Delta_{exc} = 2.5 \cdot 10^{-7} \, \Gamma_B \tag{20}$$

which leads us finally to our standard-model prediction for the difference of branching ratios

$$|\Delta Br(B^- \to K^{*-}\gamma) + \Delta Br(B^- \to \rho^-\gamma)| \sim 4 \cdot 10^{-8} \tag{21}$$

Note that we can neither give a precise value nor the sign of the U-spin breaking, since the right hand side is model-dependent. Still (21) is of some use, since a value significantly above this estimate would be a strong hint to non-CKM contributions to CP violation.

3 Inclusive Decays and $b \to s\ell^+\ell^-$

One may use similar arguments for the case of inclusive decays. For the inclusive radiative rare decays of charged B meson decays, exactly the same arguments hold for any arbitrary final state, and thus we have for the rate asymmetries the relation

$$\Delta\Gamma(B^- \to X_s\gamma) = -\Delta\Gamma(B^- \to X_d\gamma). \tag{22}$$

Concerning the validity of U-spin, similar arguments hold. Since the lowest state in the radiative decay is a vector meson[3] the invariant masses of the final states are large with respect to m_d and m_s, so we expect the U-spin to be a fairly good symmetry, very likely even better than for the exclusive channels (see below).

Going one step further one may employ the $1/m_b$ expansion for the inclusive process. To leading order the inclusive decay rate is the free b-quark decay. In particular, there is no sensitivity to the spectator quark and thus we may generalize (22) and include also neutral B mesons

$$\Delta\Gamma(B \to X_s\gamma) = -\Delta\Gamma(B \to X_d\gamma) \tag{23}$$

Furthermore, in this framework one relies on parton-hadron duality (besides in the long-distance contribution from up-quark loops which is found to be rather small [22]). So one can actually compute the breaking of U-Spin by keeping a non-vanishing strange quark mass. However, it is a formidable task to do this for the CP asymmetries and it has not yet been done. Still it

[3]There could also be a non-resonant contribution from $K\pi$ and $\pi\pi$ states with a mass lower than that of the corresponding vector meson, but this is known to be small.

is clear that the relevant parameter in the dominating contribution is m_s^2/m_b^2, which is very small.

Thus, we again parametrize the size of U-spin breaking in the same way as for the exclusive decays. We write

$$\Delta\Gamma(B \to X_s\gamma) + \Delta\Gamma(B \to X_d\gamma) = b_{inc}\Delta_{inc} \tag{24}$$

where now the typical size of b_{inc} can be roughly estimated to be of the order $|b_{inc}| \sim m_s^2/m_b^2 \sim 5 \cdot 10^{-4}$. $|\Delta_{inc}|$ is again the average of the moduli of the two CP rate asymmetries. These have been calculated (for vanishing strange quark mass) e.g. in [10] and thus we arrive at

$$|\Delta Br(B \to X_s\gamma) + \Delta Br(B \to X_d\gamma)| \sim 1 \cdot 10^{-9} \tag{25}$$

Again, any measured value in significant deviation of (25) would be an indication of new sources of CP violation. Although we give only an estimate here, we point out again that in the inclusive mode the right-hand side in (25) can be computed in a model-independent way with the help of the heavy mass expansion.

Going beyond leading order in the $1/m_b$ involves corrections of order λ_1/m_b^2 and λ_2/m_b^2 which are small and cancel in the sum of the rate asymmetries - in the limit of U-spin symmetry. Corrections of order $1/m_b^3$ involve also contributions (for example annihilation topologies), which distinguish between the charged and the neutral B mesons. These contributions are actually suppressed relative to the leading order by $\alpha_s(m_b)/m_b^3$ and are thus small; furthermore, the final states originating from annihilation carry neither strangeness nor a down quantum number. Strictly speaking these diagrams do not contribute to X_s or X_d respectively, but it is a question of the experimental set up how strongly states without s or d quantum numbers can be discriminated.

Finally, we make a few remarks on transitions of the form $b \to q\ell^+\ell^-$, $q = s, d$. Since the CKM structure of these decays is the same, one may use eqs. (6 – 10) in the same way, only with different non-hadronic contributions to the final states. Thus, we arrive at the same conclusions for the CP-violating rate differences. Of course, the use of U-spin may be questionable for the light pseudoscalars, but for the vector mesons the above arguments apply.

Moreover, since $B^\pm \to V^\pm\ell^+\ell^-$, $V = K^{*\pm}, \rho^\pm$ is a three-body decay, one may perform additional tests. Although this is not related to the CKM phases, one would end up with the prediction that the forward–backward *rate* asymmetry would be the same for K^* and ρ. This prediction could be used as an additional cross-check for the validity of our assumption of U-spin symmetry in the $K^{*\pm} - \rho^{\pm}$ system and maybe even access U-spin breaking in a model-independent way also for the exclusive modes.

4 "New Physics"

Clearly the pattern of CP violation is very peculiar in the SM. In other words, any new physics contribution is likely to have additional sources for CP violation, i.e. additional weak phases violating (21) or (25). Let us conclude with a short look at scenarios beyond the SM.

Although it has its well-known flavour problem, supersymmetry is given priority as a candidate for physics beyond the SM. Supersymmetric predictions for the CP asymmetries in $b \to s/d\gamma$ depend strongly on what is assumed for the supersymmetry-breaking sector and are thus a rather model-dependent issue. The minimal supergravity model cannot account for large CP asymmetries beyond 2% because of the constraints coming from the electron and neutron electric dipole moments [23]. However, more general models allow for larger asymmetries of the order of 10% or even larger [24, 11]. Recent studies of the $b \to d\gamma$ rate asymmetry in specific models led to asymmetries between -40% and $+40\%$ [26] or -45% and $+21\%$ [25].

In general, CP asymmetries may lead to clean evidence for new physics by a significant deviation from the SM prediction. From (2) it is obvious that a large CP asymmetry in the $b \to s\gamma$ channel or a positive CP asymmetry in the inclusive $b \to d\gamma$ channel would be a clear signal for new physics. However, if indeed a CP asymmetry in conflict with the SM is observed, this will only be an indirect hint to physics beyond the SM, and it will be difficult to identify the new structures in detail with only the information from B physics. Our test could help to discriminate between the different possibilities; It provides a definite test if generic new CP phases are present or not since it is rather unlikely that relations like (21) and (25) hold if new sources of CP violation are active.

5 Conclusion

A clean test of the SM pattern of CP violation is clearly one of the main topics of on-going experiments, in particular at the B factories. Being a direct CP asymmetry, the asymmetries in $b \to (s/d)\gamma$ can be measured without the information on the proper decay time.

Despite their smaller branching ratios, the exclusive channels are easier to identify, and we expect an experimental test of (15) in the near future.

However, for these decays the issue of U-spin breaking introduces a model dependence. Measuring the corresponding forward–backward rate asymmetry may allow for a model-independent estimation of the U-spin breaking in this channel.

The inclusive relations are theoretically cleaner, since one can actually compute the U-spin breaking in this case. Parametrically the U-spin breaking in the leading contribution will be of order m_s^2/m_b^2 and thus very small. From the experimental side, the inclusive mode is more difficult, since one has to make sure that the final state has the strange or the down quantum numbers. However, contributions which do not satify this criterion are suppressed by at least $1/m_b^3$. and thus do not contaminate the inclusive measurement $B \to (X_s + X_d)\gamma$.

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