Theoretische Teilchenphysik II

V: Prof. Kirill Melnikov , Ü: Dr. Robbert Rietkerk, Dr. Lorenzo Tancredi

Exercise Sheet 10

Due 11.1.2016

Problem 1 - Higgs boson decays

The goal of this exercise is to consider decays of the Higgs boson into various SM particles. First of all, you should recall the form of the SM Lagrangian after Spontaneous Symmetry Breaking. In particular, the coupling of the Higgs boson to a fermion f is described by the the Lagrangian

$$\mathcal{L}_{Hf\bar{f}} = -\frac{m_f}{v}\, H\, \bar{\psi}_f \psi_f\,, \tag{1}$$

where m_f is the fermion mass, $v \approx 246$ GeV is the Higgs vev, H is the Higgs field and ψ_f is the fermion field. The coupling of the Higgs boson to Z and W bosons, instead, is described by the Lagrangian

$$\mathcal{L}_{HVV} = \frac{g^2\, v^2}{8}\left(1+\frac{H}{v}\right)^2 \left(\frac{Z^\mu Z_\mu}{\cos^2\theta} + 2\, W^{+,\mu}\, W^-_\mu\right) \tag{2}$$

where θ is the Weinberg angle, g is the electroweak coupling, and the masses of the vector bosons are given in terms of the Lagrangian parameters by

$$m_W^2 = \frac{g^2\, v^2}{4}\,, \qquad \text{and} \qquad m_Z^2 = \frac{m_W^2}{\cos^2\theta}\,.$$

1. Use relations between m_W, v and g and the electric charge e, and their known numerical values from the PDG[1] to verify that $v \approx 246$ GeV.
2. Consider the decay of a Higgs boson into a fermion of mass m and derive the expression for the partial width. Estimate, numerically, the decay width of $H \to b\bar{b}$, $H \to e^+e^-$ and $H \to \tau\tau$. Take the required parameters (i.e. the fermion masses) from the PDG web page[1].
3. When the Higgs boson was still undiscovered, it was important to estimate possible values of its mass, even roughly. One of the possible arguments that allows to place an *upper* bound on the Higgs mass is sketched below.

   Consider the decay of the Higgs boson with the mass M_H into a pair of W bosons, $H \to W^+W^-$ (we assume that the Higgs mass is large enough for this decay to occur). Calculate the decay rate and take the limit of a very large Higgs boson mass. The decay rate scales as $\Gamma_H \sim M_H^3$. It is not unreasonable to assume that a particle *must have* the width smaller than a mass (ortherwise it will decay "before it is created"). Requiring that $\Gamma_H \leq M_H$, find the allowed values of M_H.
4. The major motivation for introducing the Higgs boson is to give masses to other particles in the Standard Model in a way consistent with the gauge symmetry; therefore, it can be expected that the Higgs boson does not couple to massless particles. This expectation is true at the level of the "bare" Lagrangian but it gets violated once quantum effects are considered.

   To see this, compute the matrix element that describes the interaction of the Higgs boson with gluons (non-abelian gauge bosons of strong interactions; the gauge group is $SU(3)$). Imagine that both gluons and the Higgs boson couple to a quark with the mass m_Q and obtain explicit results for the amplitude in the limit $m_Q \ll M_H$ and $m_Q \gg M_H$ (keep only the leading terms in both cases).

---

[1] http://pdg.lbl.gov/2014/listings/contents_listings.html