Theoretische Teilchenphysik II

V: Prof. Kirill Melnikov , Ü: Dr. Robbert Rietkerk, Dr. Lorenzo Tancredi

Exercise Sheet 8

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Problem 1 - Linear Sigma model

Consider a theory described by the Lagrangian

$$\mathcal{L} = i\,\overline{\psi}\,\partial\!\!\!/\,\psi - g\,\overline{\psi}_L\,\Sigma\,\psi_R - g\,\overline{\psi}_R\,\Sigma^\dagger\,\psi_L + \mathcal{L}(\Sigma) \tag{1}$$

where $\Sigma \in \mathrm{SU}(2)$ and $\mathcal{L}(\Sigma)$ depends only on Σ, such that the total Lagrangian is invariant under transformations of the group $\mathrm{SU_L}(2) \otimes \mathrm{SU_R}(2)$,

$$\psi_R' = R\,\psi_R\,, \qquad \psi_L' = L\,\psi_L\,, \tag{2}$$

$$\Sigma' = L\,\Sigma\,R^\dagger\,, \qquad \Sigma'^\dagger = R\,\Sigma^\dagger\,L^\dagger\,. \tag{3}$$

1. Find the transformation rule for Σ under infinitesimal transformations of $\mathrm{SU_L}(2) \otimes \mathrm{SU_R}(2)$.
2. Since $\Sigma \in \mathrm{SU}(2)$, it can be decomposed as

   $$\Sigma(x) = \sigma(x)\,\mathbb{I} + i\,\vec{\pi}(x)\cdot\vec{\tau}\,, \qquad \text{where} \quad \sigma, \pi_j \in \mathbb{R}\,, \quad \text{and} \quad \tau_j \;\; \text{are the Pauli matrices.}$$

   Find the transformation rules for σ and $\vec{\pi}$ under infinitesimal transformations of $\mathrm{SU_L}(2) \otimes \mathrm{SU_R}(2)$.
3. Take now

   $$\mathcal{L}(\Sigma) = \frac{1}{4}\mathrm{Tr}\left[(\partial_\mu\Sigma^\dagger)(\partial^\mu\Sigma)\right] - \frac{\lambda}{4}\left(\frac{\mathrm{Tr}[\Sigma^\dagger\Sigma]}{2} - \mathrm{F}_\pi^2\right)^2\,.$$

   Prove that $\mathcal{L}(\Sigma)$ is invariant under $\mathrm{SU_L}(2) \otimes \mathrm{SU_R}(2)$.
4. Express $\mathcal{L}(\Sigma)$ in terms of σ and $\vec{\pi}$.
5. Break now the $\mathrm{SU_L}(2)\otimes\mathrm{SU_R}(2)$ symmetry by choosing the vacuum $<\sigma>= F_\pi$. Write the Lagrangian in the "broken phase". What is the spectrum of the theory?
6. Show that the Lagrangian in the broken phase is invariant under the "diagonal subgroup" of $\mathrm{SU_L}(2)\otimes\mathrm{SU_R}(2)$, namely under transformations of the type

   $$\Sigma \to L\,\Sigma\,R^\dagger\,, \qquad \text{where} \quad L = R\,.$$

7. Finally prove that the number of Goldstone bosons agrees with the number of broken symmetry generators.