Theoretische Teilchenphysik II

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Exercise Sheet 9

Due 21.12.2016

Problem 1 - Spontaneous symmetry breaking

Construct a field theory with spontaneously broken gauge symmetry SU(2) where the masses of all 3 gauge bosons are different.

Problem 2 - Gauge dependence in theories with spontaneously broken symmetries

Consider a U(1) gauge theory with the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi^* D^\mu\phi - V(\phi^*\phi) - \frac{1}{2\xi}\left(\partial_\mu A^\mu - \xi\, e\, v\, \phi_2\right)^2 , \tag{1}$$

where v is the vev defined below, $\phi = (\phi_1 + i\,\phi_2)/\sqrt{2}$ and

$$V(\phi^*\phi) = -\frac{\mu^2}{2}|\phi|^2 + \frac{\lambda}{4!}|\phi|^4 . \tag{2}$$

Note that the last term in (1) is the gauge-fixing term (and ξ is the gauge-fixing parameter). This term allows us to quantise the theory but, on the other hand, physical quantities must be independent on ξ.

1. Break the U(1) symmetry by choosing $\phi_1 = v + \phi_1'$ and compute the spectrum of the theory. What is the mass of the ϕ_2 particle?

2. Calculate the propagators of the fields A and ϕ_2. To do this easily identify the terms in the Lagrangian that are quadratic in the corresponding field, perform a Fourier transform and invert the corresponding expression. What is the $\xi-$dependence of the two propagators? What happens to the propagators when the limit $\xi \to 0$ is formally taken?

3. Derive the Feynman rules that describe the interactions of ϕ_1', ϕ_2 and A_μ.

4. Use these Feynman rules in order to show that the scattering amplitude for

   $$\phi_1' + \phi_1' \to A + A$$

   computed at lowest order in perturbation theory, is independent on ξ[1].

[1] Note that there are two diagrams, one with the exchange of ϕ_2 and one with the exchange of A. The ξ dependence should cancel between the two of them.