Physics of Strong Interactions

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Exercise Sheet 7

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Electromagnetic current

In class we have shown that, starting from the leading term in the chiral Lagrangian coupled to external electromagnetic field, one derives the following expression for the electromagnetic current

$$J^\mu_{\rm em} = [\vec{\pi} \times \partial^\mu \vec{\pi}] \left(1 - \frac{\vec{\pi}^2}{3F_\pi^2}\right). \tag{1}$$

Calculate additional contributions to the electromagnetic current that originate from the following terms in next-to-leading chiral Lagrangian

$$\begin{aligned} \mathcal{L}^{(4)} &= \alpha_4 \mathrm{Tr}\left[D_\mu U (D^\mu U)^+\right] \mathrm{Tr}\left[\chi U^+ + U\chi^+\right] + \alpha_5 \mathrm{Tr}\left[D_\mu U (D^\mu U)^+ (\chi U^+ + U\chi^+)\right] \\ &+ i\alpha_9 \mathrm{Tr}[\hat{F}_{\mu\nu}\left(D^\mu U (D^\nu U)^+ + (D^\nu U)^+ (D^\mu U)\right)], \end{aligned} \tag{2}$$

where $\chi = m_\pi^2 I_2$ is a matrix source function, with I_2 the 2×2 identity matrix.
Recall also that $D_\mu = \partial_\mu U + eA_\mu[\hat{Q}, U]$ with $\hat{Q} = 1/6 + 1/2\tau_3$, and $\hat{F}_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)\, e\hat{Q}$.

Pion scattering amplitude to higher terms in the chiral expansion

In the previous exercise you computed the pion scattering $\pi^a\pi^b \to \pi^c\pi^d$ to first non-vanishing order in E/F_π expansion. In this exercise, we will discuss what changes if we want to compute the pion scattering amplitude to next-to-leading order term in that expansion. We will work in exact chiral limit (massless quarks). The Lagrangian reads

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U)(\partial^\mu U)^+\right] + \alpha_1 \mathrm{Tr}\left[\partial_\mu U)(\partial^\mu U)^+\right]^2 + \alpha_2 \mathrm{Tr}\left[\partial_\mu U)(\partial_\nu U)^+\right] \mathrm{Tr}\left[\partial^\mu U)(\partial^\nu U)^+\right], \tag{3}$$

where $\alpha_{1,2}$ are arbitrary phenomenological constants. Our goal is to compute the scattering amplitude $0 \to \pi^a\pi^b\pi^c\pi^d$ to $\mathcal{O}(E^4)$ where $E \sim \sqrt{s} \sim \sqrt{t} \sim \sqrt{u}$ is a typical energy of the pions in that process.

1. The leading term in the chiral Lagrangian contributes to $\mathcal{O}(E^4)$ terms in the amplitude through one-loop corrections. Calculate the corresponding contributions to the scattering amplitudes using dimensional regularization. Separate divergent and finite terms.

2. The contribution of the two subleading terms in the chiral Lagrangian contain four derivatives; therefore they directly contribute to $\mathcal{O}(E^4)$. Calculate their contribution to the scattering amplitude.

3. Show that by adjusting the parameters α_1 and α_2 you can remove all the divergences in the the scattering amplitude.

4. Which parts of the amplitude are predicted uniquely? Which parts depend on the unknown constants $\alpha_{1,2}$. Is it possible to determine $\alpha_{1,2}$ from measurements of various pion scattering amplitudes?