Theoretische Teilchenphysik II

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Exercise Sheet 13

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Problem 1 - One-loop UV divergences of two- and three-point functions in QCD

In this exercise we will compute the UV divergent part of one-loop corrections to two- and three-point functions in QCD.

1. We start by considering the gluon self-energy $-i\,\Sigma_g(q)$, where q^μ is the gluon momentum.

   a) Draw all one-loop Feynman diagrams which contribute to the gluon self-energy. You should find 4 different diagrams. Do not forget to consider, together with fermions and gluons, also ghost contributions!

   b) Note that the four diagrams do not have and IR divergence in $d \to 4$ (or $\epsilon \to 0$), such that all poles must be only of UV nature.

   c) Compute the four contributions separately *in dimensional regularisation*, keeping only the $1/\epsilon$ divergent pieces. Note that only 2 diagrams must be computed. One can be, in fact, recycled from QED (*with an appropriate overall normalisation!*), while another one is identically equal to zero in dimensional regularisation. Why?

   d) Verify that upon summing the four contributions one recovers that the gluon self-energy is transverse, i.e.

$$-i\,\Sigma_g(q) = -i\,q^2\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\frac{\Pi(q^2)}{\epsilon}\,\delta_{ab}\,, \tag{1}$$

   where a, b are the color indices of the two external gluons. Note that this ensures that the mass of the gluons remains zero also under radiative corrections.

2. Consider now the one-loop corrections to the fermions self-energy $-i\,\Sigma_f(q)$. In this case there is only one Feynman diagram contributing at one-loop. Compute its value in *dimensional regularization* neglecting the finite piece, i.e. retain only the terms proportional to $1/\epsilon$!

3. Consider finally the one-loop corrections to the quark-gluon vertex in QCD.

   a) Draw the two Feynman diagrams contributing at one-loop order.

   b) Compute the two contribution, again retaining only the divergent pieces proportional to $1/\epsilon$! After having carried out the color algebra, notice that the divergent contributions come from the region where the loop momentum becomes very large. In particular, since the diagrams diverge *logarithmically* as $k \to \infty$, you can neglect all external momenta w.r.t the loop momentum k.