Physics of Strong Interactions

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Exercise Sheet 5

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Problem 1 - Pion scattering in the Linear σ Model

Previously you have seen a model of pions, protons and neutrons called the linear sigma model. The Lagrangian for this model is given by,

$$\mathcal{L}_\pi = i\overline{\psi}\hat{\partial}\psi - gF_\pi\overline{\psi}\psi - g\sigma'\overline{\psi}\psi + ig\overline{\psi}\vec{\pi}\cdot\vec{\tau}\gamma^5\psi + \frac{1}{2}(\partial_\mu\sigma')^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - \frac{\lambda}{4}(\sigma'^2 + \vec{\pi}^2 + 2F_\pi\sigma')^2, \quad (1)$$

with a doublet of protons and neutrons, $\psi = \begin{pmatrix} p \\ n \end{pmatrix}$ of mass gF_π, a triplet of massless pions, $\vec{\pi}$, and a scaler field σ' with mass $m_{\sigma'} = \sqrt{2\lambda F_\pi^2}$.

1) Write down the vertices and propagators that can appear in this model.

2) Using these vertices, construct the tree level diagrams that contribute to $\pi_1\pi_3 \to \pi_1\pi_3$.

3) Compute the scattering amplitude $\mathcal{M}(\pi_1(p_1)\pi_3(p_2) \to \pi_1(p_3)\pi_3(p_4))$ in the limit of small collision energy. i.e. $s = (p_1 + p_2)^2 \to 0$.

Problem 2 - Pion scattering in a Non-Linear σ Model

In class, you will discuss another representation of the σ model called the exponential representation, or non-linear σ model. Here, the fields are written in terms of S and U with S defined by the transformation

$$\Sigma = \sigma + i\pi = (v + S)U \quad \text{and} \quad U = \exp(i\vec{\tau}\cdot\vec{\pi}'/v), \quad (2)$$

such that $\vec{\pi}' = \vec{\pi} + \ldots$. In this case, the Lagrangian becomes

$$\mathcal{L}_U = \frac{1}{2}(\partial_\mu S)^2 + \frac{(v+S)^2}{4}\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger) - \frac{\lambda}{4}(S + 2Sv)^2 + \overline{\psi}i\hat{\partial}\psi - g(v+S)(\overline{\psi}_L U\psi_R + \overline{\psi}_R U^\dagger\psi_L). \quad (3)$$

The pions are now hidden in the U matrices with the relationship between $\vec{\pi}$ and $\vec{\pi}'$ being complex. To compute the same pion scattering amplitude as in problem 1, we need to re-write the relevant terms in the Lagrangian.

1) The only term in the Lagrangian that has pion-pion interactions is $\frac{(v+S)^2}{4}\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger)$. Re-write U in terms of $\sin(\frac{|\vec{\pi}|}{v})$ and $\cos(\frac{|\vec{\pi}|}{v})$ and use the result to expand this term in the Lagrangian to $\mathcal{O}(\vec{\pi}^4)$. What kinds of terms do you end up with?

2) We want to compute the low energy limit $E_i \ll v$ of the $\pi_1\pi_3$ scattering amplitude using the Lagrangian in Equation 3. Show that the only relevant terms in the expanded Lagrangian give a 4-pion interaction. i.e. The diagram with an S exchange can be safely neglected.

3) Compute the amplitude $\mathcal{M}(\pi_1(p_1)\pi_3(p_2) \to \pi_1(p_3)\pi_3(p_4))$ and show that it is the same as the result from problem 1.