Physics of Strong Interactions

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Exercise Sheet 6

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Pion scattering amplitude

The modified leading $SU(2)$ chiral Lagrangian is given by the following expression

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\mathrm{Tr}\left[(\partial_\mu U)(\partial^\mu U)^+\right] + \frac{m_\pi^2 F_\pi^2}{4}\mathrm{Tr}\left[U + U^+\right], \tag{1}$$

where $U = e^{i\pi^a\tau^a/F_\pi}$ and $\tau^{1,2,3}$ are the Pauli matrices.

Use this Lagrangian to compute the four-gluon scattering amplitude

$$0 \to \pi^a + \pi^b + \pi^c + \pi^d. \tag{2}$$

1) Start by expanding $\mathcal{L}_2$ to relevant (fourth) order in the pion fields.

2) Derive the Feynman rules for $\mathcal{L}_2$.

3) Compute the scattering amplitude.

Uniqueness of the chiral Lagrangian

When the chiral Lagrnagian is derived, it is important to remove all the redundant operators. Unfortuantely, establishing equivalence of operators that, at first sight, may look very different, requires some effort.

To get familiar with this situation, prove the following relation between the three operators that may contribute to the chiral Lagrangian at next-to-leading order in E/F_π expansion

$$\mathrm{Tr}\left[\partial_\mu U \partial^\nu U^+ \partial^\mu U \partial_\nu U^+\right] = \frac{1}{2}\mathcal{L}_4^b - \frac{1}{4}\mathcal{L}_4^{(a)}, \tag{3}$$

where

$$\mathcal{L}_4^{(a)} = \mathrm{Tr}\left[\partial_\mu U \partial^\mu U^+\right]\mathrm{Tr}\left[\partial_\nu U \partial^\nu U^+\right], \quad \mathcal{L}_4^{(b)} = \mathrm{Tr}\left[\partial_\mu U \partial^\nu U^+\right]\mathrm{Tr}\left[\partial^\mu U \partial_\nu U^+\right]. \tag{4}$$