Physics of Strong Interactions

V: Prof. Kirill Melnikov,   Ü: Dr. Matthew Dowling

Exercise Sheet 4

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Problem 1 - Pion Decay

In class, we looked at how to obtain the pion decay constant f_π from the decay $\pi^- \to e^- \overline{\nu}_e$. In this problem, we want to compute the decay rate of a pion to a muon or an electron and associated neutrino. We will need the following constants,

$$m_e = 0.511\text{MeV} \quad m_\mu = 105.66\text{MeV} \quad m_{\pi^-} = 139.57\text{MeV}$$

1) Compute the decay rate of a pion to a lepton and neutrino in terms of the pion decay constant f_π.
2) Assuming that these are the only two modes of decay, compute the branching ratios $\mathcal{B}(\pi^- \to e^- \overline{\nu}_e)$ and $\mathcal{B}(\pi^- \to \mu^- \overline{\nu}_\mu)$. Compare these values to the current measurements of the branching ratios.

$$\mathcal{B}(\pi^- \to \mu^- \overline{\nu}_\mu) = 99.98770\%, \qquad \mathcal{B}(\pi^- \to e^- \overline{\nu}_e) = 1.230 \times 10^{-4}\%$$

3) Use the decay rate $\Gamma(\pi^- \to e^- \overline{\nu}_e)$ with the pion lifetime, $\tau_{\pi^-} = 2.6033 \times 10^{-8}$s, to find a value for f_π. The current accepted value is $f_\pi = 130.41(20)$MeV. What effects are the likely cause of the difference between your value of f_π and the accepted value?

Problem 2 - Nöther Currents

The Gell-Mann-Low Σ-model is invariant under $SU(2)_L \otimes SU(2)_R$ transformations. Find, explicitly, the Nöther currents associated with these symmetries and express them through π, σ and ψ fields.

$$\mathcal{L}_\pi = i\overline{\psi}\hat{\partial}\psi - g\overline{\psi}\psi\sigma - ig\pi^a\overline{\psi}\gamma_5\tau^a\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - F_\pi^2)^2. \tag{1}$$

Problem 3 - Neutron Decay

In this problem we want to compute the decay of a neutron into a proton and leptons, $n \to p + e^- + \overline{\nu}_e$ using the matrix element of the weak current

$$V^\alpha = \langle p|\overline{u}\gamma^\alpha d|n\rangle. \tag{2}$$

Note that we neglect the axial part of the weak coupling discussed in class and concentrate only on the vector part.

1) Suppose that the matrix elements of the electromagnetic current $J_\mu^{\text{em}} = 2/3\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$ are known at (almost) zero momentum transfer.

$$\langle p|J_\mu^{em}|p\rangle = \overline{u}\{F_p(0)\gamma_\mu + \frac{\mu_p}{2m_p}\sigma_{\mu\beta}q^\beta\}u, \tag{3}$$

$$\langle n|J_\mu^{em}|n\rangle = \overline{u}\{F_n(0)\gamma_\mu + \frac{\mu_n}{2m_n}\sigma_{\mu\beta}q^\beta\}u, \tag{4}$$

with

$$F_p(0) = 1, \quad F_n(0) = 0, \quad \mu_p = 1.79, \quad \mu_n = -1.91$$

Write the current $J_a^\mu = \overline{\psi}\gamma^\mu \frac{\tau_a}{2}\psi$ in terms of $+, -$ and 3 components such that $V^\alpha = \langle p|\overline{u}\gamma^\alpha d|n\rangle = \langle p|J_+^\alpha|n\rangle$. Note that τ^a are the Pauli matrices and $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$.

2) The electromagnetic current can be written as a linear combination of the singlet isospin current J_s^μ (the one that does not change under isospin transformations) and the third component of the iso-vector current J_3^μ. Find an expression for the isospin-singlet current.

   It can be shown that charges and currents satisfy generic commutation relations

$$[Q_a, j_b^\mu] = i\epsilon_{abc}J_c^\mu, \qquad [Q_a, J_s^\mu] = 0. \tag{5}$$

   Use these equations as well as the fact that we are interested in matrix elements with nearly zero momentum transfers to show that

$$\langle p|J_s^\mu|p\rangle = \langle n|J_s^\mu|n\rangle, \quad \langle p|J_3^\mu|p\rangle = -\langle n|J_3^\mu|n\rangle. \tag{6}$$

   Express the matrix elements $\langle n|J_3^\mu|n\rangle$ and $\langle p|J_3^\mu|p\rangle$ in terms of $\langle p|J_{em}^\mu|p\rangle$ and $\langle n|J_{em}^\mu|n\rangle$ using the above equations.

3) Use the fact that the proton and neutron belong to the same $SU(2)$ iso-spin multiplet to express the matrix element V_α through the parameters of the electromagnetic currents $F_{p,n}(0), \mu_{p,n}$.