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Some higher moments of deep inelastic structure functions at next-to-next-to leading order of perturbative QCD

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Abstract

We present the analytic next-to-next-to-leading QCD calculation of some higher moments of deep inelastic structure functions in the leading twist approximation. We give results for the moments N=1,3,5,7,9,11,13 of the structure function F_3. Similarly we present the moments N=10,12 for the flavour singlet and N=12,14 for the non-singlet structure functions F_2 and F_L. We have calculated both the three-loop anomalous dimensions of the corresponding operators and the three-loop coefficient functions of the moments of these structure functions.

1 Introduction

The determination of the next-to-next-to-leading (NNL) order QCD approximation for the structure functions of deep inelastic scattering has become important for the understanding of perturbative QCD and necessary for an accurate comparison of perturbative QCD with the increasing precision of experiments. Such calculations however are rather complicated and hence a complete NNL result does not exist as of yet. The one-loop anomalous dimensions were calculated in [1, 2]. In [3] (see also the references therein) the complete one-loop coefficient functions were obtained. Anomalous dimensions at 2-loop order were obtained in [4, 5, 6, 7, 8, 9, 10, 11, 12]. The 2-loop coefficient functions were calculated in [13, 14, 15, 16, 17, 18, 19, 20], [21].

Analytical results of the 3-loop anomalous dimensions and coefficient functions of the moments of F_2 and F_L are only known for the moments N=2,4,6,8 in both the singlet and non-singlet case and additionally for N=10 in the non-singlet case from [22] and [23]. In addition the Gross-Llevellyn Smith sum rule, which corresponds to the first moment of $F_3^{\nu \mathrm{p}+\bar{\nu}\mathrm{p}}$ has been calculated at this order [24].

For a complete reconstruction of the x-dependence of the structure functions via an inverse Mellin-transformation one would need the moments for all N (that is either all even or all odd integer values). Additionally one needs them both for F_2 and F_3 in order to untangle the various quark and gluon contributions. The determination of the NNL approximation for generic N is work in progress [25], but probably will not be finished in the near future.

The available moments of F_2 have been used by a number of authors to make a reconstruction of the complete structure functions at NNL by a variety of means [23, 26, 27, 28]. Additionally they can be used to obtain a better value of α_S [29] It should be clear that it is important to have as large a number of moments as possible. First, these results can be immediately used to increase the precision of phenomenological investigations of deep inelastic scattering [30]. Second, the moments will be a very important check for the new methods and programs needed for the determination of the 3-loop results for arbitrary N. Unfortunately it is not very easy to increase the number of moments, because each new moment requires roughly five times the computer resources that its predecessor needs. With the advent of better computers this means that by now it has been possible to obtain two more moments for the singlet case and one additional moment for the non singlet F_2 case. This should allow for instance a somewhat better determination of α_S. More important however is the determination of the first seven odd moments of $F_3^{\nu \mathrm{p}+\bar{\nu}\mathrm{p}}$ to three loops. To this end we used the the same programs as in [23], state of the art computers and a new version of the symbolic manipulation program FORM [31] that supports now 64-bit architectures and to some extend parallel computers (see also [32]). We could push the limit in these calculations to include two new moments (N=10,12) in the calculation of the flavour singlet structure functions F_L and F_2, and two new moments (N = 12,14) for the flavour nonsinglet structure functions F_L and F_2. Additionally we have computed the moments N=3,5,7,9,11,13 of the structure function F_3. We do not expect more moments to become available before the complete results for all N will be presented.

2 The formalism

This calculation follows the one presented in [23] (see also [21]) in every detail, so we only will give a very short review on the methods used.

We need to calculate the hadronic part of the amplitude for unpolarized deep inelastic scattering

which is given by the hadronic tensor

$$\begin{aligned} W_{\mu\nu}(x,Q^2) &= \frac{1}{4\pi}\int \mathrm{d}^4 z \mathrm{e}^{\mathrm{i}q\cdot z}\langle p,\mathrm{nucl}|J_\mu(z)J_\nu(0)|\mathrm{nucl},p\rangle \\ &= (g_{\mu\nu}-q_\mu q_\nu)\frac{1}{2x}F_L(x,Q^2) \\ &\quad +\left(-g_{\mu\nu}-p_\mu p_\nu\frac{4x^2}{q^2}-(p_\mu q_\nu+p_\nu q_\mu)\frac{2x}{q^2}\right)\frac{1}{2x}F_2(x,Q^2) \\ &\quad +\mathrm{i}\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho q^\sigma}{p\cdot q}F_3(x,Q^2) \end{aligned} \tag{1}$$

where the J^μ are either electromagnetic or weak hadronic currents and $x = Q^2/(2\,p\cdot q)$ is the Bjorken scaling variable with $0 < x \leq 1$. $Q^2 = -q^2$ is the transfered momentum and $|p,\mathrm{nucl}\rangle$ is the nucleon state with momentum p. In these equations spin averaging is assumed. The longitudinal structure function F_L is related to the structure function F_1 by $F_L = F_2 - 2xF_1$. For electron-nucleon scattering J^μ is the electromagnetic quark current and F_3 vanishes. For neutrino-nucleon scattering J^μ is an electroweak quark current which has an axial vector contribution and F_3, which describes parity violating effects that arise from vector and axial-vector interference, will not vanish.

Using the dispersion relation technique one can relate the hadronic tensor to the following 4-point Green functions:

$$W^{\mu\nu}(p,q) = \frac{1}{2\pi}\mathrm{Im}T^{\mu\nu}(p,q), \quad T_{\mu\nu}(p,q) = \mathrm{i}\int \mathrm{d}^4 z \mathrm{e}^{\mathrm{i}q\cdot z}\langle p,\mathrm{nucl}|T\left[J_\mu(z)J_\nu(0)\right]|\mathrm{nucl},p\rangle.$$

Applying a formal operator product expansion in terms of local operators to the time-ordered product of two quark currents leads to:

$$\begin{aligned} \mathrm{i}\int \mathrm{d}^4 z \mathrm{e}^{\mathrm{i}q\cdot z}T\left[J_{\nu_1}(z)J_{\nu_2}(0)\right] = \sum_{N,j}\left(\frac{1}{Q^2}\right)^N \Bigg[&\left(g_{\nu_1\nu_2}-\frac{q_{\nu_1}q_{\nu_2}}{q^2}\right)q_{\mu_1}q_{\mu_2}C^j_{L,N}(\frac{Q^2}{\mu^2},a_s) \\ &-\left(g_{\nu_1\mu_1}g_{\mu_2\nu_2}q^2-g_{\nu_1\mu_1}q_{\nu_2}q_{\mu_2}-g_{\nu_2\mu_2}q_{\nu_1}q_{\mu_1}+g_{\nu_1\nu_2}q_{\mu_1}q_{\mu_2}\right)C^j_{2,N}(\frac{Q^2}{\mu^2},a_s) \\ &+\mathrm{i}\,\epsilon_{\nu_1\nu_2\mu_1\nu_3}g^{\nu_3\nu_4}q_{\nu_4}q_{\mu_2}C^j_{3,N}(\frac{Q^2}{\mu^2},a_s)\Bigg]\times q_{\mu_3}\ldots q_{\mu_N}O^{j,\{\mu_1,\ldots\mu_N\}}(0) + \text{higher twists}, \\ & j=\alpha,\psi,G \end{aligned} \tag{2}$$

Here we have introduced the notation $a_s = \alpha_s/(4\pi) = g^2/(4\pi)^2$ and everything is assumed to be renormalized. The sum over N runs over the standard set of the spin-N twist-2 irreducible flavour non-singlet quark operators and the singlet quark and gluon operators:

$$\begin{aligned} O^{\alpha,\{\mu_1,\ldots,\mu_N\}} &= \bar\psi\lambda^\alpha\gamma^{\{\mu_1}D^{\mu_2}\ldots D^{\mu_N\}}\psi, \quad \alpha = 1,2,\ldots,(n_f^2-1) \\ O^{\Psi,\{\mu_1,\ldots,\mu_N\}} &= \bar\psi\gamma^{\{\mu_1}D^{\mu_2}\ldots D^{\mu_N\}}\psi \\ O^{G,\{\mu_1,\ldots,\mu_N\}} &= G^{\{\mu\mu_1}D^{\mu_2}\ldots D^{\mu_{N-1}}G^{\mu_N\mu\}} \end{aligned}$$

Application of this OPE to Eq. (1) leads to an expansion for the unphysical values $x \to \infty$. From the proper analytical continuation to the physical region $0 < x \leq 1$ one finds for the moments of the structure functions F_2, F_L and F_3:

$$M_{k,N-2} = \int_0^1 \mathrm{d}x x^{N-2}F_k(x,Q^2) = \sum_{i=\alpha,\psi,G} C^i_{k,N}(\frac{Q^2}{\mu^2},a_s)A^i_{\mathrm{nucl},N}\,, \quad k=2,L \tag{3}$$

$$M_{3,N-1} = \int_0^1 \mathrm{d}x x^{N-1}F_3(x,Q^2) = \sum_{i=\alpha} C^{i,N}(\frac{Q^2}{\mu^2},a_s)A^i_{\mathrm{nucl},N} \tag{4}$$

with the spin averaged matrix elements

$$\langle p, \mathrm{nucl}|O^{j,\{\mu_1\cdots\mu_N\}}|p, \mathrm{nucl}\rangle = p^{\{\mu_1}\cdots p^{\mu_N\}} A^j_{\mathrm{nucl},N}(\frac{p^2}{\mu^2}) \tag{5}$$

In the derivation of (3) one needs the symmetry properties of $T_{\mu\nu}$ under $x \to -x$. This is why one can only find either even or odd moments from these equations, dependent on the process under consideration. For F_3 we will only consider the flavor non-singlet contributions, due to the properties of the operators O^ψ and O^G under charge conjugation there should not be a singlet contribution (see e.g. [33]).

The scale-dependence of the coefficient functions is then covered by the renormalization group equations:

$$\left[\mu^2\frac{\partial}{\partial\mu^2} + \beta(a_s)\frac{\partial}{\partial a_s} - \gamma_N^{\mathrm{ns}}\right] C^{\mathrm{ns}}_{i,N}\left(\frac{Q^2}{\mu^2}, a_s\right) = 0, \qquad i = 2, 3, L \tag{6}$$

$$\sum_{k=\psi,G}\left[\left\{\mu^2\frac{\partial}{\partial\mu^2} + \beta(a_s)\frac{\partial}{\partial a_s}\right\}\delta^{jk} - \gamma_N^{jk}\right] C^k_{i,N}\left(\frac{Q^2}{\mu^2}, a_s\right) = 0, \qquad i = 2, L;\ j = \psi, G \tag{7}$$

The non-singlet coefficient functions and anomalous dimensions don't depend on the index α, and we have adopted the conventional collective denotation "ns" for them.

3 The even moments of F_2 and F_L

Equation (2) is a relation between operators and does not depend on the hadronic states to which the OPE is applied.

Using the method of projectors [34] one can find both the coefficient functions and the anomalous dimensions for the even moments of F_2 and F_L as defined in Eq. (6) from the following 4-point Green functions:

$$T^{q\gamma q\gamma}_{\mu\nu} = \mathrm{i}\int \mathrm{d}^4 z \mathrm{e}^{\mathrm{i}q\cdot z}\langle p, \mathrm{quark}|T\left[J_\mu(z)J_\nu(0)\right]|\mathrm{quark}, p\rangle \tag{8}$$

$$T^{g\gamma g\gamma}_{\mu\nu} = \mathrm{i}\int \mathrm{d}^4 z \mathrm{e}^{\mathrm{i}q\cdot z}\langle p, \mathrm{gluon}|T\left[J_\mu(z)J_\nu(0)\right]|\mathrm{gluon}, p\rangle \tag{9}$$

Applying to Eqs. (8) the projectors

$$P_N \equiv \left[\frac{q^{\{\mu_1}\cdots q^{\mu_N\}}}{N!}\frac{\partial^N}{\partial p^{\mu_1}\cdots\partial p^{\mu_N}}\right]\Bigg|_{p=0} \tag{10}$$

and, to project out the different Lorentz projections (these projectors are valid in $D = 4 - 2\epsilon$ and for the leading twist approximation):

$$P_L = -\frac{q^2}{(p\cdot q)^2}p^\mu p^\nu$$
$$P_2 = -\left(\frac{3-2\epsilon}{2-2\epsilon}\frac{q^2}{(p\cdot q)^2}p^\mu p^\nu + \frac{1}{2-2\epsilon}g^{\mu\nu}\right),$$

as well as the corresponding flavour projections results in the equations:

$$\begin{aligned}
T^{q\gamma q\gamma,\mathrm{s}}_{k,N}(\frac{Q^2}{\mu^2}, a_s, \epsilon) &= \left(C^\psi_{k,N}(\frac{Q^2}{\mu^2}, a_s, \epsilon)Z^{\psi\psi}_N(a_s, \frac{1}{\epsilon}) + C^G_{k,N}(\frac{Q^2}{\mu^2}, a_s, \epsilon)Z^{G\psi}_N(a_s, \frac{1}{\epsilon})\right) \quad A^{\psi,\mathrm{tree}}_{\mathrm{quark},N}(\epsilon) \\
T^{g\gamma g\gamma,\mathrm{s}}_{k,N}(\frac{Q^2}{\mu^2}, a_s, \epsilon) &= \left(C^\psi_{k,N}(\frac{Q^2}{\mu^2}, a_s, \epsilon)Z^{\psi G}_N(a_s, \frac{1}{\epsilon}) + C^G_{k,N}(\frac{Q^2}{\mu^2}, a_s, \epsilon)Z^{GG}_N(a_s, \frac{1}{\epsilon})\right) \quad A^{G,\mathrm{tree}}_{\mathrm{gluon},N}(\epsilon) \\
T^{q\gamma q\gamma,\mathrm{ns}}_{k,N}(\frac{Q^2}{\mu^2}, a_s, \epsilon) &= C^{\mathrm{ns}}_{k,N}(\frac{Q^2}{\mu^2}, a_s, \epsilon)Z^{\mathrm{ns}}_N(a_s, \frac{1}{\epsilon})A^{\mathrm{ns,tree}}_{\mathrm{quark},N}(\epsilon) \qquad k = 2, L
\end{aligned} \tag{11}$$

From these equations the coefficient functions and from the Z_N^{ij} the anomalous dimensions can be calculated in the usual way.

It should be mentioned that in the Eqs. (11) on the left hand side after applying the projectors (10) we are left with only diagrams of the massless propagator type, a problem solved at 3-loop order long ago [35] and implemented in an efficient way in the FORM package MINCER [36]. On the right hand side only the tree level diagrams contributing to the Matrix elements survive.

It turns out that much computing time can be saved when calculating additionally Green functions with external ghosts to get rid of the unphysical polarization states of the external gluons instead of using the very complicated projection onto physical states. Also, from Eqs. (11) one can determine the Z_N^{GG} and $Z_N^{G\psi}$ only to order α_s^2. To obtain the α_s^3-contributions one can calculate additionally Greens functions with external scalar fields ϕ that couple to gluons only at tree level. Altogether, to obtain the coefficient functions and anomalous dimensions for the even moments with N=10,12 of F_2 and F_L the following diagrams had to be calculated (q=quark, g=gluon, γ=photon, h=ghost, ϕ=scalar field):

	tree	1-loop	2-loops	3-loops	Lorentz projections
$q\gamma q\gamma$	1	3	27	413	2
$q\phi q\phi$		1	24	697	1
$g\gamma g\gamma$		2	20	366	2
$h\gamma h\gamma$			2	53	2
$g\phi g\phi$	1	11	241	1266	1
$h\phi h\phi$		11	241	1266	1
Total	3	23	399	10846	

The $q^{\{\mu_1}\cdots q^{\mu_N\}}$ in Eq. (10) are the harmonic (i.e. symmetrical and traceless) part of the tensor $q^{\mu_1}\cdots q^{\mu_N}$. The number of terms in these harmonic tensors explodes as N increases and this is the real limitation in these calculations considering the computing time as well as disk-space usage. In spite of a very efficient implementation of these tensors (see [37]) for $N = 12$, singlet and $N = 14$, nonsinglet, individual diagrams had a disk space usage up to and over 100 GB. Altogether the calculation of all the above diagrams for $N = 10, 12$ took approximately 5 weeks on a Compaq Server with 8 Alpha 21264 processors running at 700 MHz, 4 GB of RAM and 12×17 GB of disk-space. The $N = 14$ nonsinglet calculation took comparable resources.

4 The odd moments of F_3

The coefficient functions and anomalous dimensions of the odd moments of the structure function F_3 can be obtained in the same way as the non-singlet part of F_2 and F_L but now considering the time-ordered product of one vector current V_μ and one axial vector current A_ν. The axial current introduces the appearance of a γ_5 and some care has to be taken to treat it correctly within the framework of dimensional regularization. We adopt the definition used in [24]:

$$\gamma_\mu\gamma_5 = \mathrm{i}\frac{1}{6}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$$

Projecting out the flavour non-singlet part and the corresponding Lorentz structure with:

$$P_3 = -\mathrm{i}\frac{1}{(1-2\epsilon)(2-2\epsilon)}\epsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{p\cdot q}$$

one finds products of metric tensors which have to be considered as D-dimensional objects. Since this definition of γ_5 in D dimensions violates the axial Ward identity one needs to renormalize A_μ with a renormalization constant Z_A and additionally apply a finite renormalization with Z_5, both of these constants are given to 3-loop order in [24]. Combining all this finally leads to

$$Z_A(a_s,\frac{1}{\epsilon})Z_5(a_s,\epsilon)T^{\rm ns}_{3,N}(\frac{Q^2}{\mu^2},a_s,\epsilon) = C_{3,N}(\frac{Q^2}{\mu^2},a_s,\epsilon)Z_N^{-1}(a_s,\frac{1}{\epsilon})A^{\rm ns}_{N,\rm tree}(\epsilon)$$

Due to the γ_5 insertion at one of the vertices, some of the symmetries that were used to minimize the number of diagrams could not be applied in this case and to determine the $T^{\rm ns}_{3,N}$ 1076 (= 1 + 4 + 55 + 1016) diagrams had to be evaluated, which took about 6 weeks for the moments N=1,3,5,7,9,11,13.

5 Results

Using the strategies sketched in the previous section we find the following results for the coefficient functions and anomalous dimensions. Again following Ref. [23], we present the combined singlet and non-singlet results for F_2 and F_L in terms of flavour factors which are defined in the following table for n_f number of flavours:

	fl_2	fl_{11}	fl_{02}	fl^g_2	fl^g_{11}
non-singlet	1	$\frac{3}{n_f}\sum_{f=1}^{n_f} e_f$	0	-	-
singlet	1	$\frac{1}{n_f}\frac{\left(\sum_{f=1}^{n_f} e_f\right)^2}{\sum_{f=1}^{n_f} e_f^2}$	1	1	$\frac{1}{n_f}\frac{\left(\sum_{f=1}^{n_f} e_f\right)^2}{\sum_{f=1}^{n_f} e_f^2}$

The numerical values of the anomalous dimensions for the spin-even operators contributing to F_2 and F_L that now are known to NNL order are:

$$\begin{aligned}
\gamma_2^{\psi\psi} &= 3.555555556\,a_s + a_s^2\,(48.32921811 - 3.160493827\,n_f - 1.975308642\,\mathrm{fl}_{02}\,n_f)\\
&\quad + a_s^3\,(859.4478372 - 133.4381617\,n_f - 1.229080933\,n_f^2\\
&\quad + \mathrm{fl}_{02}\,(-42.21182429\,n_f - 3.445816187\,n_f^2))\\
\gamma_4^{\psi\psi} &= 6.977777778\,a_s + a_s^2\,(86.28665021 - 6.553580247\,n_f - 0.1060246914\,\mathrm{fl}_{02}\,n_f)\\
&\quad + a_s^3\,(1515.562363 - 244.728592\,n_f - 2.108515775\,n_f^2\\
&\quad + \mathrm{fl}_{02}\,(-5.17013312\,n_f - 0.6789278464\,n_f^2))\\
\gamma_6^{\psi\psi} &= 9.003174603\,a_s + a_s^2\,(108.0184697 - 8.62925674\,n_f - 0.02080408883\,\mathrm{fl}_{02}\,n_f)\\
&\quad + a_s^3\,(1891.827779 - 307.4236889\,n_f - 2.570638992\,n_f^2\\
&\quad + \mathrm{fl}_{02}\,(-2.526091192\,n_f - 0.2884556035\,n_f^2))\\
\gamma_8^{\psi\psi} &= 10.45820106\,a_s + a_s^2\,(123.7764525 - 10.14583662\,n_f - 0.006586485074\,\mathrm{fl}_{02}\,n_f)\\
&\quad + a_s^3\,(2164.091836 - 352.3116596\,n_f - 2.882493484\,n_f^2\\
&\quad + \mathrm{fl}_{02}\,(-1.682156519\,n_f - 0.1620816452\,n_f^2))\\
\gamma_{10}^{\psi\psi} &= 11.5969216\,a_s + a_s^2\,(136.2741775 - 11.34594534\,n_f - 0.00269944007\,\mathrm{fl}_{02}\,n_f)\\
&\quad + a_s^3\,(2379.919952 - 387.6422968\,n_f - 3.115523145\,n_f^2\\
&\quad + \mathrm{fl}_{02}\,(-1.256562245\,n_f - 0.1054295071\,n_f^2))\\
\gamma_{12}^{\psi\psi} &= 12.53336293\,a_s + a_s^2\,(146.6771964 - 12.34063169\,n_f - 0.001302012432\,\mathrm{fl}_{02}\,n_f)\\
&\quad + a_s^3\,(2559.641948 - 416.9091301\,n_f - 3.300349343\,n_f^2\\
&\quad + \mathrm{fl}_{02}\,(-0.9957297081\,n_f - 0.07498848634\,n_f^2))\\
\gamma_{14}^{\psi\psi,NS} &= 13.32896733\,a_s + a_s^2\,(155.6071962 - 13.19066078\,n_f)\\
&\quad + a_s^3\,(2714.031720 - 441.9494048\,n_f - 3.452881831\,n_f^2)
\end{aligned}$$

$$
\begin{aligned}
\gamma_2^{\psi G} &= -0.6666666667\, a_s\, n_f - 7.543209877\, a_s^2\, n_f \\
&\quad + a_s^3\, \left(-37.62337275\, n_f + 12.11248285\, n_f^2\right) \\
\gamma_4^{\psi G} &= -0.3666666667\, a_s\, n_f + 1.290703704\, a_s^2\, n_f \\
&\quad + a_s^3\, \left(33.58149273\, n_f + 6.06027262\, n_f^2\right) \\
\gamma_6^{\psi G} &= -0.2619047619\, a_s\, n_f + 2.761104812\, a_s^2\, n_f \\
&\quad + a_s^3\, \left(33.41602135\, n_f + 3.537682102\, n_f^2\right) \\
\gamma_8^{\psi G} &= -0.2055555556\, a_s\, n_f + 3.243957223\, a_s^2\, n_f \\
&\quad + a_s^3\, \left(28.7612615\, n_f + 2.225433112\, n_f^2\right) \\
\gamma_{10}^{\psi G} &= -0.1696969697\, a_s\, n_f + 3.407168695\, a_s^2\, n_f \\
&\quad + a_s^3\, \left(23.93704198\, n_f + 1.449828678\, n_f^2\right) \\
\gamma_{12}^{\psi G} &= -0.1446886447\, a_s\, n_f + 3.438705999\, a_s^2\, n_f \\
&\quad + a_s^3\, \left(19.63230379\, n_f + 0.9524545446\, n_f^2\right)
\end{aligned}
$$

$$
\begin{aligned}
\gamma_2^{G\psi} &= -3.555555556\, a_s + a_s^2\, \left(-48.32921811 + 5.135802469\, n_f\right) \\
&\quad + a_s^3\, \left(-859.4478372 + 175.649986\, n_f + 4.674897119\, n_f^2\right) \\
\gamma_4^{G\psi} &= -0.9777777778\, a_s + a_s^2\, \left(-16.1752428 + 0.6182716049\, n_f\right) \\
&\quad + a_s^3\, \left(-315.276255 + 39.82571027\, n_f + 1.801843621\, n_f^2\right) \\
\gamma_6^{G\psi} &= -0.5587301587\, a_s + a_s^2\, \left(-9.496317796 + 0.08884857647\, n_f\right) \\
&\quad + a_s^3\, \left(-188.9088124 + 19.67944546\, n_f + 1.087843741\, n_f^2\right) \\
\gamma_8^{G\psi} &= -0.3915343915\, a_s + a_s^2\, \left(-6.757603506 - 0.07061952353\, n_f\right) \\
&\quad + a_s^3\, \left(-134.7055042 + 12.3754454\, n_f + 0.7536013741\, n_f^2\right) \\
\gamma_{10}^{G\psi} &= -0.3016835017\, a_s + a_s^2\, \left(-5.297576945 - 0.1348941718\, n_f\right) \\
&\quad + a_s^3\, \left(-104.911278 + 8.796702078\, n_f + 0.5579674847\, n_f^2\right) \\
\gamma_{12}^{G\psi} &= -0.2455322455\, a_s + a_s^2\, \left(-4.398625917 - 0.1639529655\, n_f\right) \\
&\quad + a_s^3\, \left(-86.18107998 + 6.735609285\, n_f + 0.4293379075\, n_f^2\right)
\end{aligned}
$$

$$
\begin{aligned}
\gamma_2^{GG} &= 0.6666666667\, a_s\, n_f + 7.543209877\, a_s^2\, n_f \\
&\quad + a_s^3\, \left(37.62337275\, n_f - 12.11248285\, n_f^2\right) \\
\gamma_4^{GG} &= a_s^2\, \left(128.178 - 13.64948148\, n_f\right) + a_s\, \left(12.6 + 0.6666666667\, n_f\right) \\
&\quad + a_s^3\, \left(2066.19278 - 401.3127939\, n_f - 10.43150645\, n_f^2\right) \\
\gamma_6^{GG} &= a_s^2\, \left(183.0538144 - 20.46668466\, n_f\right) + a_s\, \left(17.78571429 + 0.6666666667\, n_f\right) \\
&\quad + a_s^3\, \left(2987.042058 - 566.6373298\, n_f - 10.78060861\, n_f^2\right) \\
\gamma_8^{GG} &= a_s^2\, \left(219.6240988 - 24.69926432\, n_f\right) + a_s\, \left(21.26666667 + 0.6666666667\, n_f\right) \\
&\quad + a_s^3\, \left(3609.35419 - 673.9430658\, n_f - 11.20133837\, n_f^2\right) \\
\gamma_{10}^{GG} &= a_s^2\, \left(247.6655484 - 27.82178573\, n_f\right) + a_s\, \left(23.92337662 + 0.6666666667\, n_f\right) \\
&\quad + a_s^3\, \left(4089.236943 - 755.1340541\, n_f - 11.57068198\, n_f^2\right) \\
\gamma_{12}^{GG} &= a_s^2\, \left(270.6428892 - 30.31377688\, n_f\right) + a_s\, \left(26.08168498 + 0.6666666667\, n_f\right) \\
&\quad + a_s^3\, \left(4483.563048 - 821.1236576\, n_f - 11.88665683\, n_f^2\right)
\end{aligned}
$$

The corresponding coefficient functions read:

$$
\begin{aligned}
C^{\psi}_{2,2} &= 1 + 0.4444444444\,a_s + a_s^2\,\left(17.69376589 - 5.333333333\,n_f - 2.189300412\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(442.7409693 - 165.1971095\,n_f - 24.09201335\,\mathrm{fl}_{11}\,n_f + 6.030272415\,n_f^2\right.\\
&\quad \left. + \mathrm{fl}_{02}\,\left(-79.04486142\,n_f + 3.325504478\,n_f^2\right)\right)\\
C^{\psi}_{2,4} &= 1 + 6.066666667\,a_s + a_s^2\,\left(142.3434719 - 16.98791358\,n_f + 0.4858308642\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(4169.267888 - 901.2351626\,n_f - 18.21884618\,\mathrm{fl}_{11}\,n_f + 23.35503924\,n_f^2\right.\\
&\quad \left. + \mathrm{fl}_{02}\,\left(16.64834849\,n_f - 2.208630689\,n_f^2\right)\right)\\
C^{\psi}_{2,6} &= 1 + 11.17671958\,a_s + a_s^2\,\left(302.398735 - 28.0130504\,n_f + 0.4868787285\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(10069.63085 - 1816.322929\,n_f - 16.14271761\,\mathrm{fl}_{11}\,n_f + 42.66273116\,n_f^2\right.\\
&\quad \left. + \mathrm{fl}_{02}\,\left(24.11778813\,n_f - 1.525489143\,n_f^2\right)\right)\\
C^{\psi}_{2,8} &= 1 + 15.52989418\,a_s + a_s^2\,\left(470.807419 - 37.9248228\,n_f + 0.3859585393\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(17162.37245 - 2787.297692\,n_f - 15.09203827\,\mathrm{fl}_{11}\,n_f + 61.91177997\,n_f^2\right.\\
&\quad \left. + \mathrm{fl}_{02}\,\left(22.33201938\,n_f - 1.036308122\,n_f^2\right)\right)\\
C^{\psi}_{2,10} &= 1 + 19.30061568\,a_s + a_s^2\,\left(639.210663 - 46.86131842\,n_f + 0.3045901308\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(24953.13497 - 3770.10212\,n_f - 14.45874451\,\mathrm{fl}_{11}\,n_f + 80.52097973\,n_f^2\right.\\
&\quad \left. + \mathrm{fl}_{02}\,\left(19.53359559\,n_f - 0.7372434464\,n_f^2\right)\right)\\
C^{\psi}_{2,12} &= 1 + 22.62841097\,a_s + a_s^2\,\left(804.5854321 - 54.99446579\,n_f + 0.2451231747\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(33171.45501 - 4746.440949\,n_f - 14.03541028\,\mathrm{fl}_{11}\,n_f + 98.3483124\,n_f^2\right.\\
&\quad \left. + \mathrm{fl}_{02}\,\left(16.98652635\,n_f - 0.5471547625\,n_f^2\right)\right)\\
C^{\psi,NS}_{2,14} &= 1 + 2.561093284\,a_s + a_s^2\,\left(965.8132564 - 62.46549093\,n_f\right)\\
&\quad + a_s^3\,\left(41657.11568 - 5708.215623\,n_f - 13.73240102\,\mathrm{fl}_{11}\,n_f + 115.3919490\,n_f^2\right)
\end{aligned}
$$

$$
\begin{aligned}
C^{G}_{2,2} &= -0.5\,a_s\,n_f - 8.918338961\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(-130.7340963\,n_f + 29.37933515\,n_f^2 - 0.9007972776\,\mathrm{fl}^g_{11}\,n_f^2\right)\\
C^{G}_{2,4} &= -0.7388888889\,a_s\,n_f - 14.27158692\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(-346.4612756\,n_f + 46.52017564\,n_f^2 - 1.611816512\,\mathrm{fl}^g_{11}\,n_f^2\right)\\
C^{G}_{2,6} &= -0.7051587302\,a_s\,n_f - 20.06849828\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(-715.0372438\,n_f + 61.28545096\,n_f^2 - 1.496036938\,\mathrm{fl}^g_{11}\,n_f^2\right)\\
C^{G}_{2,8} &= -0.6440873016\,a_s\,n_f - 23.17873524\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(-996.5038709\,n_f + 68.66467304\,n_f^2 - 1.286400915\,\mathrm{fl}^g_{11}\,n_f^2\right)\\
C^{G}_{2,10} &= -0.5861279461\,a_s\,n_f - 24.76678064\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(-1201.206903\,n_f + 72.23614791\,n_f^2 - 1.094394334\,\mathrm{fl}^g_{11}\,n_f^2\right)\\
C^{G}_{2,12} &= -0.5358430591\,a_s\,n_f - 25.51669345\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(-1351.047836\,n_f + 73.7936445\,n_f^2 - 0.9344248731\,\mathrm{fl}^g_{11}\,n_f^2\right)
\end{aligned}
$$

$$
\begin{aligned}
C^{\psi}_{L,2} &= 1.777777778\,a_s + a_s^2\,\left(56.75530152 - 4.543209877\,n_f - 3.950617284\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(2544.598087 - 421.6908885\,n_f - 7.736698288\,\mathrm{fl}_{11}\,n_f + 11.8957476\,n_f^2\right.\\
&\quad \left.+\mathrm{fl}_{02}\,\left(-213.9253076\,n_f + 17.91326528\,n_f^2\right)\right)\\
C^{\psi}_{L,4} &= 1.066666667\,a_s + a_s^2\,\left(47.99398931 - 3.413333333\,n_f - 0.6945185185\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(2523.73902 - 383.0520013\,n_f - 5.058869512\,\mathrm{fl}_{11}\,n_f + 10.88895473\,n_f^2\right.\\
&\quad \left.+\mathrm{fl}_{02}\,\left(-55.5530456\,n_f + 2.348005487\,n_f^2\right)\right)\\
C^{\psi}_{L,6} &= 0.7619047619\,a_s + a_s^2\,\left(40.9961976 - 2.69569161\,n_f - 0.2524824533\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(2368.193775 - 340.0691069\,n_f - 3.705612526\,\mathrm{fl}_{11}\,n_f + 9.472190428\,n_f^2\right.\\
&\quad \left.+\mathrm{fl}_{02}\,\left(-24.01322539\,n_f + 0.7652692585\,n_f^2\right)\right)\\
C^{\psi}_{L,8} &= 0.5925925926\,a_s + a_s^2\,\left(35.87664406 - 2.231471683\,n_f - 0.1217397796\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(2215.210875 - 305.4730329\,n_f - 2.913702563\,\mathrm{fl}_{11}\,n_f + 8.337149534\,n_f^2\right.\\
&\quad \left.+\mathrm{fl}_{02}\,\left(-12.97185267\,n_f + 0.344362391\,n_f^2\right)\right)\\
C^{\psi}_{L,10} &= 0.4848484848\,a_s + a_s^2\,\left(32.01765947 - 1.908598248\,n_f - 0.06856377261\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(2081.213222 - 278.0172177\,n_f - 2.397641695\,\mathrm{fl}_{11}\,n_f + 7.452505612\,n_f^2\right.\\
&\quad \left.+\mathrm{fl}_{02}\,\left(-7.947555677\,n_f + 0.1841598535\,n_f^2\right)\right)\\
C^{\psi}_{L,12} &= 0.4102564103\,a_s + a_s^2\,\left(29.0058065 - 1.671007435\,n_f - 0.04265241396\,\mathrm{fl}_{02}\,n_f\right)\\
&\quad + a_s^3\,\left(1965.791047 - 255.8431044\,n_f - 2.035689631\,\mathrm{fl}_{11}\,n_f + 6.751061503\,n_f^2\right.\\
&\quad \left.+\mathrm{fl}_{02}\,\left(-5.284573837\,n_f + 0.1098463658\,n_f^2\right)\right)\\
C^{\psi,NS}_{L,14} &= 0.3555555556\,a_s + a_s^2\,\left(26.5848844 - 1.488624298\,n_f\right)\\
&\quad + a_s^3\,\left(1866.009187 - 237.5642566\,n_f - 1.768138102\,\mathrm{fl}_{11}\,n_f + 6.182499654\,n_f^2\right)
\end{aligned}
$$

$$
\begin{aligned}
C^{G}_{L,2} &= 0.6666666667\,a_s\,n_f + 12.94776709\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(407.280632\,n_f - 20.23959748\,n_f^2 - 0.388939664\,\mathrm{fl}^g_{11}\,n_f^2\right)\\
C^{G}_{L,4} &= 0.2666666667\,a_s\,n_f + 13.81659259\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(767.7125421\,n_f - 36.78419232\,n_f^2 - 0.3984298404\,\mathrm{fl}^g_{11}\,n_f^2\right)\\
C^{G}_{L,6} &= 0.1428571429\,a_s\,n_f + 10.26095364\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(694.5092121\,n_f - 27.9895081\,n_f^2 - 0.3055276256\,\mathrm{fl}^g_{11}\,n_f^2\right)\\
C^{G}_{L,8} &= 0.08888888889\,a_s\,n_f + 7.733545104\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(592.3307972\,n_f - 21.30333681\,n_f^2 - 0.2322211886\,\mathrm{fl}^g_{11}\,n_f^2\right)\\
C^{G}_{L,10} &= 0.06060606061\,a_s\,n_f + 6.023053074\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(504.5424832\,n_f - 16.70544537\,n_f^2 - 0.1805482229\,\mathrm{fl}^g_{11}\,n_f^2\right)\\
C^{G}_{L,12} &= 0.04395604396\,a_s\,n_f + 4.830676204\,a_s^2\,n_f\\
&\quad + a_s^3\,\left(433.9050534\,n_f - 13.47418408\,n_f^2 - 0.1439099345\,\mathrm{fl}^g_{11}\,n_f^2\right)
\end{aligned}
$$

The numerical values of the anomalous dimensions for the odd moments of F_3 read:

$$
\begin{aligned}
\gamma_1^{\rm ns} &= 0 \\
\gamma_3^{\rm ns} &= 5.555555556\,a_s + a_s^2\,(70.88477366 - 5.12345679\,n_f) \\
&\quad + a_s^3\,(1244.913602 - 196.4738081\,n_f - 1.762002743\,n_f^2) \\
\gamma_5^{\rm ns} &= 8.088888889\,a_s + a_s^2\,(98.19940741 - 7.68691358\,n_f) \\
&\quad + a_s^3\,(1720.942172 - 278.1581739\,n_f - 2.366211248\,n_f^2) \\
\gamma_7^{\rm ns} &= 9.780952381\,a_s + a_s^2\,(116.4158903 - 9.437457798\,n_f) \\
&\quad + a_s^3\,(2036.492478 - 330.8816595\,n_f - 2.739358023\,n_f^2) \\
\gamma_9^{\rm ns} &= 11.05820106\,a_s + a_s^2\,(130.3414045 - 10.77682428\,n_f) \\
&\quad + a_s^3\,(2277.19805 - 370.5905277\,n_f - 3.006446616\,n_f^2) \\
\gamma_{11}^{\rm ns} &= 12.08581049\,a_s + a_s^2\,(141.6907901 - 11.86441897\,n_f) \\
&\quad + a_s^3\,(2473.311857 - 402.691565\,n_f - 3.212753995\,n_f^2) \\
\gamma_{13}^{\rm ns} &= 12.94606135\,a_s + a_s^2\,(151.2989044 - 12.78102552\,n_f) \\
&\quad + a_s^3\,(2639.409887 - 429.7314605\,n_f - 3.37996738\,n_f^2)
\end{aligned}
$$

and the coefficient functions are:

$$
\begin{aligned}
C_{3,1}^{\rm ns} &= 1 - 4\,a_s + a_s^2\,(-73.33333333 + 5.333333333\,n_f) \\
&\quad + a_s^3\,(-2652.154437 + 513.3100408\,n_f - 11.35802469\,n_f^2) \\
C_{3,3}^{\rm ns} &= 1 + 1.666666667\,a_s + a_s^2\,(14.25404015 - 6.742283951\,n_f) \\
&\quad + a_s^3\,(-839.7638717 - 45.09953407\,n_f + 1.747689309\,n_f^2) \\
C_{3,5}^{\rm ns} &= 1 + 7.748148148\,a_s + a_s^2\,(173.000629 - 19.39801646\,n_f) \\
&\quad + a_s^3\,(4341.081057 - 961.2756356\,n_f + 22.24125078\,n_f^2) \\
C_{3,7}^{\rm ns} &= 1 + 12.72248677\,a_s + a_s^2\,(345.9910777 - 30.52332666\,n_f) \\
&\quad + a_s^3\,(11119.00053 - 1960.237096\,n_f + 43.10377964\,n_f^2) \\
C_{3,9}^{\rm ns} &= 1 + 16.9152381\,a_s + a_s^2\,(520.0059615 - 40.35464229\,n_f) \\
&\quad + a_s^3\,(18771.99642 - 2975.924131\,n_f + 63.17127673\,n_f^2) \\
C_{3,11}^{\rm ns} &= 1 + 20.54831329\,a_s + a_s^2\,(690.8719666 - 49.17096968\,n_f) \\
&\quad + a_s^3\,(26941.47987 - 3984.411605\,n_f + 82.24581704\,n_f^2) \\
C_{3,13}^{\rm ns} &= 1 + 23.76237745\,a_s + a_s^2\,(857.1778817 - 57.18099124\,n_f) \\
&\quad + a_s^3\,(35426.82868 - 4976.080869\,n_f + 100.3509187\,n_f^2)
\end{aligned}
$$

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A Conventions

Here we give the complete expressions for the newly computed moments and coefficient functions.

The notation of the color factors is as usual: The Casimir operators of the fundamental and adjoint representation are denoted by C_F and C_A and their values for the color group SU(3) are $\frac{4}{3}$ and 3, respectively. Additionally we are using the symmetric structure constants of SU(3) for which $d^{abc}d^{abc} = \frac{40}{3}$. For the trace normalization of the fundamental representation we have inserted $T_F = \frac{1}{2}$. For a generic SU(n) group the number of generators equals $N_A = n^2 - 1$.

The values of the Riemann ζ function are written as $\zeta_n = \zeta(n)$. The only check for the newly computed 3-loop results is from a large n_f-expansion of [38] where the n_f^2 terms are calculated. These terms are in agreement with ours. To test the new parallel version of FORM we have also recalculated the lower moments for F_2 and F_L and found complete agreement with [23] except for one missing term. In $C_{2,2}^G$ there should be the extra term $a_s^3 n_f C_F^2(-\frac{160}{3}\zeta_5)$. The numerical values given in this reference are correct.

It should be noted that the terms in $d^{abc}d^{abc}$ enter for the first time at the three loop level and help in the determination of $P_{qq}^S - P_{q\overline{q}}^S$.

B Results for the moments of F_2 and F_L

$$
\begin{aligned}
\gamma_{10}^{GG} &= a_s C_A \left[+\frac{18421}{2310}\right] + a_s n_f \left[+\frac{2}{3}\right] \\
&+ a_s^2 C_A^2 \left[+\frac{339202487377}{12326391000}\right] + a_s^2 C_F n_f \left[+\frac{9284182}{4492125}\right] + a_s^2 C_A n_f \left[-\frac{17481908}{1715175}\right] \\
&+ a_s^3 C_A^3 \left[+\frac{239083526238286750523}{1578596520362400000}\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{3374081335517123191}{36243287457300000} + \frac{17831164}{190575}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{3009386129483453}{3883209370425000} - \frac{1344}{3025}\zeta_3\right] \\
&+ a_s^3 C_A^2 n_f \left[+\frac{43228502203851731}{2196562876200000} - \frac{17746492}{190575}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[-\frac{453912946493}{420260754375}\right] + a_s^3 C_A n_f^2 \left[-\frac{2752314359}{815051160}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{12}^{GG} &= a_s C_A \left[+\frac{71203}{8190}\right] + a_s n_f \left[+\frac{2}{3}\right] + a_s^2 C_A^2 \left[+\frac{165198392441 57}{549353259000}\right] \\
&+ a_s^2 C_A n_f \left[-\frac{23220103}{2108106}\right] + a_s^2 C_F n_f \left[+\frac{74823503}{36540504}\right] \\
&+ a_s^3 C_A^3 \left[+\frac{6993119873800651614841}{42112541869725600000}\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{49693541388602890695713}{498238162032042432000} + \frac{58085396}{585585}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{4699124115250144376149}{5480619782352466752000} - \frac{158}{507}\zeta_3\right] \\
&+ a_s^3 C_A^2 n_f \left[+\frac{74338654569222233539}{3871314390303360000} - \frac{57902906}{585585}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[-\frac{50627726543561953}{45626205314289600}\right] \\
&+ a_s^3 C_A n_f^2 \left[-\frac{2635361358193}{759677078160}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{10}^{G\psi} &= a_s C_F \left[-\frac{112}{495}\right] \\
&+ a_s^2 C_F C_A \left[-\frac{133349533}{80858250}\right] + a_s^2 C_F^2 \left[+\frac{88631998}{121287375}\right] + a_s^2 C_F n_f \left[-\frac{74368}{735075}\right] \\
&+ a_s^3 C_F C_A^2 \left[-\frac{165894431274725803}{48324383276400000} - \frac{645584}{81675}\zeta_3\right] \\
&+ a_s^3 C_F^2 C_A \left[-\frac{4224421791031474951}{217459724743800000} + \frac{645584}{27225}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[+\frac{6457897459084371893}{326189587115700000} - \frac{1291168}{81675}\zeta_3\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{18846629176433}{47069204490000} + \frac{1792}{495}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[+\frac{529979902254031}{1294403123475000} - \frac{1792}{495}\zeta_3\right] + a_s^3 C_F n_f^2 \left[+\frac{152267426}{363862125}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{12}^{G\psi} &= a_s C_F \left[-\frac{79}{429}\right] + a_s^2 C_F C_A \left[-\frac{70863259553}{50645138544}\right] + a_s^2 C_F^2 \left[+\frac{9387059226553}{13927413099600}\right] \\
&+ a_s^2 C_F n_f \left[-\frac{14257247}{115945830}\right] \\
&+ a_s^3 C_F C_A^2 \left[-\frac{24751969909767240119153}{14947144860961272960000} - \frac{46179137}{6441435}\zeta_3\right] \\
&+ a_s^3 C_F^2 C_A \left[-\frac{11598659918312280997 4023}{5872092623949071520000} + \frac{46179137}{2147145}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[+\frac{21659273057249922158947 03}{11303778301101962676000 0} - \frac{92358274}{6441435}\zeta_3\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{64190493078139789}{195540879918384000} + \frac{1264}{429}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[+\frac{1401404001326440151}{13981172914164456000} - \frac{1264}{429}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{13454024393417}{41782239298800}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{10}^{\psi G} &= a_s n_f \left[-\frac{28}{165}\right] + a_s^2 C_F n_f \left[-\frac{379479917}{125779500}\right] + a_s^2 C_A n_f \left[+\frac{373810079}{150935400}\right] \\
&+ a_s^3 C_F C_A n_f \left[+\frac{926990216580622991}{24162191638200000} - \frac{643396}{27225}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{1091980048536213833}{54364931185950000} + \frac{17712}{3025}\zeta_3\right] \\
&+ a_s^3 C_A^2 n_f \left[-\frac{21025430857658971}{1022738270400000} + \frac{483988}{27225}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{1584713325754369}{2588806246950000}\right] + a_s^3 C_A n_f^2 \left[+\frac{1669885489}{7906140000}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{12}^{\psi G} &= a_s n_f \left[-\frac{79}{546}\right] \\
&+ a_s^2 C_F n_f \left[-\frac{9256843807}{3197294100}\right] + a_s^2 C_A n_f \left[+\frac{653436358741}{268572704400}\right] \\
&+ a_s^3 C_F C_A n_f \left[+\frac{40460325300210081486419 59}{10463001402672891072000 0} - \frac{171207527}{8198190}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{29601183661211541861450 47}{14386626928675225224000 0}\right] + \frac{2563}{507}\zeta_3\right] \\
&+ a_s^3 C_A^2 n_f \left[-\frac{10876559659107463644949}{54353254039859174400 0} + \frac{129763817}{8198190}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{1490819476931356358 81}{24911908101602121600 0}\right] \\
&+ a_s^3 C_A n_f^2 \left[+\frac{226617401255197}{4399220898072000}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{10}^{\psi\psi} &= a_s C_F \left[+\frac{12055}{1386}\right] \\
&+ a_s^2 C_F C_A \left[+\frac{19524247733}{523908000}\right] + a_s^2 C_F^2 \left[-\frac{9579051036701}{1331250228000}\right] \\
&+ a_s^2 C_F n_f \left[-\frac{2451995507}{288149400}\right] + a_s^2 \mathrm{fl}_{02} C_F n_f \left[-\frac{27284}{13476375}\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{94091568579766453}{435681892800000} + \frac{151796299}{8004150}\zeta_3\right] \\
&+ a_s^3 C_F^2 C_A \left[-\frac{16389982059548833}{465937579800000} - \frac{151796299}{2668050}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{2207711300808736405687}{127866318149354400000} + \frac{151796299}{4002075}\zeta_3\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{9007773127403}{389001690000} - \frac{48220}{693}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{75522073210471127}{1230075210672000} + \frac{48220}{693}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[-\frac{27995901056887}{11981252052000}\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F C_A n_f \left[-\frac{1028766412107043}{5177612493900000} - \frac{12544}{27225}\zeta_3\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F^2 n_f \left[+\frac{209966063746798}{485401171303125} + \frac{12544}{27225}\zeta_3\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F n_f^2 \left[-\frac{33230913134}{420260754375}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{12}^{\psi\psi} &= a_s C_F \left[+\frac{423424}{45045}\right] \\
&+ a_s^2 C_F C_A \left[+\frac{19487270392267}{486972486000}\right] + a_s^2 C_F^2 \left[-\frac{5507868301548461}{731189187729000}\right] \\
&+ a_s^2 C_F n_f \left[-\frac{90143221429}{9739449720}\right] + a_s^2 \mathrm{fl}_{02} C_F n_f \left[-\frac{249775}{255783528}\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{1395004186652448755863}{6016642459027200000} + \frac{25648239313}{1352701350}\zeta_3\right] \\
&+ a_s^3 C_F^2 C_A \left[-\frac{98204412073910020058227}{2634913356900224400000} - \frac{25648239313}{450900450}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{8163014101177279144633005 7}{4747586886462824323920000} + \frac{25648239313}{676350675}\zeta_3\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{25478252190337435009}{1052912430329760000} - \frac{3387392}{45045}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{35346062280941906036867}{526982671380044880000} + \frac{3387392}{45045}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[-\frac{651555853387858071}{26322810758244000}\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F C_A n_f \left[-\frac{69697489543846494691}{332158774688028288000} - \frac{12482}{39039}\zeta_3\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F^2 n_f \left[+\frac{86033255402443256197}{2192247912940986700 80} + \frac{12482}{39039}\zeta_3\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F n_f^2 \left[-\frac{2566080055386457}{45626205314289600}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{14}^{\mathrm{ns}} &= a_s C_F \left[+\frac{180121}{18018}\right] \\
&+ a_s^2 C_F C_A \left[+\frac{288858136265399}{6817614804000}\right] \\
&+ a_s^2 C_F n_f \left[-\frac{481761665447}{48697248600}\right] \\
&+ a_s^2 C_F^2 \left[-\frac{228191423813134 07}{2924756750916000}\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{92531316363319241549}{3685193506154160000} - \frac{720484}{9009}\zeta_3\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{126653245164236390142889}{515927090861582400000} + \frac{3663695353}{193243050}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[-\frac{681671662577670 19}{26322810758244000}\right] \\
&+ a_s^3 C_F^2 C_A \left[-\frac{960013337169036214883 87}{245925246644020944000 0} - \frac{3663695353}{64414350}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{379085447979756145127 33}{526982671380044880000} + \frac{720484}{9009}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{405523957460648712422117 09}{237379344323141216196000 0} + \frac{3663695353}{96621525}\zeta_3\right]
\end{aligned}
$$

$$
\begin{aligned}
C^{\psi}_{2,10} &= 1 + a_s C_F \left[+\frac{2006299}{138600}\right] + a_s^2 C_F C_A \left[+\frac{61240931938241 87}{29045459520000} - \frac{104674}{1155}\zeta_3\right] \\
&+ a_s^2 C_F n_f \left[-\frac{561457267429757}{15975002736000}\right] + a_s^2 C_F^2 \left[+\frac{558708799987324013}{14760902528064000} + \frac{88798}{1155}\zeta_3\right] \\
&+ a_s^2 \mathrm{fl}_{02} C_F n_f \left[+\frac{3584203788491}{15689734830000}\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{216642449260393572 14987}{23550349033411200000} + \frac{10519793104}{42567525}\zeta_3 - \frac{24110}{693}\zeta_4\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{709221119965457939095237}{235503490334112000000} - \frac{14713925739913}{6243237000}\zeta_3\right. \\
&\qquad \left. + \frac{151796299}{16008300}\zeta_4 + \frac{190858}{231}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{570844280478515519 11}{996360920644320000} + \frac{48220}{18711}\zeta_3\right] \\
&+ a_s^3 C_F^2 C_A \left[+\frac{163500093049269333896088 29}{836943173341228800000} + \frac{1430215936081}{6163195500}\zeta_3\right. \\
&\qquad \left. - \frac{151796299}{5336100}\zeta_4 - \frac{22658}{99}\zeta_5\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{152138746003699406101 0049}{272006531335899360000 0} - \frac{3997754476}{42567525}\zeta_3 + \frac{24110}{693}\zeta_4\right] \\
&+ a_s^3 C_F^3 \left[-\frac{324777953237092062377 0610131}{921558128166027031680 00000} + \frac{2182208825245282}{1622461215375}\zeta_3\right. \\
&\qquad \left. + \frac{151796299}{8004150}\zeta_4 - \frac{75212}{99}\zeta_5\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F C_A n_f \left[+\frac{3566946294536415188593}{861140509985448000000} - \frac{79527463}{56600775}\zeta_3 - \frac{6272}{27225}\zeta_4\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F n_f^2 \left[-\frac{148475806971656561}{2446421903367750 00} + \frac{33008}{735075}\zeta_3\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F^2 n_f \left[+\frac{422577250875954453771617}{587728398065068260000 00} - \frac{1294910501 2}{11037151125}\zeta_3 + \frac{6272}{27225}\zeta_4\right] \\
&+ a_s^3 \mathrm{fl}_{11} n_f \frac{d^{abc} d^{abc}}{n} \left[+\frac{3753913187503}{352066176000} + \frac{81388}{606375}\zeta_3 - \frac{448}{33}\zeta_5\right]
\end{aligned}
$$

$$
\begin{aligned}
C^{\psi}_{2,12} &= 1 + a_s C_F \left[+ \frac{183473419}{10810800} \right] \\
&+ a_s^2 C_F C_A \left[+ \frac{21388499873332252399}{87742702527480000} - \frac{1477711}{15015}\zeta_3 \right] \\
&+ a_s^2 C_F^2 \left[+ \frac{91271109157024077989 41}{131745667845011220000} + \frac{1261726}{15015}\zeta_3 \right] \\
&+ a_s^2 C_F n_f \left[- \frac{57904356630607013}{1403883240439680} \right] + a_s^2 \mathrm{fl}_{02} C_F n_f \left[+ \frac{1035559286632 69}{563286485361600} \right] \\
&+ a_s^3 C_F C_A n_f \left[- \frac{83712626229337204073275967}{75885504678726462720000} + \frac{36842282041}{133783650}\zeta_3 \right. \\
&\qquad \left. - \frac{1693696}{45045}\zeta_4 \right] + a_s^3 C_F C_A^2 \left[+ \frac{16318162036768790786440405 4279}{45531302807235877632000000} \right. \\
&\qquad \left. - \frac{641004330821357}{243486243000}\zeta_3 + \frac{25648239313}{2705402700}\zeta_4 + \frac{2642336}{3003}\zeta_5 \right] \\
&+ a_s^3 C_F n_f^2 \left[+ \frac{2003755100099657438423887}{28457064254522423520000} + \frac{3387392}{1216215}\zeta_3 \right] \\
&+ a_s^3 C_F^2 C_A \left[+ \frac{48431919865268491573003338041 09}{17091312791266167566112000 00} + \frac{4948562279669051}{97491891697200}\zeta_3 \right. \\
&\qquad \left. - \frac{25648239313}{901800900}\zeta_4 - \frac{241094}{1287}\zeta_5 \right] \\
&+ a_s^3 C_F^2 n_f \left[- \frac{84838981067097560083179 8557}{11300041514867223577600 0} - \frac{1326399435709}{12174312150}\zeta_3 \right. \\
&\qquad \left. + \frac{1693696}{45045}\zeta_4 \right] + a_s^3 C_F^3 \left[+ \frac{13177036739377353353636388 9790633}{34216808208114867467356224 00000} \right. \\
&\qquad \left. + \frac{42634681331415644}{24926904127125}\zeta_3 + \frac{25648239313}{1352701350}\zeta_4 - \frac{88004}{99}\zeta_5 \right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F C_A n_f \left[+ \frac{86565903158314138806418902631}{26931765610480021619328000000} - \frac{3109476222803}{3165321159000}\zeta_3 \right. \\
&\qquad \left. - \frac{6241}{39039}\zeta_4 \right] + a_s^3 \mathrm{fl}_{02} C_F n_f^2 \left[- \frac{1960867603733060624851}{4404069467961803640000} + \frac{2780}{95823}\zeta_3 \right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F^2 n_f \left[+ \frac{33528586559068805780843402423}{52901682449157185323680000 00} - \frac{751723347541}{791330289750}\zeta_3 \right. \\
&\qquad \left. + \frac{6241}{39039}\zeta_4 \right] \\
&+ a_s^3 \mathrm{fl}_{11} n_f \frac{d^{abc} d^{abc}}{n} \left[+ \frac{809917806143013559}{67966375276800000} + \frac{622064791}{851350500}\zeta_3 - \frac{200}{13}\zeta_5 \right]
\end{aligned}
$$

$$
\begin{aligned}
C_{2,14}^{\rm ns} &= 1 + a_s C_F \left[+ \frac{90849502}{4729725} \right] \\
&+ a_s^2 C_F C_A \left[+ \frac{1345455725874078602801}{4913591341538880000} - \frac{315626}{3003} \zeta_3 \right] \\
&+ a_s^2 C_F n_f \left[- \frac{1644267296654871017}{35097081010992000} \right] \\
&+ a_s^2 C_F^2 \left[+ \frac{31002322638187643268973}{301132955074311360000} + \frac{271010}{3003} \zeta_3 \right] \\
&+ a_s^3 C_F C_A n_f \left[- \frac{28812973254576289068812626927}{22575937641921122659200000} + \frac{31112773830559}{103481653275} \zeta_3 \right. \\
&\left. - \frac{360242}{9009} \zeta_4 \right] \\
&+ a_s^3 C_F C_A^2 \left[+ \frac{78236211563500471751258156 27621}{18963787619213743033728000 00} - \frac{83168919211026563}{28974862917000} \zeta_3 \right. \\
&\left. + \frac{3663695353}{386486100} \zeta_4 + \frac{2738146}{3003} \zeta_5 \right] \\
&+ a_s^3 C_F n_f^2 \left[+ \frac{23614661638288534402180 87}{28457064254522423520000} + \frac{720484}{243243} \zeta_3 \right] \\
&+ a_s^3 C_F^2 C_A \left[+ \frac{64116556842577537005311631547547}{17091312791266167566112000000} - \frac{13682992796062061}{81243243081000} \zeta_3 \right. \\
&\left. - \frac{3663695353}{128828700} \zeta_4 - \frac{865562}{9009} \zeta_5 \right] \\
&+ a_s^3 C_F^2 n_f \left[- \frac{35845184538120717524004377 4137}{37734067201496733587520000 0} - \frac{12775152582499}{103481653275} \zeta_3 + \frac{360242}{9009} \zeta_4 \right] \\
&+ a_s^3 C_F^3 \left[+ \frac{12924407779985031268428280066600921}{72710717442244093368131976000000} + \frac{3556746663996971701}{1695029480644500} \zeta_3 \right. \\
&\left. + \frac{3663695353}{193243050} \zeta_4 - \frac{9512108}{9009} \zeta_5 \right] \\
&+ a_s^3 f l_{11} n_f \frac{d^{abc} d^{abc}}{n} \left[+ \frac{637395762233410021}{49587712574400000} + \frac{78376866703}{65553988500} \zeta_3 - \frac{352}{21} \zeta_5 \right]
\end{aligned}
$$

$$
\begin{aligned}
C^{\psi}_{L,10} &= a_s C_F \left[+\frac{4}{11}\right] \\
&+ a_s^2 C_F C_A \left[+\frac{89670761}{8731800} - \frac{48}{11}\zeta_3\right] \\
&+ a_s^2 C_F n_f \left[-\frac{163679}{114345}\right] \\
&+ a_s^2 C_F^2 \left[-\frac{1999510607}{528273900} + \frac{96}{11}\zeta_3\right] \\
&+ a_s^2 \mathrm{fl}_{02} C_F n_f \left[-\frac{415796}{8085825}\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{176183576988227323}{1699159381920000} + \frac{55485434}{1216215}\zeta_3\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{2366034921481985137}{6796637527680000} - \frac{95022195887}{187297110}\zeta_3 + \frac{3760}{11}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{63272639}{11320155}\right] \\
&+ a_s^3 C_F^2 C_A \left[-\frac{323139848004267269}{3354750574560000} + \frac{22904191}{17325}\zeta_3 - \frac{14240}{11}\zeta_5\right] \\
&+ a_s^3 C_F^2 n_f \left[+\frac{9048874326307637}{190368782604000} - \frac{1174256}{15015}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{887562386698645967383}{3166213592269728000} - \frac{357031607224}{468242775}\zeta_3 + \frac{13440}{11}\zeta_5\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F C_A n_f \left[-\frac{68379915239899511}{434919448948760000} - \frac{36224}{147015}\zeta_3\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F n_f^2 \left[+\frac{21670644503}{156897348300}\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F^2 n_f \left[-\frac{319520059852805113}{282697642166940000} + \frac{1373248}{1911195}\zeta_3\right] \\
&+ a_s^3 \mathrm{fl}_{11} n_f \frac{d^{abc}d^{abc}}{n} \left[-\frac{5073093424963}{528099264000} - \frac{1820773}{363825}\zeta_3 + \frac{160}{11}\zeta_5\right]
\end{aligned}
$$

$$
\begin{aligned}
C^{\psi}_{L,12} &= a_s C_F \left[+\frac{4}{13}\right] + a_s^2 C_F C_A \left[+\frac{7126442885209}{811620810000} - \frac{48}{13}\zeta_3\right] \\
&+ a_s^2 C_F n_f \left[-\frac{2201663}{1756755}\right] + a_s^2 C_F^2 \left[-\frac{12296141077867}{5275535265000} + \frac{96}{13}\zeta_3\right] \\
&+ a_s^2 \mathrm{fl}_{02} C_F n_f \left[-\frac{16072451}{502431930}\right] + a_s^3 C_F n_f^2 \left[+\frac{5203557911}{1027701675}\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{61335054761825219657}{658070268956100000} + \frac{2151978544}{52026975}\zeta_3\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{148978189688273389644 11}{526456215164880000 00} - \frac{5467652405147}{10145260125}\zeta_3 + \frac{5600}{13}\zeta_5\right] \\
&+ a_s^3 C_F^2 C_A \left[+\frac{150896405958473529481879 1}{263491335690022440000000} + \frac{3065115509662}{2029052025}\zeta_3 - \frac{21600}{13}\zeta_5\right] \\
&+ a_s^3 C_F^2 n_f \left[+\frac{12143703744427185053}{316848648015900000} - \frac{47440688}{675675}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{160913446784796686877078 41}{428173420496286465000000} - \frac{10180155542576}{10145260125}\zeta_3 + 1600\zeta_5\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F C_A n_f \left[-\frac{592278098533386728681}{576664539388938000000} - \frac{561855512}{3381753375}\zeta_3\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F n_f^2 \left[+\frac{435058406339681}{5280810800265000}\right] \\
&+ a_s^3 \mathrm{fl}_{02} C_F^2 n_f \left[-\frac{65983065928499265747263}{856346840992572930000 00} + \frac{1570446544}{3381753375}\zeta_3\right] \\
&+ a_s^3 \mathrm{fl}_{11} n_f \frac{d^{abc}d^{abc}}{n} \left[-\frac{503821438649257451}{62302510670400000} - \frac{302982523}{70945875}\zeta_3 + \frac{160}{13}\zeta_5\right]
\end{aligned}
$$

$$
\begin{aligned}
C^{\mathrm{ns}}_{L,14} &= a_s C_F \left[+\frac{4}{15}\right] \\
&+ a_s^2 C_F C_A \left[+\frac{3736751546509}{486972486000} - \frac{16}{5}\zeta_3\right] \\
&+ a_s^2 C_F n_f \left[-\frac{2263109}{2027025}\right] \\
&+ a_s^2 C_F^2 \left[-\frac{6706232197}{4969107000} + \frac{32}{5}\zeta_3\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{144797446441225084969 43}{170858971648965600000} + \frac{16947716309}{447972525}\zeta_3\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{170413131627782238959 1041}{751779475255448640000 0} - \frac{47503400282843}{82785322620}\zeta_3 + \frac{1552}{3}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{422957746}{91216125}\right] \\
&+ a_s^3 C_F^2 C_A \left[+\frac{101651397887847124868 3819}{526982671380044880000 0} + \frac{51799344112951}{30435780375}\zeta_3 - 2016\zeta_5\right] \\
&+ a_s^3 C_F^2 n_f \left[+\frac{583451898649142756 83}{1864532428708950000} - \frac{438969448}{6891885}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{832850600729143201689 0749}{179174108269215259200 00} - \frac{640509641943719}{517408266375}\zeta_3 + \frac{5888}{3}\zeta_5\right] \\
&+ a_s^3 fl_{11} n_f \frac{d^{abc}d^{abc}}{n} \left[-\frac{110339419075223314037}{157936864549464000 00} - \frac{17420356529}{4682427750}\zeta_3 + \frac{32}{3}\zeta_5\right]
\end{aligned}
$$

$$
\begin{aligned}
C^G_{2,10} &= a_s n_f \left[-\frac{4352}{7425}\right] \\
&+ a_s^2 C_A n_f \left[-\frac{651112454591}{185952412800} - \frac{54}{55}\zeta_3\right] \\
&+ a_s^2 C_F n_f \left[-\frac{72533010722807}{6973215480000} + \frac{108}{55}\zeta_3\right] \\
&+ a_s^3 C_A n_f^2 \left[+\frac{30367858943250477461}{1739677797950400000} - \frac{18965}{137214}\zeta_3\right] \\
&+ a_s^3 C_A^2 n_f \left[-\frac{114563089982050063118447}{669775952210904000000} + \frac{427785744377}{7924108500}\zeta_3 \right. \\
&\qquad \left. + \frac{241994}{27225}\zeta_4 + \frac{8}{33}\zeta_5\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{1108504139753184460611 77}{2487739251069072000000} + \frac{7772983651}{7358100750}\zeta_3 \right. \\
&\qquad \left. - \frac{321698}{27225}\zeta_4 + \frac{244}{11}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{25164738348656825457229}{1399353328726353000000} - \frac{2762978063}{1226350125}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{26284376777719892724358177}{2350913592260273040000 00} + \frac{1782724946402}{77260057875}\zeta_3 \right. \\
&\qquad \left. + \frac{8856}{3025}\zeta_4 - \frac{1048}{33}\zeta_5\right] \\
&+ a_s^3 \text{fl}^g_{11} n_f^2 \frac{d^{abc}d^{abc}}{N_A} \left[-\frac{11661390042871}{128024064000} - \frac{29154339}{107800}\zeta_3 + \frac{4408}{11}\zeta_5\right]
\end{aligned}
$$

$$
\begin{aligned}
C^G_{2,12} &= a_s n_f \left[-\frac{8110049}{15135120}\right] \\
&+ a_s^2 C_A n_f \left[-\frac{39540735563386331}{10558130155372800} - \frac{11}{13}\zeta_3\right] \\
&+ a_s^2 C_F n_f \left[-\frac{6258789011950819}{598533455520000} + \frac{22}{13}\zeta_3\right] \\
&+ a_s^3 C_A n_f^2 \left[+\frac{1102976289541525512630679}{62778008416037346432000} - \frac{1574273}{8049132}\zeta_3\right] \\
&+ a_s^3 C_A^2 n_f \left[-\frac{18295361799349570193991309242431}{10283037778546917345561600000} + \frac{6726830671058291}{132943488678000}\zeta_3\right. \\
&\qquad \left. + \frac{129763817}{16396380}\zeta_4 + \frac{20}{21}\zeta_5\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{969948676266548715044545722 3019}{188522359273360151335296000000} + \frac{789241683301607}{132943488678000}\zeta_3\right. \\
&\qquad \left. - \frac{171207527}{16396380}\zeta_4 + \frac{4390}{273}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{250161845928752032895603515 01}{13465882805240010809664000 00} - \frac{13373036672}{7193911725}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{19438251371991420503794932083 5327}{15553094640052212485161920000 00} + \frac{542538591728921}{33235872169500}\zeta_3\right. \\
&\qquad \left. + \frac{2563}{1014}\zeta_4 - \frac{2220}{91}\zeta_5\right] \\
&+ a_s^3 \mathrm{fl}^g_{11} n_f^2 \frac{d^{abc}d^{abc}}{N_A} \left[-\frac{1656600471440498533}{10297935648000000} - \frac{82799792129}{180589500}\zeta_3 + \frac{62436}{91}\zeta_5\right]
\end{aligned}
$$

$$
\begin{aligned}
C^G_{L,10} &= a_s n_f \left[+\frac{2}{33}\right] \\
&+ a_s^2 C_A n_f \left[+\frac{2460678191}{1056547800}\right] \\
&+ a_s^2 C_F n_f \left[-\frac{509195549}{704365200}\right] \\
&+ a_s^3 C_A n_f^2 \left[-\frac{140853814103239}{21965628762000} - \frac{4}{33}\zeta_3\right] \\
&+ a_s^3 C_A^2 n_f \left[+\frac{127219094296097749861}{1623699278087040000} + \frac{5906419}{24012450}\zeta_3 - \frac{80}{11}\zeta_5\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{1480618456217076384 77}{2638511326891440000} - \frac{210786157}{31216185}\zeta_3 + \frac{320}{11}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{491491787586698683}{188465094777960000} - \frac{136}{429}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[+\frac{9053411269935853949}{376930189555920000} + \frac{860883476}{156080925}\zeta_3 - \frac{320}{11}\zeta_5\right] \\
&+ a_s^3 \mathrm{fl}^g_{11} n_f^2 \frac{d^{abc}d^{abc}}{N_A} \left[+\frac{112883693141257}{4526565120000} + \frac{347273501}{4365900}\zeta_3 - \frac{1280}{11}\zeta_5\right]
\end{aligned}
$$

$$
\begin{aligned}
C^G_{L,12} &= a_s n_f \left[+\frac{4}{91}\right] + a_s^2 C_A n_f \left[+\frac{4978992299}{2685727044}\right] + a_s^2 C_F n_f \left[-\frac{98593150597}{179847793125}\right] \\
&+ a_s^3 C_A n_f^2 \left[-\frac{116919410865341069}{22683482755683750} - \frac{8}{91}\zeta_3\right] \\
&+ a_s^3 C_A^2 n_f \left[+\frac{96912603479263207273229}{1467873372990024000000} - \frac{16493287}{717341625}\zeta_3 - \frac{480}{91}\zeta_5\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{2084160567995424969918773}{46709827690503978000000} - \frac{9006563627}{2152024875}\zeta_3 + \frac{1920}{91}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{49342300647723867029}{24328035255470821875} - \frac{1696}{6825}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[+\frac{28171159611508188485 3551}{15569942563501326000000} + \frac{2347741862}{717341625}\zeta_3 - \frac{1920}{91}\zeta_5\right] \\
&+ a_s^3 \mathrm{fl}^g_{11} n_f^2 \frac{d^{abc}d^{abc}}{N_A} \left[+\frac{2232852976776993919}{56638646064000000} + \frac{115891012697}{993242250}\zeta_3 - \frac{15776}{91}\zeta_5\right]
\end{aligned}
$$

C Results for the moments of F_3

$$
\begin{aligned}
\gamma_3^{\rm ns} &= a_s C_F \left[+\frac{25}{6}\right] + a_s^2 C_F C_A \left[+\frac{535}{27}\right] + a_s^2 C_F n_f \left[-\frac{415}{108}\right] + a_s^2 C_F^2 \left[-\frac{2035}{432}\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{62249}{3888} - \frac{100}{3}\zeta_3\right] + a_s^3 C_F C_A^2 \left[+\frac{889433}{7776} + \frac{55}{3}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[-\frac{2569}{1944}\right] + a_s^3 C_F^2 C_A \left[-\frac{311213}{15552} - 55\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{203627}{7776} + \frac{100}{3}\zeta_3\right] + a_s^3 C_F^3 \left[-\frac{244505}{15552} + \frac{110}{3}\zeta_3\right] \\
&+ a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[+\frac{205}{288}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_5^{\rm ns} &= a_s C_F \left[+\frac{91}{15}\right] + a_s^2 C_F C_A \left[+\frac{73223}{2700}\right] + a_s^2 C_F n_f \left[-\frac{7783}{1350}\right] + a_s^2 C_F^2 \left[-\frac{2891}{500}\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{38587}{2000} - \frac{728}{15}\zeta_3\right] + a_s^3 C_F C_A^2 \left[+\frac{305342801}{1944000} + \frac{1414}{75}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[-\frac{215621}{121500}\right] + a_s^3 C_F^2 C_A \left[-\frac{63892213}{2430000} - \frac{1414}{25}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{5494973}{135000} + \frac{728}{15}\zeta_3\right] + a_s^3 C_F^3 \left[-\frac{51831073}{3037500} + \frac{2828}{75}\zeta_3\right] \\
&+ a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[+\frac{931}{4050}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_7^{\rm ns} &= a_s C_F \left[+\frac{1027}{140}\right] + a_s^2 C_F C_A \left[+\frac{67710257}{2116800}\right] + a_s^2 C_F n_f \left[-\frac{3745727}{529200}\right] \\
&+ a_s^2 C_F^2 \left[-\frac{106801937}{16464000}\right] + a_s^3 C_F C_A n_f \left[-\frac{2257057261}{106686720} - \frac{2054}{35}\zeta_3\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{219582793861}{1185408000} + \frac{92741}{4900}\zeta_3\right] + a_s^3 C_F n_f^2 \left[-\frac{1369936511}{666792000}\right] \\
&+ a_s^3 C_F^2 C_A \left[-\frac{629922436973}{20744640000} - \frac{278223}{4900}\zeta_3\right] + a_s^3 C_F^2 n_f \left[-\frac{3150205788689}{62233920000} + \frac{2054}{35}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{151689902637457}{8712748800000} + \frac{92741}{2450}\zeta_3\right] + a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[+\frac{56527729}{508032000}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_9^{\rm ns} &= a_s C_F \left[+\frac{1045}{126}\right] + a_s^2 C_F C_A \left[+\frac{242855129}{6804000}\right] + a_s^2 C_F n_f \left[-\frac{19247947}{2381400}\right] \\
&+ a_s^2 C_F^2 \left[-\frac{6993510271}{1000188000}\right] + a_s^3 C_F C_A n_f \left[-\frac{63405201707}{2813028750} - \frac{4180}{63}\zeta_3\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{744184331602493}{3600676800000} + \frac{1253219}{66150}\zeta_3\right] + a_s^3 C_F n_f^2 \left[-\frac{20297329837}{9001692000}\right] \\
&+ a_s^3 C_F^2 C_A \left[-\frac{5022136344247}{150028200000} - \frac{1253219}{22050}\zeta_3\right] + a_s^3 C_F^2 n_f \left[-\frac{1630263834317}{28005264000} + \frac{4180}{63}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{13829238556849837}{793949234400000} + \frac{1253219}{33075}\zeta_3\right] + a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[+\frac{13172190779}{200037600000}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{11}^{\rm ns} &= a_s C_F \left[+\frac{31408}{3465}\right] + a_s^2 C_F C_A \left[+\frac{1448235599}{37422000}\right] + a_s^2 C_F n_f \left[-\frac{512808781}{57629880}\right] \\
&+ a_s^2 C_F^2 \left[-\frac{2454220717793}{332812557000}\right] + a_s^3 C_F C_A n_f \left[-\frac{1031510572686647}{43568189280000} - \frac{251264}{3465}\zeta_3\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{3905492444576213 03}{1742727571200000} + \frac{151689577}{8004150}\zeta_3\right] + a_s^3 C_F n_f^2 \left[-\frac{28869611542843}{11981252052000}\right] \\
&+ a_s^3 C_F^2 C_A \left[-\frac{43074459020106191}{1198125205200000} - \frac{151689577}{2668050}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{1188145134622636787}{18451128160080000} + \frac{251264}{3465}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{2214308695766900 83141}{12786631814935440000} + \frac{151689577}{4002075}\zeta_3\right] \\
&+ a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[+\frac{5083969985783}{116181838080000}\right]
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{13}^{\rm ns} &= a_s C_F \left[+\frac{874733}{90090}\right] + a_s^2 C_F C_A \left[+\frac{31236494566127}{757512756000}\right] + a_s^2 C_F n_f \left[-\frac{93360116539}{9739449720}\right] \\
&+ a_s^2 C_F^2 \left[-\frac{22445960639039759}{2924756750916000}\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{90849626920977361109}{3685193506154160000} - \frac{3498932}{45045}\zeta_3\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{410707533776382334010 27}{1719756969538608000 00} + \frac{3662719609}{193243050}\zeta_3\right] \\
&+ a_s^3 C_F n_f^2 \left[-\frac{66727681292862571}{26322810758244000}\right] \\
&+ a_s^3 C_F^2 C_A \left[-\frac{14008746817077626022846 53}{3688878699660314160000 0} - \frac{3662719609}{64414350}\zeta_3\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{36688336888519925613757}{526982671380044880000} + \frac{3498932}{45045}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{40795447722180713788820819}{23737934432314121619600 00} + \frac{3662719609}{96621525}\zeta_3\right] \\
&+ a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[+\frac{62160363128061559}{19843349648522400 00}\right]
\end{aligned}
$$

$$
\begin{aligned}
C_1^{\rm ns} &= 1 + a_s C_F \left[-3\right] \\
&+ a_s^2 C_F C_A \left[-23\right] + a_s^2 C_F n_f \left[+4\right] + a_s^2 C_F^2 \left[+\frac{21}{2}\right] \\
&+ a_s^3 C_F C_A n_f \left[+\frac{3535}{27} + 24\zeta_3 - \frac{80}{3}\zeta_5\right] + a_s^3 C_F C_A^2 \left[-\frac{10874}{27} + \frac{440}{3}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[-\frac{230}{27}\right] + a_s^3 C_F^2 C_A \left[+\frac{1241}{9} - \frac{176}{3}\zeta_3\right] + a_s^3 C_F^2 n_f \left[-\frac{133}{18} - \frac{40}{3}\zeta_3\right] \\
&+ a_s^3 C_F^3 \left[-\frac{3}{2}\right] + a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[-\frac{11}{3} + 8\zeta_3\right]
\end{aligned}
$$

$$
\begin{aligned}
C_3^{\rm ns} &= 1 + a_s C_F \left[+\frac{5}{4}\right] \\
&+ a_s^2 C_F C_A \left[+\frac{5209}{144} - 33\zeta_3\right] + a_s^2 C_F n_f \left[-\frac{4369}{864}\right] + a_s^2 C_F^2 \left[-\frac{34763}{10368} + 16\zeta_3\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{24877649}{699840} + \frac{18539}{405}\zeta_3 - \frac{50}{3}\zeta_4\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{92517547}{699840} - \frac{225337}{405}\zeta_3 + \frac{55}{6}\zeta_4 + 390\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[-\frac{12125}{69984} + \frac{100}{81}\zeta_3\right] + a_s^3 C_F^2 C_A \left[+\frac{222157399}{559872} + \frac{206}{9}\zeta_3 - \frac{55}{2}\zeta_4 - 260\zeta_5\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{4292227}{34992} + \frac{527}{9}\zeta_3 + \frac{50}{3}\zeta_4\right] \\
&+ a_s^3 C_F^3 \left[-\frac{1041473}{373248} + \frac{2183}{324}\zeta_3 + \frac{55}{3}\zeta_4 - 40\zeta_5\right] + a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[-\frac{19477}{5184} + 5\zeta_3\right]
\end{aligned}
$$

$$
\begin{aligned}
C_5^{\rm ns} &= 1 + a_s C_F \left[+\frac{523}{90}\right] \\
&+ a_s^2 C_F C_A \left[+\frac{30312449}{324000} - \frac{276}{5}\zeta_3\right] + a_s^2 C_F n_f \left[-\frac{2356859}{162000}\right] \\
&+ a_s^2 C_F^2 \left[-\frac{14729261}{1620000} + \frac{188}{5}\zeta_3\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{33317596477}{122472000} + \frac{1413442}{14175}\zeta_3 - \frac{364}{15}\zeta_4\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{142639414763}{163296000} - \frac{30424087}{28350}\zeta_3 + \frac{707}{75}\zeta_4 + 548\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{784091}{54000} + \frac{728}{405}\zeta_3\right] \\
&+ a_s^3 C_F^2 C_A \left[+\frac{179060877287}{255150000} - \frac{512039}{7875}\zeta_3 - \frac{707}{25}\zeta_4 - 180\zeta_5\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{28068155797}{127575000} + \frac{20571 2}{4725}\zeta_3 + \frac{364}{15}\zeta_4\right] \\
&+ a_s^3 C_F^3 \left[-\frac{29707273013}{2187000000} + \frac{4045486}{10125}\zeta_3 + \frac{1414}{75}\zeta_4 - 376\zeta_5\right] \\
&+ a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[-\frac{24801551}{17496000} + \frac{86}{45}\zeta_3\right]
\end{aligned}
$$

$$
\begin{aligned}
C_7^{\rm ns} &= 1 + a_s C_F \left[+\frac{48091}{5040}\right] \\
&+ a_s^2 C_F C_A \left[+\frac{502042084559}{3556224000} - \frac{4917}{70}\zeta_3\right] + a_s^2 C_F n_f \left[-\frac{20352710029}{889056000}\right] \\
&+ a_s^2 C_F^2 \left[+\frac{972223395949}{248935680000} + \frac{1836}{35}\zeta_3\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{3373161364214023}{6721263360000} + \frac{4838581}{33075}\zeta_3 - \frac{1027}{35}\zeta_4\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{5386502616041647}{3360631680000} - \frac{134407981}{88200}\zeta_3 + \frac{92741}{9800}\zeta_4 + \frac{14108}{21}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{9986153999891}{336063168000} + \frac{2054}{945}\zeta_3\right] \\
&+ a_s^3 C_F^2 C_A \left[+\frac{2316127855865945089}{1881953740800000} - \frac{450138923}{4630500}\zeta_3 - \frac{278223}{9800}\zeta_4 - \frac{4118}{21}\zeta_5\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{3455733053370931}{9801842400000} + \frac{483863}{26460}\zeta_3 + \frac{1027}{35}\zeta_4\right] \\
&+ a_s^3 C_F^3 \left[-\frac{906919428644261623}{14637417984000000} + \frac{27358836137}{37044000}\zeta_3 + \frac{92741}{4900}\zeta_4 - \frac{11224}{21}\zeta_5\right] \\
&+ a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[-\frac{1521158314519}{1920360960000} + \frac{529}{504}\zeta_3\right]
\end{aligned}
$$

$$
\begin{aligned}
C_9^{\rm ns} &= 1 + a_s C_F \left[+\frac{17761}{1400}\right] \\
&+ a_s^2 C_F C_A \left[+\frac{6247950073621}{34292160000} - \frac{2858}{35}\zeta_3\right] + a_s^2 C_F n_f \left[-\frac{363260060687}{12002256000}\right] \\
&+ a_s^2 C_F^2 \left[+\frac{26950029280781}{1008189504000} + \frac{6698}{105}\zeta_3\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{1532454569897512927}{2138802019200000} + \frac{121711969}{654885}\zeta_3 - \frac{2090}{63}\zeta_4\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{15496996833630287207}{6805279152000000} - \frac{7620848351}{3969000}\zeta_3 + \frac{1253219}{132300}\zeta_4 + \frac{49634}{63}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{3023214594598871}{68052791520000} + \frac{4180}{1701}\zeta_3\right] \\
&+ a_s^3 C_F^2 C_A \left[+\frac{1111314791170240118909}{571643448768000000} - \frac{4449188411}{41674500}\zeta_3 - \frac{1253219}{44100}\zeta_4 - \frac{2678}{9}\zeta_5\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{80521228766527611043}{157201948411200000} - \frac{22156972}{3274425}\zeta_3 + \frac{2090}{63}\zeta_4\right] \\
&+ a_s^3 C_F^3 \left[-\frac{388781612300063841847}{4001504141376000000} + \frac{95897878894}{93767625}\zeta_3 + \frac{1253219}{66150}\zeta_4 - \frac{5092}{9}\zeta_5\right] \\
&+ a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[-\frac{390642677252603}{756142128000000} + \frac{95839}{141750}\zeta_3\right]
\end{aligned}
$$

$$
\begin{aligned}
C_{11}^{\rm ns} &= 1 + a_s C_F \left[+\frac{12815983}{831600}\right] \\
&+ a_s^2 C_F C_A \left[+\frac{56653343996617}{259334460000} - \frac{104947}{1155}\zeta_3\right] + a_s^2 C_F n_f \left[-\frac{117825956269669}{3195000547200}\right] \\
&+ a_s^2 C_F^2 \left[+\frac{25436786950269217}{461278204002000} + \frac{84262}{1155}\zeta_3\right] \\
&+ a_s^3 C_F C_A n_f \left[-\frac{5339200983355026311}{5825689309440000} + \frac{206111663893}{936485550}\zeta_3 - \frac{125632}{3465}\zeta_4\right] \\
&+ a_s^3 C_F C_A^2 \left[+\frac{5458605391363920639632689}{1884027922672896000000} - \frac{8531884151173}{3745942200}\zeta_3 + \frac{151689577}{16008300}\zeta_4\right. \\
&\qquad \left. + \frac{209072}{231}\zeta_5\right] \\
&+ a_s^3 C_F n_f^2 \left[+\frac{58243229827221303967}{996360920644320000} + \frac{251264}{93555}\zeta_3\right] \\
&+ a_s^3 C_F^2 C_A \left[+\frac{37894668775960020571538737}{13600326566794968000000} - \frac{52724308943557}{576875098800}\zeta_3 - \frac{151689577}{5336100}\zeta_4\right. \\
&\qquad \left. - \frac{46958}{99}\zeta_5\right] \\
&+ a_s^3 C_F^2 n_f \left[-\frac{75040618524858901709366 87}{10880261253435974400000} - \frac{2560572011}{85135050}\zeta_3 + \frac{125632}{3465}\zeta_4\right] \\
&+ a_s^3 C_F^3 \left[-\frac{35825259686676733349510147}{35444543391001039680000 00} + \frac{156213323360401}{124804708875}\zeta_3 + \frac{151689577}{8004150}\zeta_4\right. \\
&\qquad \left. - \frac{49124}{99}\zeta_5\right] \\
&+ a_s^3 n_f \frac{d^{abc}d^{abc}}{n} \left[-\frac{1182009270543683429}{3220560551577600000} + \frac{6758}{14175}\zeta_3\right]
\end{aligned}
$$

$$
\begin{aligned}
C_{13}^{\rm ns} &= 1 + a_s C_F \left[+ \frac{84292133}{4729725} \right] \\
&+ a_s^2 C_F C_A \left[+ \frac{137082775015804598489}{545954593504320000} - \frac{1480186}{15015} \zeta_3 \right] \\
&+ a_s^2 C_F n_f \left[- \frac{301032882286150933}{7019416202198400} \right] \\
&+ a_s^2 C_F^2 \left[+ \frac{52334493073532614544 57}{60226591014862272000} + \frac{1210906}{15015} \zeta_3 \right] \\
&+ a_s^3 C_F C_A n_f \left[- \frac{87932842600424325468161 9899}{79679779912662785856000 0} + \frac{3046809425749}{12174312150} \zeta_3 - \frac{1749466}{45045} \zeta_4 \right] \\
&+ a_s^3 C_F C_A^2 \left[+ \frac{64502684578867952026928364 7601}{18591948646287983366400 0000} - \frac{37039408217872}{14203364175} \zeta_3 \right. \\
&\qquad \left. + \frac{3662719609}{386486100} \zeta_4 + \frac{3086186}{3003} \zeta_5 \right] \\
&+ a_s^3 C_F n_f^2 \left[+ \frac{20433591703164368337508 87}{28457064254522423520000} + \frac{3498932}{1216215} \zeta_3 \right] \\
&+ a_s^3 C_F^2 C_A \left[+ \frac{90814318648605197676942235 51709}{24416161130380239380160 00000} - \frac{2292102879924479}{48745945848600} \zeta_3 \right. \\
&\qquad \left. - \frac{3662719609}{128828700} \zeta_4 - \frac{6487178}{9009} \zeta_5 \right] \\
&+ a_s^3 C_F^2 n_f \left[- \frac{19506028401623230293412698913}{22196510118527490345600000} - \frac{313559848919}{6087156075} \zeta_3 + \frac{1749466}{45045} \zeta_4 \right] \\
&+ a_s^3 C_F^3 \left[- \frac{29868073029294918776349755 9489677}{42771010260143584334195280 00000} + \frac{142217314466274683}{99707616508500} \zeta_3 \right. \\
&\qquad \left. + \frac{3662719609}{193243050} \zeta_4 - \frac{235868}{693} \zeta_5 \right] \\
&+ a_s^3 n_f \frac{d^{abc} d^{abc}}{n} \left[- \frac{65696095155620706347863}{23835831597805106880000 0} + \frac{11802932}{33108075} \zeta_3 \right]
\end{aligned}
$$