Physics of Strong Interactions

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Exercise Sheet 2

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Problem 1 - The three gluon vertex

In class, we have constructed a three-gluon scattering amplitude using very general considerations. We found that

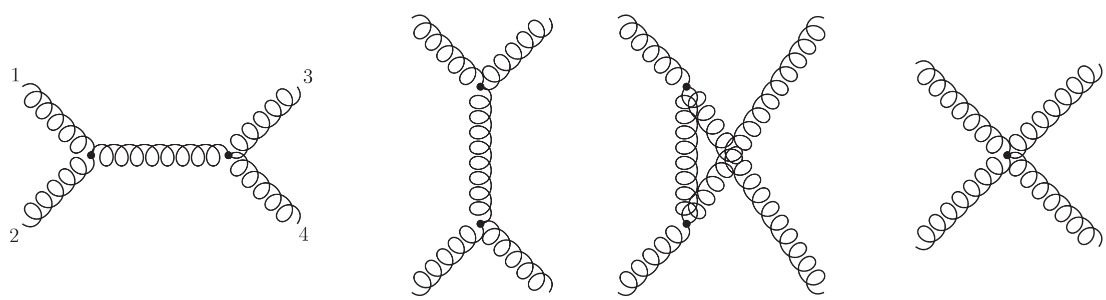
$$\mathcal{M}(1_L^a, 2_L^b, 3_R^c) = C^{abc} \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle}, \tag{1}$$

where $i^a_{L,R}$ denotes a gluon with the momentum i, color a and helicity L or R. Derive this result from the standard expression for the three-gluon vertex

$$V^{abc}(p_1,\mu_1;p_2,\mu_2;p_3,\mu_3) = f^{abc}\left((p_1-p_2)_{\mu_3} g_{\mu_1\mu_2} + (p_2-p_3)_{\mu_1} g_{\mu_2\mu_3} + (p_3-p_1)_{\mu_2} g_{\mu_1\mu_3}\right). \tag{2}$$

Problem 2 - Gluon Scattering Amplitude and the color ordering

Consider the gluon scattering process $g_1 + g_2 \to g_3 + g_4$. It can be computed from Feynman diagrams that involve the color part and the Lorentz part. Unfortunately, this is not particularly convenient, especially for more complex amplitudes. It turns out to be possible to write the full amplitude in terms of a single gauge invariant function $M(i,j,k,l)$ multiplied with various possible color traces. In this problem we will discuss how such a representation can be constructed.



1. There are four diagrams that contribute to this process at leading order. Every diagram has a color factor that looks like $f^{abe}f^{cde}$. Use the properties for generators of $SU(3)$ algebra

$$\left[T^a, T^b\right] = i\sqrt{2} f^{abc} T^c, \qquad Tr\left[T^a T^b\right] = \delta^{ab} \tag{3}$$

to re-write the color structure as traces over the generators T^{a_i}. How many unique traces are possible?

2. Show that the amplitude can be written in terms of six color-stripped amplitudes,

$$\begin{aligned}\mathcal{M}(1^{a_1}, 2^{a_2}, 3^{a_3}, 4^{a_4}) \;=\;& \mathrm{Tr}\left[T^{a_1}T^{a_2}T^{a_3}T^{a_4}\right] M(1,2,3,4) + \mathrm{Tr}\left[T^{a_1}T^{a_2}T^{a_4}T^{a_3}\right] M(1,2,4,3) \\ +\;& \mathrm{Tr}\left[T^{a_1}T^{a_3}T^{a_2}T^{a_4}\right] M(1,3,2,4) + \mathrm{Tr}\left[T^{a_1}T^{a_3}T^{a_4}T^{a_2}\right] M(1,3,4,2) \\ +\;& \mathrm{Tr}\left[T^{a_1}T^{a_4}T^{a_2}T^{a_3}\right] M(1,4,2,3) + \mathrm{Tr}\left[T^{a_1}T^{a_4}T^{a_3}T^{a_2}\right] M(1,4,3,2).\end{aligned} \tag{4}$$

Note that the amplitude is described by a single function $M(i,j,k,l)$ that needs to be computed for the different values of the argument.

3. Show that

$$M(1,2,3,4)+M(2,1,3,4)+M(2,3,1,4)+M(2,3,4,1)=0 \tag{5}$$

4. Check that $M(1,2,3,4)$ is gauge invariant. Why should it be?