Theoretische Teilchenphysik II

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Exercise Sheet 12

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Problem 1 - Spinor-helicity methods and the forward-backward asymmetry

The goal of this exercise is to apply spinor-helicity methods to perform realistic computations.

1. Consider the annihilation of a e^+e^- pair to a $\mu^+\mu^-$ pair in the Standard Model (assume all fermion masses to be zero), $e^-(p_1)e^+(p_2) \to \mu^-(p_3)\bar{\mu}^+(p_4)$. Such a process can occur through an exchange of a photon or a Z boson. The interaction of charged leptons with photons is given by the vertex $ie\gamma_\mu$, while their interaction with Z-bosons by $ie/(2\sin(2\theta_W))\,(g_V\gamma_\mu + g_A\gamma_\mu\gamma_5)$, where $g_V = (1-4\sin^2\theta_W)$ and $g_A = 1$.

2. Split the above vertices into vertices that describe interactions of vector bosons with fermions of definite helicities. Show, that the amplitude $e^+e^- \to \gamma + Z \to \mu^+\mu^-$ can be written by

$$\mathcal{M}_{Z+\gamma} = \frac{ie^2}{s}\Big(A_{LL}\bar{v}_{p_2}\gamma^\mu\omega_L u(p_1)\bar{u}(p_3)\gamma_\mu\omega_L v(p_4) + A_{RL}\bar{v}_{p_2}\gamma^\mu\omega_R u(p_1)\bar{u}(p_3)\gamma_\mu\omega_L v(p_4) + A_{LR}\bar{v}_{p_2}\gamma^\mu\omega_L u(p_1)\bar{u}(p_3)\gamma_\mu\omega_R v(p_4) + A_{RR}\bar{v}_{p_2}\gamma^\mu\omega_R u(p_1)\bar{u}(p_3)\gamma_\mu\omega_R v(p_4)\Big) \quad (1)$$

   where $\omega_{L,R} = (1 \pm \gamma_5)/2$ are projectors on different helicity states. Express the coefficients A_{ij} through the couplings $g_{A,V}$.

3. Calculate all the relevant helicity amplitudes in terms of spinor products.

4. Calculate the sum of the helicity amplitudes squared. Show that this sum can be written as

$$\sum_{\text{hel}} |\mathcal{M}|^2 = X_1(1+\cos^2\theta) + X_2\cos\theta, \quad (2)$$

   where θ is the μ^- production angle relative to e^- direction. Express $X_{1,2}$ in terms of A_{ij}.

5. Since the cross-section for $e^+e^- \to \mu^-\mu^+$ is obtained from $\sum|\mathcal{M}|^2$ by intergrating over the scattering angle and since

$$\int_{-1}^{1} \mathrm{d}\cos\theta\ \cos\theta = 0, \quad (3)$$

   the scattering cross-section is proportional to $X_1(s)$. To study $X_2(s)$, one can define a **forward-backward asymmetry**

$$A_{FB} = \frac{\int_0^1 \mathrm{d}\cos\theta\ \mathrm{d}\sigma/\mathrm{d}\cos\theta - \int_{-1}^0 \mathrm{d}\cos\theta\ \mathrm{d}\sigma/\mathrm{d}\cos\theta}{\int_0^1 \mathrm{d}\cos\theta\ \mathrm{d}\sigma/\mathrm{d}\cos\theta + \int_{-1}^0 \mathrm{d}\cos\theta\ \mathrm{d}\sigma/\mathrm{d}\cos\theta}, \quad (4)$$

   which gives the fractional difference in the number of negatively charged muons which are produced in the forward and backward hemispheres, defined w.r.t. electron direction of motion. Calculate the forward-backward asymmetry in terms of $X_{1,2}$.

6. Find A_{FB} in the small energy limit $s \ll M_Z^2$ and in the Z-resonance limit $s \to M_Z^2$, where the photon exchange can, effectively, be neglected.