Lecture 13 The Standard Model of particle physics: massive gauge bosons

We will now try to put ideas of gauge symmetry and the Higgs mechanism together, to describe a theory known as "Standard Model" of particle physics. We will start with the discussion of the boson sector of the theory. For a number of historical reasons, we would like to have a theory where 1) left-handed and right-handed fields have different interactions, 2) where weak interactions are short-range and contain couplings of charged and neutral currents and 3) where electromagnetic interactions arise naturally. It turns out that a good candidate for this is a non-abelian gauge theory with the gauge group $SU_L(2) \otimes U(1)_R$, where L- & R- refer to "left" and "right" fields. "Left" and "right" will be important in the next lecture when we discuss fermions and in later on this lecture when we will discuss the Higgs boson. For now, the two gauge groups - taken as the direct product - imply that there are 2 ~~20~~ kinetic terms in the Lagrangian:

$$\boxed{\mathcal{L}_{kin} = \mathcal{L}_{SU(2)} + \mathcal{L}_{U(1)}}$$

$$\mathcal{L}_{U(1)} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\mathcal{L}_{SU(2)} \equiv -\frac{1}{4} W^i_{\mu\nu} W^{i,\mu\nu}, \qquad D_\mu = \partial_\mu - ig\tau^i W^i_\mu \rightarrow$$

τ^i are generators of $SU(2)$: $\tau^i = \sigma^i/2$, σ^i are Pauli matrices and the field strength is

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ikj} W^k_\mu W^j_\nu, \quad i \in 1..3$$

The Lagrangian describes ~~the~~ 4 massless gauge fields, B, W^1, W^2, W^3, three of which are non-abelian. (self-interacting)

We ~~need~~ to break the gauge symmetry to produce 3 massive gauge boson. To do that, we invoke the Higgs mechanism. The Higgs boson interacts with both, the $SU(2)_L$ & the $U(1)_R$ field. It is the $SU(2)$-doublet and its ~~has~~ charge w.r.t. the $U(1)$ group is fixed by its hypercharge: Y. The corresponding covariant derivative is

$$\boxed{D_\mu = \partial_\mu - igW^i_\mu \tau^i - ig' B_\mu \frac{Y}{2}}.$$

The kinetic term for the Higgs doublet φ

$$\boxed{\mathcal{L}_{K\varphi} \equiv (D_\mu \varphi)^\dagger (D^\mu \varphi)}$$

; and the transformation rules for φ under $SU(2)_L$ & $U(1)_R$ are

$\varphi \to \Omega \varphi$, $\Omega \in SU(2)$

$\varphi \to e^{i\alpha} \varphi$, $e^{i\alpha} \in U(1)$

In general $\varphi = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$

Next, we introduce the double-well potential to break the $SU(2)_L \otimes U(1)_R$ symmetry:

$$\boxed{\Delta\mathcal{L} = -\frac{1}{2}\lambda^2 \left(\varphi^\dagger\varphi - \frac{v^2}{2}\right)^2}.$$

Now, the vacuum expectation value for the scalar field φ is $\varphi^\dagger\varphi = v^2/2$. For reasons that will become clear later, we choose the vacuum state as $\varphi_{vac} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$.

The general parametrization of the doublet, up to gauge transformations can be written as $\varphi(x) = \begin{bmatrix} 0 \\ \frac{v + h(x)}{\sqrt{2}} \end{bmatrix}$.

We can now discuss the spectrum of gauge bosons.

Indeed, taking $\varphi \to \varphi_{vac} = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}$, we find, from the Higgs kinetic term,

$$\mathcal{L}_\varphi = (D_\mu \varphi)^+ (D_\mu \varphi) \to (D_\mu \varphi_{vac})^+ (D_\mu \varphi_{vac})$$

$$D_\mu \varphi_{vac} = -ig W^i_\mu \tau^i \varphi_{vac} - ig' B_\mu \frac{Y}{2} \varphi_{vac}$$

$$(D_\mu \varphi_{vac})^+ = \varphi^+_{vac} \{ ig W^i_\mu \tau^i + ig' B_\mu \frac{Y}{2} \} \Rightarrow$$

$$[D_\mu \varphi_{vac}]^+ [D^\mu \varphi_{vac}] = g^2 W^i_\mu W^j_\mu \varphi^+_{vac} (\tau^i \tau^j) \varphi_{vac}$$

$$+ g'^2 \frac{Y^2}{4} B_\mu B^\mu \varphi^+_{vac} \cdot \varphi_{vac}$$

$$+ g' g B_\mu W^i_\mu \varphi^+_{vac} \tau^i \varphi_{vac} Y .$$

In the $O(g^2)$ term, we can use

$$W^i_\mu W^j_\mu (\varphi^+_{vac} \tau^i \tau^j \varphi_{vac} = \frac{1}{4} W^i_\mu W^j_\mu \varphi^+_{vac} \sigma^i \sigma^j \varphi_{vac} =$$

$$= \frac{1}{4} W^i_\mu W^j_\mu \varphi^+_{vac} \delta^{ij} \varphi_{vac} = \frac{1}{4} W^i_\mu W^{i\mu} |\varphi_{vac}|^2 =$$

The $O(g'g)$ term is: $\varphi^+_{vac} \tau^i \varphi_{vac} = \begin{cases} 0, & i = 1, 2 \\ -\frac{1}{2} \varphi^+_{vac} \varphi_{vac}, & i = 3 \end{cases}$

$$\Rightarrow [D_\mu \varphi_{vac}]^+ [D^\mu \varphi_{vac}] \equiv [\varphi^+_{vac} \varphi_{vac}] \cdot \Big[\frac{g^2}{4} W^i_\mu W^{i\mu} + \frac{g'^2}{4} Y^2 B_\mu B^\mu$$

$$\mp \frac{g g'}{2} B_\mu W^{(3)}_\mu \cdot Y \Big] =$$

$$= [\varphi^+_{vac} \varphi_{vac}] \left[\frac{g^2}{4} W^1_\mu W^{1\mu} + W^2_\mu W^{2\mu} + \left(\frac{g}{2} W^{3,\mu} - \frac{g'}{2} B^\mu Y \right)^2 \right]$$

Since $\varphi^+_{vac} \varphi_{vac} = v^2/2$, we obtain

$$\mathcal{L}_\varphi \to \frac{v^2 g^2}{8}\left(W^{(1)}_\mu W^{(1)\mu} + W^{(2)}_\mu W^{(2),\mu}\right) +$$

$$+ \frac{v^2}{2}\left(\frac{g}{2} W^{(3),\mu} - \frac{g'}{2} B^\mu \cdot Y\right)^2 .$$

The first term is clear: it corresponds to the situation where two gauge-bosons $W^{(1)}_\mu$ & $W^{(2)}_\mu$ get masses $\boxed{m_{1,2}^2 = \frac{v^2 g^2}{4}}$. The third term corresponds to a situation where a linear combination of one field from $SU(2)_L$ and one field from $U(1)_R$ receives ~~ma~~ a mass.

To understand how to properly interpret this, note that we must redefine the fields in such a way that kinetic terms are canonically normalized. We therefore need to "rotate" fields $(W^{(3),\mu}, B^\mu) \to (Z^\mu, A^\mu)$.

First, we write $Z^\mu = \cos\theta\, W^{(3),\mu} - \sin\theta\, B^\mu$,

where $\cos\theta \equiv \frac{g}{\sqrt{g^2+g'^2Y^2}}$ and $\sin\theta = \frac{g'Y}{\sqrt{g^2+g'^2Y^2}}$.

Then the third term in $\mathcal{L}_\varphi$ becomes

$$\frac{v^2}{2}\left(\frac{g}{2} W^{(3),\mu} - \frac{g'}{2} B^\mu Y\right)^2 \equiv \frac{v^2}{8}\left(g^2+g'^2\right) Z_\mu Z^\mu =$$

$$\equiv \frac{v^2 g^2}{8\cos^2\theta} Z_\mu Z^\mu \Rightarrow \boxed{m_Z^2 \cos^2\theta = m_{1,2}^2}$$

$$m_Z^2 = \frac{v^2 g^2}{4\cos^2\theta}$$

The second linear combination of $W^{(3),\mu}$, B^μ that is orthogonal to Z^μ is
$A^\mu = \sin\theta\, W^{(3),\mu} + \cos\theta\, B^\mu$. Let's find the kinetic, non-interacting part of the Lagrangian for Z^μ and A^μ

$$\mathcal{L}_{3,B} = -\frac{1}{4}\left[\partial_\mu W_{3\nu} - \partial_\nu W_{3\mu}\right]^2 - \frac{1}{4} B_{\mu\nu}B^{\mu\nu}$$

$$W^{(3)}_\mu = \cos\theta\, Z^\mu + \sin\theta\, A^\mu \qquad \Rightarrow \mathcal{L}_{3,B} = -\frac{1}{4}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu\right)^2$$
$$B^\mu = -\sin\theta\, Z^\mu + \cos\theta\, A^\mu \qquad\qquad - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}.$$

$\Rightarrow$ The gauge fields of the theory are

2 massive	fields	W_1^μ, W_2^μ	with masses $\frac{vg}{2}$
1 massive	field	Z^μ	with the mass $\frac{vg}{2\cos\theta}$
1 massless	field	A^μ. (photon)	

$$\cos\theta = \frac{g}{\sqrt{g^2+g'^2Y^2}} \qquad \sin\theta = \frac{g'Y}{\sqrt{g^2+g'^2Y^2}}, \text{ where}$$

g (g') are the gauge couplings of $SU(2)$ & $U(1)_R$ and Y is the charge of the Higgs doublet under $U(1)_R$.

Let us now determine the mass of the Higgs boson $h(x)$ $\left[\varphi = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}\right]$. We find

$$\varphi^\dagger\varphi - \frac{v^2}{2} = \frac{(v+h)^2 - v^2}{2} = \frac{v^2 + 2hv + h^2 - v^2}{2} = hv + \frac{h^2}{2} \Rightarrow$$

$$\Delta \mathcal{L} \to -\frac{\lambda^2 \cancel{v^2}}{2} h^2 \Rightarrow \boxed{m_h^2 = \lambda^2 v^2}$$

The vacuum expectation value v is fixed by $m_{1,2}$ & m_Z; measurement of the Higgs mass m_h fixes the self-coupling of the Higgs λ.

Higgs boson interaction with gauge bosons is easy to understand rewriting covariant derivatives through massive eigenstates $\Rightarrow$

$$D_\mu \varphi = \left[\partial_\mu - igW^i\tau^i - ig' B_\mu \frac{Y}{2}\right] \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \equiv$$

$$\equiv \left(\partial_\mu - ig \sum_{i=1,2} W^i \tau^i + \frac{ig}{2} W^3_\mu - ig' B_\mu \frac{Y}{2}\right) \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} =$$

$$\equiv \left(\partial_\mu - ig \sum_{i=1,2} W^i_\mu \tau^i + \frac{i\sqrt{g^2+g'^2}}{2} Z_\mu\right) \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} =$$

$$= \left(\partial_\mu - ig \sum_{i=1,2} W^i_\mu \tau^i + \frac{ig}{2\cos\theta} Z_\mu\right) \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

It is convenient to make 2 ~~charged~~ complex fields out of $W^{1,2}$ by writing $W^\pm = \frac{W_1 \mp iW_2}{\sqrt{2}} \Rightarrow$

$$W_1 = \frac{W^+ + W^-}{\sqrt{2}}, \quad W_2 = \frac{W^- - W^+}{i\sqrt{2}} \Rightarrow$$

$$D_\mu \varphi = \left(\partial_\mu - \frac{ig}{\sqrt{2}}\left(\tau^+ W^-_\mu + \tau^- W^+_\mu\right) + \frac{ig}{2\cos\theta} Z_\mu\right) \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

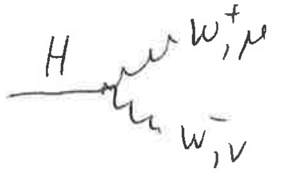
We find, by taking $(D_\mu\varphi)^+ (D^\mu \varphi)$ that the interaction of the H-boson with massive gauge bosons can be written in a simple way:

$$D_\mu \varphi \equiv \frac{1}{\sqrt{2}}\left[(\partial_\mu h)\begin{pmatrix}0\\1\end{pmatrix} - \frac{ig}{\sqrt{2}} W^-_\mu (v+h)\begin{pmatrix}1\\0\end{pmatrix} + \frac{ig}{2\cos\theta} Z_\mu (v+h)\begin{pmatrix}0\\1\end{pmatrix}\right]$$

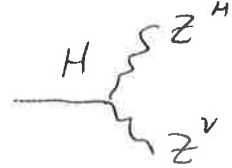
Hence $$\mathcal{L}_{\varphi kin} = (D_\mu\varphi)^\dagger (D^\mu \varphi) \equiv \frac{1}{2}\left[(\partial_\mu h)^2 + \frac{g^2 Z_\mu Z^\mu (v+h)^2}{4\cos^2\theta} + \frac{g^2 W^-_\mu W^{+\mu}(v+h)^2}{4}\right] \Rightarrow$$

$$\boxed{\mathcal{L}_{\varphi kin} = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2 v^2}{8}\left(1+\frac{h}{v}\right)^2\left(\frac{Z_\mu Z^\mu}{\cos^2\theta} + 2W^+W^-\right)}$$

It follows from this Lagrangian that the interaction of the Higgs boson h with Z's & W's is determined by the same mechanism that gives masses to gauge bosons. We have:



$$\equiv 2i\frac{m_W^2}{v} g^{\mu\nu};$$



$$= \frac{2i m_Z^2}{v} g^{\mu\nu},$$

and so couplings are fixed by masses & the vacuum expectation value.