Advanced Quantum Field Theory

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Exercise Sheet 10

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Problem 1 - Coupling of gravity to matter

Gravity couples to the energy-momentum tensor of matter.

1. Compute the energy-momentum tensor for QED and determine the interaction vertex of the graviton with an electron and the photon.
2. Enumerate and construct explicitly the "polarization vectors" of a graviton assuming that it flies along the z-axis.
3. Consider the scattering of two electrons to two muons and a graviton $e^+e^- \to \mu^+\mu^- + G$. Assuming that the graviton is soft (i.e. its momentum is much smaller than the momenta of the electrons and muons), write down a matrix element for this process. What is the consequence of the general coordinate invariance for the matrix element of the above process?

Problem 2 - Kaluza-Klein theories

Consider a five-dimensional space time characterized by a metric

$$\mathrm{d}s^2 = g^4_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - a^2\left(\mathrm{d}\theta + A_\mu \mathrm{d}x^\mu\right)^2. \tag{1}$$

This metric corresponds to a situation where a circle of radius a is attached to every point of space-time. The transformations $\theta \to \theta + \Lambda(x)$ and $A_\mu \to A_\mu - \partial_\mu \Lambda(x)$ leave the metric invariant.

The Lagrangian is given by

$$\mathcal{L}_5 = \frac{1}{16\pi G_5}\int \mathrm{d}^4x \mathrm{d}\theta \sqrt{-g_5} R_5, \tag{2}$$

where R_5 is a scalar curvature of the five-dimensional theory.

1. Show by explicit computation that $R_5 = R_4 - \frac{a^2}{4}F_{\mu\nu}F^{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
2. Show that the five-dimensional Lagrangian $\mathcal{L}_5$ can be written as a Lagrangian of a four-dimensional gravity and electromagnetism.
3. What is the relation between five- and four-dimensional Newton's constants?
4. Consider generalization of a five-dimensional space-time to a six-dimensional one by attaching a sphere to each point in the four-dimensional space. What is a four-dimensional gauge theory that gravity in such a space corresponds to?