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Light Hybrid Mesons in QCD

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Including the radiative perturbative corrections and the short distance tachyonic gluon mass effects which mimic the ones of UV renormalons, we re-estimate the decay amplitudes, masses and widths of light hybrid mesons from QCD spectral sum rules. We show that the effects are tiny and confirm the previous lowest order results. We discuss the phenomenological impacts of our results.

1. INTRODUCTION

Since the discovery of QCD, it has been emphasized [1] that exotic mesons beyond the standard octet, exist as a consequence of the non-perturbative aspects of quantum chromodynamics (QCD). Since the understanding of the nature of the η' [2], a large amount of theoretical efforts have been furnished in the past and pursued at present for predicting the spectra of the exotics using different QCD-like models [3] such as the flux tube [4] , the bags [5], the quark [6] and constituent gluon [7] models. In this paper, we present new developments of the analysis of the hybrid mesons using QCD spectral sum rules (QSSR) [8] à la SVZ [9] (for a review, see e.g. [10]) by including the radiative perturbative corrections and the short distance tachyonic gluon mass effects which mimic the ones of UV renormalons [11,12]. In this sense our results are an update of earlier results. Our predictions for the masses will be compared with the lattice results [13] and the recent experimental candidates [14].

2. QCD SPECTRAL SUM RULES (QSSR)

Description of the method

Since its discovery in 1979, QSSR has proved to be a powerful method for understanding the hadronic properties in terms of the fundamental QCD parameters such as the QCD coupling α_s, the (running) quark masses and the quark and/or gluon QCD vacuum condensates. The description of the method has been often discussed in the literature, where a pedagogical introduction can be, for instance, found in the book [10]. In practice (like also the lattice), one starts the analysis from the two-point correlator (standard notations):

$$\Pi^{\mu\nu}_{V/A}(q^2) \equiv i\int d^4x\ e^{iqx}\ \langle 0|\mathcal{T}\mathcal{O}^{\mu}_{V/A}(x)\left(\mathcal{O}^{\nu}_{V/A}(0)\right)^{\dagger}|0\rangle = -\left(g^{\mu\nu}q^2 - q^\mu q^\nu\right)\Pi^{(1)}_{V/A}(q^2) + q^\mu q^\nu \Pi^{(0)}_{V/A}(q^2), \quad (1)$$

built from the hadronic local currents $\mathcal{O}^{V/A}_{\mu}(x)$:

$$\mathcal{O}^{\mu}_{V}(x) \equiv :g\bar\psi_i\lambda_a\gamma_\nu\psi_j G^{\mu\nu}_a: , \quad \mathcal{O}^{\mu}_{A}(x) \equiv :g\bar\psi_i\lambda_a\gamma_\nu\gamma_5\psi_j G^{\mu\nu}_a: \quad (2)$$

which select the specific quantum numbers of the hybrid mesons; A and V refer respectively to the vector and axial-vector currents. The invariant $\Pi^{(1)}$ and $\Pi^{(0)}$ refer to the spin one and zero mesons. One exploits, in the sum rule approaches, the analyticity property of the correlator which obeys the well-known Källen–Lehmann dispersion relation:

$$\Pi^{(1,0)}_{V/A}(q^2) = \int_0^\infty \frac{dt}{t-q^2-i\epsilon}\,\frac{1}{\pi}\ \mathrm{Im}\ \Pi^{(1,0)}_{V/A} + \ldots \quad (3)$$

*On leave from Institute for Nuclear Research of the Russian Academy of Sciences, Moscow where ... represent subtraction terms which are polynomials in the q^2-variable. In this way, the *sum rule* expresses in a clear way the *duality* between the integral involving the spectral function $\mathrm{Im}\Pi^{(1,0)}_{V/A}(t)$ (which can be measured experimentally), and the full correlator $\Pi^{(1,0)}_{V/A}(q^2)$. The latter can be calculated directly in the QCD in the Euclidean space-time using perturbation theory (provided that $-q^2+m^2$ (m being the quark mass) is much greater than Λ^2), and the Wilson expansion in terms of the increasing dimensions of the quark and/or gluon condensates which simulate the non-perturbative effects of QCD.

Beyond the usual SVZ expansion
Using the Operator Product Expansion (OPE) [9], the two-point correlator reads for $m=0$:

$$\Pi^{(1,0)}_{V/A}(q^2) \simeq \sum_{D=0,2,\dots} \frac{1}{\left(q^2\right)^{D/2}} \sum_{dim O=D} C(q^2,\nu)\langle\mathcal{O}(\nu)\rangle \ ,$$

where ν is an arbitrary scale that separates the long- and short-distance dynamics; C are the Wilson coefficients calculable in perturbative QCD by means of Feynman diagrams techniques; $\langle\mathcal{O}(\nu)\rangle$ are the quark and/or gluon condensates of dimension D. In the massless quark limit, one may expect the absence of the terms of dimension 2 due to gauge invariance. However, it has been emphasized recently [11] that the resummation of the large order terms of the perturbative series, and the effects of the higher dimension condensates due e.g. to instantons, can be mimiced by the effect of a tachyonic gluon mass λ which generates an extra $D=2$ term not present in the original OPE. Its presence might be understood from the analogy with the short distance linear part of the QCD potential [2]. The strength of this short distance mass has been estimated from the e^+e^- data to be [12,16]:

$$\frac{\alpha_s}{\pi}\lambda^2 \simeq -(0.06 \sim 0.07)\ \mathrm{GeV}^2, \tag{4}$$

which leads to the value of the square of the (short distance) string tension: $\sigma \simeq -\frac{2}{3}\alpha_s\lambda^2 \simeq [(400\pm 20)\ \mathrm{MeV}]^2$ in an (unexpected) good agreement with the lattice result [17] of about $[(440\pm 38)\ \mathrm{MeV}]^2$. In addition to Eq. (4), the strengths of the vacuum condensates having dimensions $D\leq 6$ are also under good control, namely:

- $\langle\bar{s}s\rangle/\langle\bar{d}d\rangle \simeq 0.7\pm 0.2$ from the meson [10] and baryon systems [18];
- $\langle\alpha_s G^2\rangle \simeq (0.07\pm 0.01)\ \mathrm{GeV}^4$ from sum rules of $e^+e^- \to I=1$ hadrons [16] and heavy quarkonia [19–21], and from the lattice [22];
- $g\langle\bar{\psi}\lambda_a/2\sigma^{\mu\nu}G^a_{\mu\nu}\psi\rangle \simeq (0.8\pm 0.1)\ \mathrm{GeV}^2\langle\bar{\psi}\psi\rangle$, from the baryons [23,18], light mesons [24] and the heavy-light mesons [25];
- $\alpha_s\langle\bar{u}u\rangle^2 \simeq 5.8\times 10^{-4}\ \mathrm{GeV}^6$ from $e^+e^- \to I=1$ hadrons [16];
- $g^3\langle G^3\rangle \approx 1.2\ \mathrm{GeV}^2\langle\alpha_s G^2\rangle$ from dilute gaz instantons [26].

Spectral function
In the absence of the complete data, the spectral function is often parametrized using the "naïve" duality ansatz:

$$\frac{1}{\pi}\,\mathrm{Im}\Pi^{(1,0)}_{V/A}(t) \simeq 2M_H^4 f_H^2\delta(t-M_H^2) + \text{“QCD continuum”}\times\theta(\mathrm{t}-\mathrm{t_c})\ , \tag{5}$$

which has been tested [10] using e^+e^- and τ-decay data, to give a good description of the spectral integral in the sum rule analysis; f_H (an analogue to f_π) is the hadron's coupling to the current; $2n$ is the dimension of the correlator; while t_c is the QCD continuum's threshold.

[2] Some evidence of this term is found from the lattice analysis of the static quark potential [15], though the extraction of the continuum result needs to be clarified.

Form of the sum rules and optimization procedure

Among the different sum rules discussed in the literature [10], we shall be concerned with the following Laplace sum rule (LSR) and its ratio [9,27,20] [3]:

$$\mathcal{L}_n^{(1,0)}(\tau) = \int_0^\infty dt\ t^n \exp(-t\tau)\ \frac{1}{\pi}\ \mathrm{Im}\Pi_{V/A}^{(1,0)}(t)\ , \qquad \mathcal{R}_n \equiv -\frac{d}{d\tau}\log\mathcal{L}_n\ , \qquad (n \geq 0)\ . \tag{6}$$

The advantage of the Laplace sum rules with respect to the previous dispersion relation is the presence of the exponential weight factor which enhances the contribution of the lowest resonance and low-energy region accessible experimentally. For the QCD side, this procedure has eliminated the ambiguity carried by subtraction constants, arbitrary polynomial in q^2, and has improved the convergence of the OPE by the presence of the factorial dumping factor for each condensates of given dimensions. The ratio of the sum rules is a useful quantity to work with, in the determination of the resonance mass, as it is equal to the meson mass squared, in the usual duality ansatz parametrization. As one can notice, there are "a priori" two free external parameters (τ, t_c) in the analysis. The optimized result will be (in principle) insensitive to their variations. In some cases, the t_c-stability is not reached due to the too naïve parametrization of the spectral function. In order to restore the t_c-stability of the results one can either fix the t_c-values by the help of FESR (local duality) [28,29] or improve the parametrization of the spectral function by introducing threshold effects with the help of chiral perturbation theory The results discussed below satisfy these stability criteria.

3. QCD EXPRESSION OF THE TWO-POINT FUNCTION

A QCD analysis of the two-point function have been done in the past by different groups [30,31], where (unfortunately) the non-trivial QCD expressions were wrong leading to some controversial predictions [10]. The final correct QCD expression is given in [32,33]. In this paper, we extend the analysis by taking into account the non-trivial α_s correction and the effect of the new $1/q^2$ term not taken into account into the SVZ expansion. The corrected QCD expressions of the correlator are given in [10] to lowest order of perturbative QCD but including the contributions of the condensates of dimensions lower or equal than six. The new terms appearing in the OPE are presented in the following [4]:

- The perturbative QCD expression including the NLO radiative corrections reads:

$$\begin{aligned}
\frac{1}{\pi}\mathrm{Im}\Pi^{(1)}_{V/A}(t)_{pert} &= \frac{\alpha_s}{60\pi^3}t^2\left\{1+\frac{\alpha_s}{\pi}\left[\frac{121}{16}-\frac{257}{360}n_f+\left(\frac{35}{36}-\frac{n_f}{6}\right)\log\frac{\nu^2}{t}\right]\right\} \\
\frac{1}{\pi}\mathrm{Im}\Pi^{(0)}_{V/A}(t)_{pert} &= \frac{\alpha_s}{120\pi^3}t^2\left\{1+\frac{\alpha_s}{\pi}\left[\frac{1997}{432}-\frac{167}{360}n_f+\left(\frac{35}{36}-\frac{n_f}{6}\right)\log\frac{\nu^2}{t}\right]\right\}
\end{aligned} \tag{7}$$

- The anomalous dimension of the current can be easily deduced to be:

$$\nu\frac{\mathrm{d}}{\mathrm{d}\nu}\mathcal{O}^\mu_V = -\frac{16}{9}\frac{\alpha_s}{\pi}\mathcal{O}^\mu_V. \tag{8}$$

- The lowest order correction due to the (short distance) tachyonic gluon mass reads:

$$\begin{aligned}
\frac{1}{\pi}\mathrm{Im}\Pi^{(1)}_{V/A}(t)_\lambda &= -\frac{\alpha_s}{60\pi^3}\frac{35}{4}\lambda^2 t \\
\frac{1}{\pi}\mathrm{Im}\Pi^{(0)}_{V/A}(t)_\lambda &= \frac{\alpha_s}{120\pi^3}\frac{15}{2}\lambda^2 t
\end{aligned} \tag{9}$$

[3] FESR or τ-like sum rules are complement to the Laplace sum rules and will be used if necessary, though the final results are independent on the form of the sum rules used.

[4] The results described below [Eqs. (7) to (9)] have been obtained with the help of program packages GEFICOM (see, e.g. [34]) and MINCER [35] written in FORM [36]. More details on the derivation of these results will be published elsewhere. Note that the results for the NLO radiative corrections are derived in neglecting some possible mixings of our operators with those containing more γ-matrixes like $g\bar\psi_i\lambda_a\gamma_\mu\sigma_{\nu\lambda}\psi_j G_a^{\nu\lambda}$ which could in principle mix with $\mathcal{O}^\mu_V$. We expect that effects due to the mixings will be small.

- The (corrected) contributions of the dimension-four and -six terms reads in the limit $m^2=0$ [10]:

$$\begin{aligned}
\Pi^{(1)}_V(q^2)_{NP} &= -\frac{1}{9\pi}\Big[\alpha_s\langle G^2\rangle+8\alpha_s m\langle\bar\psi\psi\rangle\Big]\log-\frac{q^2}{\nu^2} \\
&\quad +\frac{1}{q^2}\Big[\frac{16\pi}{9}\alpha_s\langle\bar\psi\psi\rangle^2+\frac{1}{48\pi^2}g^3\langle G^3\rangle-\frac{83}{432}\frac{\alpha_s}{\pi}mg\langle\bar\psi G\psi\rangle\Big] \\
\Pi^{(0)}_A(q^2)_{NP} &= \left[\frac{1}{6\pi}\Big[\alpha_s\langle G^2\rangle+8\alpha_s m\langle\bar\psi\psi\rangle\Big]+\frac{11}{18}\frac{\alpha_s}{\pi}\frac{1}{q^2}mg\langle\bar\psi G\psi\rangle+\mathcal{O}\left(\frac{1}{q^2}\right)\right]\log-\frac{q^2}{\nu^2}\ ,
\end{aligned} \tag{10}$$

  where one can notice the miraculous cancellation of the log-coefficient of the dimension-six condensates for $\Pi^{(1)}_V$.

4. PROPERTIES OF LIGHT HYBRIDS

The $\tilde\rho(1^{-+})$

The experimental (resp. theoretical) situation has been reviewed in [14] (resp. [8,3]). The sum rule analysis of the spectrum is based on the 2-point correlator $\Pi(q^2)_{V/A}$ associated to the hybrid currents.

- One expects, from different QCD-like approaches, that the lightest exotic state is the one with the quantum numbers 1^{-+} [5]. From the analysis of the moments $\mathcal{R}_{0,1}$, we notice that the effect of the perturbative corrections (slightly decrease) and of the new dimension-two contribution (slightly increase) are almost negligible. This means that perturbation theory expansion in α_s converges well. The main uncertainties come from the value of t_c because the result does not show t_c stability. The τ-stability of $\mathcal{R}_0$ also disappears if one considers the value of the subtraction constant proposed in [30]:

$$q^2\Pi^{(1)}_V|_{q^2=0}\approx\frac{16\pi}{9}\alpha_s\langle\bar\psi\psi\rangle^2\ , \tag{11}$$

  as it would cancel the effect of $\langle\bar\psi\psi\rangle^2$ appearing in the OPE. However, we shall check "a posteriori" that by approximating it with the sum rule estimated quantity $2M^4_{\tilde\rho}f^2_{\tilde\rho}$ from the following Eq. (13), this result is inaccurate and, therefore, we shall not include this term in our analysis. An independent measurement of this quantity, e.g. on the lattice is required.

- Using the different QCD input parameters given previously and the value $\Lambda=(0.35\pm0.05)$ GeV, the positivity ($\equiv t_c\to\infty$) of the $\mathcal{R}_0$ moment leads to the rigorous upper bound:

$$M_{\tilde\rho}\leq 1.9\ \mathrm{GeV}\ , \tag{12}$$

  which excludes some of range spanned by the quenched lattice estimates of (1.9 ± 0.2) GeV [13].

- For reasonnable finite values of $t_c\simeq 3.5$ GeV^2 (beginning of τ-stability) to 4.5 GeV^2 as also fixed by the Finite Energy Sum Rule constraints [33,10], we obtain at the stability point $\tau\approx(0.5\sim0.6)$ GeV^{-2} of $\mathcal{R}_0$, the common solution of $\mathcal{R}_0$ and $\mathcal{R}_1$:

$$M_{\tilde\rho}\approx(1.6\sim1.7)\ \mathrm{GeV}\ ,\qquad f_{\tilde\rho}\approx(25\sim50)\ \mathrm{MeV}\ ,\qquad (M_{\tilde\rho'}\approx\sqrt{t_c})-M_{\tilde\rho}\approx 200\ \mathrm{MeV}\ , \tag{13}$$

  where the $\tilde\rho'$ is the radial excitation. One can consider this result as an improvement of the available sum rule results ranging from 1.4 to 2.1 GeV, [30–33,10]. Though, we cannot absolutely exclude the presence of the $1.4\sim1.6$ GeV experimental candidates [14], we expect from your analysis that this observed state is a hybrid which can have a small $\bar qq\bar qq$ component through mixing.

- $\tilde\rho'$-$\tilde\rho$ mass-splitting is much smaller than $M_{\rho'}-M_\rho\simeq 700$ MeV, and can signal a rich population of 1^{-+} states above 1.6 GeV.

[5]From QCD spectral sum rules, we also expect to a good approximation that the 1^{--} is almost degenerate with the 1^{-+}.

- The hadronic widths have been computed in [33,37]. Given our new values of the mass and decay constant, the updated values are:

$$\begin{aligned} \Gamma(\tilde\rho\to\rho\pi) &\approx 274 \text{ MeV}\ , \quad \Gamma(\tilde\rho\to\gamma\pi)\approx 3\text{ MeV}\ ,\\ \Gamma(\tilde\rho\to\eta'\pi) &\approx 3\text{ MeV}\ , \qquad\qquad \Gamma(\tilde\rho\to\pi\pi,\ \bar KK,\ \eta_8\eta_8)\approx\mathcal{O}(m_q^2)\ . \end{aligned} \tag{14}$$

- One can measure the $SU(3)$ breakings and the mass of the $\tilde\phi(\bar ss)$ from the difference of the ratio of moments, which gives [10]:

$$M^2_{\tilde\phi}-M^2_{\tilde\rho}\simeq\frac{20}{3}\overline{m}_s^2-\frac{160\pi^2}{9}m_s\langle\bar ss\rangle\tau\approx 0.3\text{ GeV}^2 \quad\Longrightarrow\quad M_{\tilde\phi}\approx(1.7\sim1.8)\text{ GeV}\ . \tag{15}$$

  The quenched lattice results are in the range of (2.0 ± 0.2) GeV [13], which is slightly higher than our result.

The $\tilde\eta(0^{--})$
Similar analysis can be done for the pseudoscalar channel. In this case, the most convenient sum rule to work with is $\mathcal{R}_1$, which presents both τ and t_c stabilities. Using the previous QCD input parameters, stabilities are reached for $\tau\simeq0.12$ GeV^{-2} and $t_c\simeq7.8$ GeV^2 showing again that the mass-splitting between the radial excitation and the ground state is tiny. At these values one obtains [6]:

$$M_{\tilde\eta}\simeq2.8\text{ GeV}\approx\sqrt{t_c}\ , \tag{16}$$

which we consider as an update of the previous results in [33,10]. Again the effects of the correction terms are small. The relatively higher value of the mass of the 0^{--} meson than the one of the 1^{-+}, is mainly due to the relative strength of the perturbative and non-perturbative terms.

5. CONCLUSIONS

There are some progresses in the long run study and experimental search for the exotics. Before some definite conclusions, one still needs improvments of the present data, and some improved lattice unquenched estimates which should complement the QCD spectral sum rule (QSSR) results. In this paper we have updated previous sum rule analysis of the light hybrids [30–33,10] by including the perturbative radiative corrections and the new effect due to the (short distance) tachyonic gluon mass not included in the original SVZ expansion. However, these effects are negligible which are reassuring for the validity of the approximation used. Our result which is $M_{\tilde\rho}\approx(1.6\sim1.7)$ GeV can be renconciled with the existence of the 1^{-+} states at $(1.4\sim1.6)$ GeV seen recently in hadronic machines (BNL and Crystal Barrel) [14], but in the same time predicts the existence of a 1^{--} hybrid almost degenerate with the 1^{-+}, and which could manifest in $e^+e^-\to$ hadrons by mixing with the radial excitations of the ρ and ω mesons. In addition, the relatively low value of the continuum threshold indicates that we expect a rich population of (axial-) vector hybrids in the region above 1.8 GeV.
In our analysis, the 0^- mass is about 2.8 GeV, which is in the range of the different charmonium states, such that it could mix with these charmonium states as well. Moreover, the small splitting between the continuum threshold and the lowest ground state indicates that rich population of pseudoscalar hybrids is expected in the 3 GeV mass range.
Light hybrid mesons (1^{--} and 0^{-+}) might be (partly) responsible of the anomalous behaviour of the $e^+e^-\to$ hadrons cross section observed in the region below 4 GeV.

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[6] In the case $\lambda=0$, i.e. of the ordinary SVZ expansion, the stability is obtained for $\tau\simeq0.15$ GeV^{-2} at which, one can deduce $M_{\tilde\eta}\simeq3.2$ GeV$\approx\sqrt{t_c}$.

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