Theoretische Teilchenphysik II

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Exercise Sheet 7

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Problem 1 - Additional term in the non-abelian Lagrangian

Consider a non-abelian gauge field theory with an arbitrary gauge group. At first sight, it appears possible to add to the standard gauge field Lagrangian the term

$$\delta\mathcal{L} = \epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left[F^{\mu\nu}F^{\rho\sigma}\right] .$$

Show that this term is, in fact, a total derivative

$$\delta\mathcal{L} = \partial_\mu J^\mu.$$

Find the expression for the vector current J^μ and explain why such terms do not change the equations of motion.

Problem 2 - Equations of motions in non-abelian field theory

Consider a non-abelian gauge theory where gauge fields couple to scalar fields in the fundamental representation of the gauge group

$$\mathcal{L} = \mathcal{L}_A + \left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right) - m^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2 ,$$

where $\mathcal{L}_A$ is the standard Lagrangian for the gauge fields.

1. Show that the equations of motion for the gauge field can be written in a form

$$\left(D_\mu F^{\mu\nu}\right)^a = gJ^a_\nu \tag{1}$$

   and find the current J^a_ν. Note that in Eq.(1) the covariant derivative acts on $F^{\mu\nu}$ which lives in the adjoint representation!

2. Show that $D_\mu D_\nu F^{\mu\nu} = 0$. Use this result to write down the conservation equation for the current J^a_μ.

3. Find the equations of motion for the scalar field ϕ.