Physics of Strong Interactions

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Exercise Sheet 9

Due 15.01.2016

Power Counting

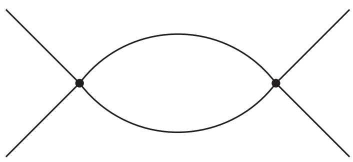
Suppose the coupling of terms in the Lagrangian go like F^2/Λ_{CSB}^{2n}, where Λ_{CSB} is the chiral symmetry breaking scale, $2n$ is the number of derivatives. In this problem, we construct a power-counting argument to show that if a divergent loop diagram is cut off at momentum of order $\Lambda_{CSB} \approx 4\pi F$, it contributes terms to the effective action of the same order of magnitude as the bare interaction terms. We restrict ourself to the derivative interactions because terms with derivatives replaced by $B_0 M_Q$ act the same way.

1) Show that a vertex has the form

$$(2\pi)^4 \delta^4(\Sigma p_i) F^2 \left(\frac{p^2}{\Lambda_{CSB}^2}\right)^n \left(\frac{1}{F}\right)^m, \tag{1}$$

where p is a typical momentum and m is the number of π lines emanating from the vertex.

2) We start by considering a concrete example.



Call the loop momenta k and external momenta p. For each internal meson line, include a factor

$$\frac{1}{k^2} \frac{d^4k}{(2\pi)^4}. \tag{2}$$

The 4-fermion vertex has contributions from the leading order and NLO lagrangians. Use the power counting arguments to compute each contribution and show that there are contributions of the same order in each term.

3) Now, we would like to make the argument for a general loop diagram. Take N such vertices for various n and m. Suppose there are l internal meson lines. Consider a contribution in which there is a k^{2j} from the momentum factors and the rest involves only external momenta, p^{2M}, for $M = N + \Sigma n - j$. Show that there are terms that contribute at the same order of magnitude as the bare interaction terms.

The Axial $U(1)$ Problem

In class we looked at $SU(3)_L \times SU(3)_R$ symmetries to build up our chiral theory. The transformations we have imposed, e.g. $\psi_L \to L\psi_L$, are actually $U(3)$ transformations. Since we can write $U(3) = SU(3) \times U(1)$, we have an extra symmetry that needs to be considered.

Suppose the axial $U(1)$ were a symmetry of QCD. There would be an $SU(3)$ singlet Goldstone boson field, π_0, which in the $SU(3)$ symmetry limit transforms under a $U(1)$ transformation by a translation $\delta\pi_0 = c_0$. In the presence of $SU(3) \times SU(3)$ symmetry breaking, the analog of

$$\mathcal{L} = \frac{F^2}{4}\mathrm{tr}\left[D^\mu \Sigma^\dagger D_\mu \Sigma\right] + B_0 \mathrm{tr}\left[\Sigma^\dagger M_Q\right] + B_0 \mathrm{tr}\left[\Sigma M_Q\right] \tag{3}$$

is

$$\begin{aligned} \mathcal{L}(\pi) &= \frac{1}{2}\partial^\mu \pi_0 \partial_\mu \pi_0 + \frac{F^2}{4}\mathrm{tr}\left[\partial^\mu \Sigma^\dagger \partial_\mu \Sigma\right] \\ &\quad + B_0 \mathrm{tr}\left[M_Q \Sigma^\dagger e^{-i\lambda\pi_0/F}\right] + B_0 \mathrm{tr}\left[M_Q \Sigma e^{i\lambda\pi_0/F}\right], \end{aligned} \tag{4}$$

where λ is an unknown constant that determines the strength of the spontaneous breaking of the $U(1)$ symmetry. In Equation 4 M_Q is transformed with the extra $U(1)$ transformations. The new Goldstone field π_0 appears in order to compensate for this transformation meaning that there are now 9 Goldstone bosons.

Equation 4 cannot describe the pseudo-scalar mesons in our world for any value of λ. Show this by computing the mass spectrum as done in class and demonstrate that it does not describe our physical world. Show that the lightest meson has a mass less than km_{pi} for some k. What is k?