Theoretische Teilchenphysik II

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Exercise Sheet 5

Due 23.11.2016

Problem 1 - Renormalization of the Yukawa Theory (see Peskin-Schröder, Ex. 10.2)

Consider the pseudo scalar Yukawa Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2} m^2 \phi^2 + \overline{\psi}\left(i\partial\!\!\!/ - M\right)\psi - i\,g\,\overline{\psi}\,\gamma^5\,\psi\,\phi \tag{1}$$

where ϕ is a real scalar field and ψ is a Dirac fermion. Notice that this Lagrangian is invariant under the parity transformation $\psi(t,\vec{x}) \to \gamma^0\,\psi(t,-\vec{x})$, $\phi(t,\vec{x}) \to -\phi(t,-\vec{x})$, in which the field ϕ carries odd parity.

1. Determine the superficially divergent amplitudes and work out the Feynman rules for renormalized perturbation theory for this Lagrangian. Include all necessary counterterm vertices.
2. Show that the theory contains a superficially divergent 4ϕ amplitude. This means that the theory cannot be renormalized unless one includes a scalar self-interaction

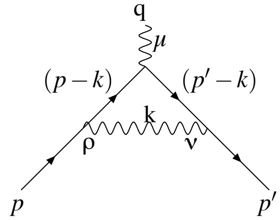
   $$\delta\mathcal{L} = \frac{\lambda}{4!}\phi^4,$$

   and a counterterm of the same form. Are there any further interactions required?
3. Compute the divergent part (the pole as $d \to 4$) of each counterterm, to the one-loop order of perturbation theory, implementing a sufficient set of renormalization conditions. You don't need to worry about finite parts of the counterterms. Since the divergent part must have a fixed dependence on the external momenta, you can simplify this calculation by choosing the momenta in the simplest possible way.

Problem 2 - The value of Z_1 in 1-loop QED

In the following exercise we will calculate the QED renormalization constant Z_1 at 1-loop order.

Let us consider the 1-loop correction to the QED vertex as shown below.



As it was discussed in class, the value of Z_1 is fixed by imposing that the amputated electron-photon interaction vertex be equal to $-i\,e\gamma^\mu$ for zero photon momentum, i.e. $q^\mu = 0$.

1. Using standard techniques for computing Feynman integrals, compute the 1-loop value of the QED vertex in dimensional regularisation, in the limit of $q^\mu \to 0$, $-i\,e\,\Lambda(p,p')_{p'\to p}$.
2. Imposing the renormalization condition $-i\,e\,\Lambda(p,p')_{p'\to p} \;\to\; -i\,e\,\gamma^\mu$, extract the value of δ_1, and therefore that of Z_1.
3. Compare this value with the one for Z_2 obtained in class. Are they equal or different? Why?