Lecture 12 The Higgs mechanism

We discussed the Goldstone phenomenon and the spontaneous symmetry breaking in the previous lecture. In this lecture we will talk about a similar phenomenon ~~but~~ in a gauge theory.

We will first consider a simple concrete example:

$$\mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

where ϕ is a complex field $\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad D_\mu\phi = \partial_\mu\phi + ieA_\mu\phi$$

and $V(\phi^*\phi) \equiv -\frac{1}{2}\mu^2|\phi|^2 + \frac{\lambda}{4!}(|\phi|^2)^2 \quad \mu^2 > 0.$

If we switch of A_μ-ϕ interaction, we'll have a case considered in the previous lecture, so we know that spontaneous symmetry breaking occurs. Let's repeat that calculation now with gauge fields. We choose

$\phi = \rho e^{i\theta}$, i.e. parametrization of the field involves radial component ρ and phase θ. $V(\phi^*\phi) = -\frac{1}{2}\mu^2\rho^2 + \frac{\lambda}{4!}\rho^4$, so there is non-vanishing value of ρ which minimizes the potential.

The covariant derivative is

$$D_\mu \phi = D_\mu e^{i\theta} \rho = \cancel{\partial_\mu (e^{i\theta} \rho)} = e^{i\theta}\left(\partial_\mu \rho + i(\partial_\mu \theta + eA_\mu)\rho\right) \Rightarrow$$

$$(D_\mu \phi)^* (D_\mu \phi) = (\partial_\mu \rho)^2 + \rho^2 (\partial_\mu \theta + eA_\mu)^2$$

Hence, $\mathcal{L} = (\partial_\mu \rho)^2 + \rho^2(\partial_\mu \theta + eA_\mu)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\rho)$

~~If~~ ~~spontaneous~~ We see that the cross-talk of $\partial_\mu \theta$ and A_μ happens because of the term $\partial_\mu \theta + eA_\mu$. If $e = 0$, $\mathcal{L} = (\partial_\mu \rho)^2 + \rho^2 (\partial_\mu \theta)^2 - V(\rho)$ and, on account of the spontaneous symmetry breaking $\rho = v + \sigma$, we will have one massive particle (σ) and one massless particle (θ) in the spectrum. However, if $e \neq 0$, we have a freedom of redefining the gauge field $A_\mu \to A_\mu - \frac{1}{e} \partial_\mu \theta$. This is a gauge transformation that is legitimate and has no bearing on $F_{\mu\nu} \Rightarrow$

$$\mathcal{L} = (\partial_\mu \rho)^2 + \rho^2 e^2 A_\mu^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\rho)$$

Now, if the ~~gau~~ symmetry is spontaneously broken : $\rho = v + \sigma$, the term with $\rho^2 e^2 A_\mu^2$ becomes $v^2 e^2 A_\mu^2$. Hence, after the symmetry breaking, the gauge field

Lagrangian becomes $\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + v^2 e^2 A_\mu^2$

The equations of motion

$$\delta \mathcal{L}_A = -\frac{1}{2} F_{\mu\nu} \left(\partial_\mu \delta A_\nu - \partial_\nu \delta A_\mu \right) + 2v^2 e^2 A_\mu \delta A_\mu =$$

$$= \left(\partial_\mu F_{\mu\nu} \right) \delta A_\nu + 2v^2 e^2 A_\mu \delta A_\mu \quad \Rightarrow$$

The equation of motion for the field A_μ is

$$\partial_\mu F_{\mu\nu} + 2v^2 e^2 A_\nu = 0 \quad \Rightarrow$$

$$\Box A_\nu - \partial_\nu (\partial_\mu A_\mu) + 2v^2 e^2 A_\nu = 0$$

Taking ∂_ν, we find $\partial_\nu A_\nu = \phi \Rightarrow$

$\Box A_\nu + 2v^2 e^2 A_\nu = 0 \Rightarrow$ The field A_ν is a spin-one field with 3 independent polarizations and the mass $\boxed{m_A^2 \equiv 2v^2 e^2}$. Hence we have a theory of a massive scalar particle and a massive gauge field, when we have spontaneous breaking of the gauge symmetry. No massless degrees of freedom; Goldstone bosons disappear ("get eaten"), gauge bosons become massive

Note that the number of degrees of freedom that we have before and after symmetry breaking remains the same. Indeed:

before we had: a massless gauge fields (2 polariz.) and a complex scalar field (2 real fields), so 4 "degrees of freedom".

after we have: a massive vector field (3 polarizat.) and 1 real field, so 4 degrees of freedom.

We continue now with the discussion of a similar phenomenon in non-abelian case

We consider a theory with the gauge group $SU(2)$ and a doublet of scalar fields (complex) $\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$ that transform under fundamental representation of $SU(2)$. The Lagrangian of the theory is $\mathcal{L} = -\frac{1}{4} F'^{a}_{\mu\nu} F'^{a\mu\nu} + (D_\mu \varphi)^+ (D^\mu \varphi) - \left[-\frac{\mu^2 \varphi^+ \varphi}{2} + \frac{\lambda}{4!} (\varphi^+ \varphi)^2 \right]$

Similar to the previous discussion, the minimal value of the scalar ~~[illegible]~~ potential energy is reached if $(\varphi^+ \varphi)_{vac} = \frac{6\mu^2}{\lambda} = v^2$ We now consider small ~~[illegible]~~ fluctuations of the field φ around its vacuum value. We choose $\varphi_{vac} = \begin{pmatrix} 0 \\ v \end{pmatrix}$ and write
$$\varphi(x) = \begin{bmatrix} \xi_1(x) + i\xi_2(x) \\ v + \xi_3(x) + i\xi_4(x) \end{bmatrix}$$

We would like to make use of the gauge invariance. In principle, we can rewrite the theory through a different field $\varphi'(x)$ by means of gauge transform
$$\varphi'(x) = \Omega(x)\varphi(x), \quad \Omega(x) \in SU(2)$$
We can show now that if $\varphi'(x) = \begin{bmatrix} \xi_1 + i\xi_2 \\ v + \xi_3 + i\xi_4 \end{bmatrix}$,

then $\varphi(x)$ can be taken as $\varphi(x) = \begin{bmatrix} 0 \\ v + h(x) \end{bmatrix}$

To see this, consider infinitesimal transformations $\Omega(x) = 1 + i\varepsilon^a(x)\tau^a$, where τ^a are the $su(2)$ generators, $\tau^a = \sigma^a/2$, $a \in 1..3$, where $\sigma^{1..3}$ are the Pauli matrices.

We have now

$$\Omega(x) = \begin{pmatrix} 1 + i\varepsilon_3/2 & \frac{\varepsilon_2}{2} + i\varepsilon_1/2 \\ -\frac{\varepsilon_2}{2} + \frac{i\varepsilon_1}{2} & 1 - i\varepsilon_3/2 \end{pmatrix} \Rightarrow \left(\begin{matrix}\text{to first order}\\ \text{in } \varepsilon \sim h\end{matrix}\right)$$

$$\Omega(x)\begin{pmatrix}0\\ v+h(x)\end{pmatrix} \approx \begin{pmatrix}0\\ v+h(x)\end{pmatrix} + \begin{pmatrix} i\varepsilon_3/2 & \frac{\varepsilon_2}{2} + i\varepsilon_1/2 \\ -\frac{\varepsilon_2}{2} + \frac{i\varepsilon_1}{2} & -i\varepsilon_3/2 \end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix} =$$

$$= \begin{pmatrix}0\\ v+h(x)\end{pmatrix} + \begin{pmatrix}(\varepsilon_2 + i\varepsilon_1)/2\\ -i\frac{\varepsilon_3}{2}v\end{pmatrix} = \begin{pmatrix}(\varepsilon_2 + i\varepsilon_1)/2\\ v+h(x) - i\varepsilon_3 v/2\end{pmatrix}$$

Hence, we conclude that by a gauge transformation, the field φ can be always choosen to be $\varphi(x) = \begin{pmatrix}0\\ v+h(x)\end{pmatrix}$.

We will now investigate the covariant derivative of the field φ:

$$D_\mu\varphi(x) = \left[\partial_\mu + ig\tau^a A_\mu^a\right]\begin{pmatrix}0\\ v+h(x)\end{pmatrix} = \cancel{\partial_\mu}$$

$$\left(\partial_\mu + ig\tau^a A_\mu^a\right)\begin{pmatrix}0\\ h(x)\end{pmatrix} + ig\tau^a A_\mu^a\begin{pmatrix}0\\ v\end{pmatrix}$$

Hence, the quadratic ~~peal~~ in A_μ^a part becomes

$$(D_\mu\varphi)^\dagger(D_\mu\varphi) \rightarrow (0, v)\left(-ig\tau^b A_\mu^b\right)\left(ig\tau^a A_\mu^a\right)\begin{pmatrix}0\\ v\end{pmatrix} =$$

$$= g^2 A^a_\mu A^b_\mu \,(0, v)\, [\tau^a \tau^b] \begin{pmatrix} 0 \\ v \end{pmatrix} \equiv$$

$$\equiv \frac{g^2}{8} A^a_\mu A^b_\mu \,(0, v)\, [\sigma^a\sigma^b + \sigma^b\sigma^a] \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{g^2}{8} 2 A^a_\mu A^a_\mu v^2 =$$

$$= \frac{g^2 v^2}{4} A^a_\mu A^{a\mu}$$

Hence, the gauge part of L becomes $\mathcal{L}_A = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{g^2 v^2}{4} A^a_\mu A^{a\mu}$, so that we have <u>three</u> ($a = 1,..,3$) massive bosons with ~~the~~ equal masses $\underline{m_a^2 = \frac{g^2 v^2}{2}}$, $a = 1..3$

The field h has the mass which is fully fixed by the properties of the scalar potential $\left[-\frac{\mu^2 \varphi^+\varphi}{2} + \frac{\lambda}{4}(\varphi^+\varphi)^2\right]$ which is, for the parametrization $\varphi = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$ is the same as for the simple scalar field, so that $m_h^2 = 2\mu^2$. Again, let us count degrees of freedom: <u>before</u> the symmetry breaking:

$$\underbrace{3 \times 2}_{\text{massless gauge fields}} + \underbrace{4}_{\text{scalars}} = \underline{10}$$

after the symmetry breaking:

$$\underbrace{3 \times 3}_{\text{massive gauge fields}} + \underbrace{1}_{\text{scalar}} \equiv \underline{10}$$

agree.

> So, as follows from the above examples, we have the mechanism to get rid of massless scalar particles <u>and</u>, at the same time, give masses to gauge bosons both abelian and non-abelian. This is the Higgs mechanism.

It is important to emphasize that the spectrum of the theory strongly depends on the details of the symmetry breaking. To see this consider again a gauge theory where scalar fields are in the adjoint representation

We have

$$\mathcal{L} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + Tr\left((D_\mu \varphi)^2\right) - \left(-\mu^2 Tr(\varphi^2) + \frac{\lambda}{3!}\left[Tr(\varphi^2)\right]^2\right),$$

where $\varphi = \varphi^a \tau^a$ and

$D_\mu \varphi = \partial_\mu \varphi + g[\varphi, \hat{A}_\mu]$. The vacuum expectation value of the field φ is given by the condition $\varphi_1^2 + \varphi_2^2 + \varphi_3^2 = \frac{6\mu^2}{\lambda} \equiv v^2$.

~~By~~ Let us choose vacuum in the "third" direction $\varphi_{vac} = v\tau^3$. The spectrum of gauge bosons follows from $D_\mu \varphi\big|_{\varphi \to \varphi_{vac}} = ig\,[\varphi_{vac}, \hat{A}_\mu] =$

$$\equiv igv\,[\tau^{(3)}, \hat{A}_\mu] = igv\left\{A^{(1)}_\mu [\tau^3, \tau^1] + A^{(2)}_\mu [\tau^3, \tau^2]\right\}$$

$$= igv\left\{A^{(1)}_\mu i\varepsilon^{312}\tau^2 + A^{(2)}_\mu i\varepsilon^{321}\tau^1\right\} =$$

$$\pm gv\left[A^{(1)}_\mu \tau^{(2)} + A^{(2)}_\mu \tau^{(1)}\right] \Rightarrow$$

$$Tr\left[(D_\mu \varphi_{vac})^2\right] = \frac{g^2 v^2}{2}\left(A^{(1)}_\mu \cdot A^{(1)}_\mu + A^{(2)}_\mu \cdot A^{(2)}_\mu\right).$$

Hence, we obtain <u>two</u> <u>massive gauge fields</u> $A^{(1)}_\mu, A^{(2)}_\mu$ while the field $A^{(3)}_\mu$ <u>remains massless</u>

The mass of the gauge bosons $A^{(1)}_\mu$, $A^{(2)}_\mu$ is $m^2_{1,2} = g^2 v^2$. There is also one massive particle in the theory – the Higgs boson.

Let us talk about hadron physics, for a while.

Let us consider a theory of two <u>massless</u> fermions, that we assume to belong to an $SU(2)$ doublet. [This $SU(2)$ has nothing to do with the $SU(2)$ of EW interactions.] So $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$;

$L = \bar{\psi}\, i \hat{\partial} \psi$ and $\psi \to U\psi$, $U = \exp\left(\frac{i\epsilon^a \tau^a}{2}\right)$ is the symmetry of the Lagrangian.

We write $\psi = \psi_L + \psi_R$; the Lagrangian splits $L = \bar{\psi}_L i\hat{\partial}\psi_L + \bar{\psi}_R i \hat{\partial} \psi_R$ and the $SU(2)$ transformations can be applied to left- & right-fields separately. We then write

$$\begin{cases} \delta\psi_L = i\epsilon_L^a T^a \psi_L \\ \delta\psi_R = i\epsilon_R^a T^a \psi_R \end{cases} \Rightarrow \text{symmetry group is } SU_L(2) \otimes SU_R(2).$$

The variation of the field ψ is then

$$\delta\psi = \delta\psi_L + \delta\psi_R = i\epsilon_L^a T^a \psi_L + i\epsilon_R^a T^a \psi_R =$$

$$= i\epsilon_L^a T^a \frac{(1+\gamma_5)}{2}\psi + i\epsilon_R^a T^a \frac{(1-\gamma_5)}{2}\psi$$

$$\equiv i\frac{(\epsilon_L^a + \epsilon_R^a)}{2} T^a \psi + i\frac{(\epsilon_L^a - \epsilon_R^a)}{2} T^a \gamma_5 \psi$$

$$\boxed{\delta\psi \equiv i\epsilon^a T^a \psi - i\epsilon_5^a T^a \gamma_5 \psi}$$, where

$$\epsilon^a = \frac{\epsilon_R^a + \epsilon_L^a}{2}; \quad \& \quad \epsilon_5^a = \frac{\epsilon_R^a - \epsilon_L^a}{2};$$

We would like to associate the two fermions with neutrons and proton; and the global symmetry with the isospin. However, we can not just write the mass term ~~becau~~ $\Delta\mathcal{L}_m = m\bar{\Psi}\Psi$ because it will explicitely violate $SU_L(2) \otimes SU_R(2)$ "chiral" symmetry. We also can not describe proton & neutron with massless fields, since it is very clearly unphysical. So, we will try to use ideas of the spontaneous symmetry breaking where the Lagrangian will have an $SU_L(2) \otimes SU_R(2)$ symmetry, but the vacuum will not.

We write the Lagrangian as:

$$\mathcal{L} = i\bar{\Psi}\partial\Psi - g\bar{\Psi}_L \Sigma \Psi_R - g\bar{\Psi}_R \Sigma^+ \Psi_L + \mathcal{L}(\Sigma),$$

where Σ is an $SU(2)$ matrix. Under $SU(2)_L \otimes SU_R(2)$ transformations, $\Psi_R \Rightarrow \hat{R}\Psi_R$, $\Psi_L \Rightarrow \hat{L}\Psi_L$, so if we want the interaction terms to be invariant, we will need $\begin{cases} \Sigma \to L\Sigma R^+ \\ \Sigma^+ \to R\Sigma^+ L^+ \end{cases}$

under $SU_2(2) \otimes SU_R(2)$.

Taking infinitesimal left & right transformations, we find

$$\delta\Sigma = \delta L\,\Sigma + \Sigma\,\delta R^+ \; ; \begin{cases} L = \exp(il^a\tau^a/2) \\ R = \exp(ir^a\tau^a/2) \end{cases}$$

$$\Rightarrow \delta L \simeq (1 + il^a\tau^a/2) - 1$$

$$\delta R = (1 + ir^a\tau^a/2) - 1 \Rightarrow \delta R^+ = (1 - ir^a\tau^a/2) - 1$$

$$\Rightarrow$$

$$\delta\Sigma = i\left(\frac{\ell^a\tau^a}{2}\Sigma - \Sigma\frac{r^a\tau^a}{2}\right)$$

We can take Σ in the form $\boxed{\Sigma = \sigma + i\vec{\pi}\cdot\vec{\tau},}$ so that σ & $\pi_{i=1..3}$ are the 4 real fields. (It is not obvious that $SU_L(2)$ & $SU_R(2)$ transformation maintain this reality property, but they do as we will show in a second) so

$$\delta\Sigma = \delta\sigma + i\,\delta\vec{\pi}\cdot\vec{\tau} = i\left(\frac{\epsilon_L^a\tau^a}{2}(\sigma + i\vec{\pi}\vec{\tau}) - (\sigma + i\vec{\pi}\vec{\tau})\frac{\epsilon_R^a\tau^a}{2}\right)$$

$$= \frac{i}{2}\left[(\epsilon_L^a - \epsilon_R^a)\tau^a\sigma + i\left(\epsilon_L^a\tau^a\vec{\pi}\vec{\tau} - \vec{\pi}\cdot\vec{\tau}\,\epsilon_R^a\tau^a\right)\right]$$

$$= \frac{i}{2}\Big[(\epsilon_L^a - \epsilon_R^a)\tau^a\cdot\sigma + i\Big(\epsilon_L^a\pi^b(\delta^{ab} + i\epsilon^{abc}\tau^c)$$

$$- \pi^b\epsilon_R^a(\delta^{ab} + i\epsilon^{abc}\tau^c)\Big)\Big]$$

$$= \frac{i}{2}\left[(\epsilon_L^a - \epsilon_R^a)\tau^a\sigma + i\vec{\pi}(\vec{\epsilon}_L - \vec{\epsilon}_R) - \left[(\vec{\epsilon}_L + \vec{\epsilon}_R)\times\vec{\pi}\right]\cdot\vec{\tau}\right]$$

$$= \frac{\vec{\pi}\cdot(\vec{\epsilon}_R - \vec{\epsilon}_L)}{2} + i\left\{-\frac{(\vec{\epsilon}_R - \vec{\epsilon}_L)}{2}\sigma - \left[\left(\frac{\vec{\epsilon}_R + \vec{\epsilon}_L}{2}\right)\times\vec{\pi}\right]\right\}\vec{\tau}$$

Hence
$$\begin{cases} \delta\sigma = \vec{\pi}\cdot\vec{\epsilon}_5\,; \\ \delta\vec{\pi} = -\vec{\epsilon}_5\cdot\sigma + \vec{\pi}\times\vec{\epsilon}\,; \end{cases} \qquad \text{where } \vec{\epsilon}_5 = \frac{\vec{\epsilon}_R - \vec{\epsilon}_L}{2},\quad \vec{\epsilon} = \frac{\vec{\epsilon}_R + \vec{\epsilon}_L}{2}.$$

We now use the $\Sigma = \sigma + i\vec{\pi}\cdot\vec{\tau}$ parametrization in the Lagrangian and find

$$\mathcal{L} = i\bar{\psi}i\hat{\partial}\psi - g\bar{\psi}\psi\sigma + ig\bar{\psi}\,\vec{\pi}\cdot\vec{\tau}\gamma_5\psi + \mathcal{L}_2(\Sigma).$$

It is clear from this Lagrangian that

if σ gets a vacuum expectation value, the term $g\bar{\Psi}\Psi\sigma$ becomes a mass term for the fermion field. To arrange such vacuum expectation value, we use $\mathcal{L}_\Sigma(\Sigma)$.

Recall, that the Lagrangian should be invariant under $SU(2)_L \otimes SU(2)_R$ transformations. This is possible if we write $\mathcal{L}_\Sigma(\Sigma)$ as a function of $Tr(\Sigma^+\Sigma)$ since $\Sigma^+\Sigma = (\sigma - i\vec{\pi}\cdot\vec{\tau})(\sigma + i\vec{\pi}\vec{\tau}) \equiv (\sigma^2 + \vec{\pi}^2)\cdot\hat{\mathbb{1}}_2$, where $\hat{\mathbb{1}}_2$ is the $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ matrix. Hence $\frac{1}{2}Tr(\Sigma^+\Sigma) \equiv \sigma^2 + \vec{\pi}^2$. We then write the kinetic term as $\frac{1}{4}\,tr\left((\partial_\mu\Sigma^+)(\partial^\mu\Sigma)\right) \equiv \frac{1}{2}\partial^\mu\sigma\,\partial_\mu\sigma + \frac{1}{2}(\partial^\mu\vec{\pi})(\partial_\mu\vec{\pi})$ & the interaction potential

$V(\varphi^2) \equiv \frac{\lambda}{4}\left[\varphi^2 - F_\pi^2\right]^2$ through $\frac{1}{2}tr(\Sigma^+\Sigma) = \varphi^2$

As the result $V(\sigma^2, \vec{\pi}^2) \equiv \frac{\lambda}{4}\left[\sigma^2 + \vec{\pi}^2 - F_\pi^2\right]^2$

Now, things become clearly connected to the Higgs discussion: the minimum of the potential is at $\sigma^2 + \vec{\pi}^2 = F_\pi^2$. We take the vacuum fields to be $\langle\sigma\rangle = F_\pi$ and $\langle\vec{\pi}\rangle = \emptyset$; ~~and~~ introduce σ' such that $\sigma = F_\pi + \sigma'$ and rewrite the Lagrangian in terms of the σ' field:

$$\mathcal{L} = \bar{\psi} i \hat{\partial} \psi - g F_\pi \bar{\psi} \psi - g \bar{\psi} \psi \sigma' + i g \bar{\psi} \vec{\pi} \cdot \vec{\tau} \gamma_5 \psi$$
$$+ \frac{1}{2} \partial^\mu \sigma' \partial_\mu \sigma' + \frac{1}{2} (\partial^\mu \vec{\pi}) (\partial_\mu \vec{\pi}) - \frac{\lambda}{4} \left[\vec{\pi}^2 + 2\sigma' F_\pi + \sigma'^2 \right]^2 .$$

It follows from this Lagrangian that the fermion field(s) got a mass $m \equiv g F_\pi$ and that the σ' field got a mass $m_\sigma^2 = 2\lambda F_\pi^2$. The three $\vec{\pi}$ - fields (pions) remained massless.

It is instructive to compute the current associated with chiral (ϵ_5) transformations. Fields transform as ($\epsilon_5 \neq 0$, $\vec{\epsilon} = \emptyset$)

$$\delta \sigma = \vec{\pi} \cdot \epsilon_5 \qquad \delta \vec{\pi} = - \epsilon_5 \sigma \qquad \delta \psi = - i \vec{\epsilon}_5 \vec{\tau}/2 \gamma_5 \psi$$

The current

$$J_5^{\mu, a} = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \sigma)} \delta \sigma + \frac{\delta \mathcal{L}}{\delta(\partial_\mu \vec{\pi}^a)} \delta \pi^a + \frac{\delta \mathcal{L}}{\delta(\partial_\mu \psi)} \delta \psi$$

$$= (\partial^\mu \sigma) \pi^a - (\partial^\mu \pi^a) \sigma - i \bar{\psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}^a}{2} \psi .$$

Let's write it through the "shifted" fields, with vanishing vacuum expectation value

$$J_5^{\mu, a} = (\partial^\mu \sigma') \pi^a - (\partial^\mu \pi^a) F_\pi - (\partial^\mu \pi^a) \sigma'$$
$$- i \bar{\psi} \gamma^\mu \gamma_5 \frac{\vec{\tau}^a}{2} \psi \approx - F_\pi \partial^\mu \pi^a + \ldots$$

where the elipses stand for contributions from <u>massive fields</u>. A linear term in the current implies that the matrix element of an axial current between a vacuum and a pion is simple

$$\langle 0 | J_5^{\mu a} | \pi^b \rangle = i F_\pi p^\mu \delta^{ab}$$

Such matrix element, as it turns out can be probed in pion decays (pions are not really massless, although they *are* in this model). Hence, the F_π can be obtained in this way. The constant g is obtained from the pion-nucleon scattering. Then the σ-model predicts the relation of three independent quantities

$$\boxed{m_N = g F_\pi}$$