Advanced Quantum Field Theory

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Exercise Sheet 1

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Problem 1 - Properties of kinks

In the lecture we have seen that the solution for a Kink at rest centered in $x = x_0$ can be written as

$$\phi_k(x) = \frac{\mu}{g}\,\tanh\left(\frac{\mu}{\sqrt{2}}(x - x_0)\right), \tag{1}$$

with $\mu = v\,g$.

1. Explain how, by boosting this solution to a reference frame which moves with velocity β w.r.t. the kink one can get a corresponding solution for a moving Kink

$$\phi_k(x,\beta) = \frac{\mu}{g}\,\tanh\left(\frac{\mu}{\sqrt{2}}\frac{(x - x_0 - \beta\,t)}{\sqrt{1-\beta^2}}\right). \tag{2}$$

2. Prove that (2) satisfies the equations of motion.
3. Calculate the classical energy and the classical momentum of the moving kink (2).
4. Show that for the moving kink the relativistic relation between energy, momentum and mass hold.

Problem 2 - Stability of the static kink solution

The stability of the static kink solution (1) follows from the properties of the solutions of the differential equation

$$\hat{\mathcal{O}}\,\chi(x) = \omega^2\,\chi(x)\,, \tag{3}$$

where

$$\hat{\mathcal{O}} = -\frac{d}{dx^2} + \frac{\partial^2 V}{\partial \phi^2}(\phi_k)\,, \qquad V(\phi) = \frac{g^2}{4}(\phi^2 - v^2)^2\,, \tag{4}$$

and ϕ_k is the static kink solution (1).

1. Calculate explicitly the operator $\hat{\mathcal{O}}$.
2. Prove now that, in the two-dimensional model under study, the operator $\hat{\mathcal{O}}$ always admits one mode with *zero* eigenvalue, i.e. a *zero mode*

$$\hat{\mathcal{O}}\,\chi_0(x) = 0\,. \tag{5}$$

3. Show that the solution for the zero mode appropriately normalised reads

$$\chi_0(x) = \sqrt{\frac{3\,m}{8}}\,\frac{1}{[\cosh{(m\,x/2)}\,]^2}\,. \tag{6}$$

4. Solve the differential equation (3) and find the spectrum of the operator $\hat{\mathcal{O}}$. Note that (3) is equivalent to a Schrödinger equation with a potential whose spectrum can be determined exactly.
5. Discuss the consequences of the spectrum determined in point 2. for the stability of the kink solution.