Physics of Strong Interactions

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Exercise Sheet 13

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Callan Gross relation in the parton model

In class we showed that if, within the parton model, partons are described by spin one-half particles, then the Callan-Gross relation is satsified. Re-derive the Callan-Gross relation assuming that quarks are spin-zero particles.

Neutrino scattering

In this problem, we first motivate the presence of additional deep inelastic form factors that are proportional to differences of quark and antiquark distribution functions. Then we define these functions formally and work out their properties.

(a) We want to analyze neutrino-proton scattering using operators. Define

$$J_+^\mu = \overline{u}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) d, \qquad J_-^\mu = \overline{d}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) u. \tag{1}$$

Let

$$W^{\mu\nu(\nu)} = 2i \int d^4x e^{iq\cdot x} \langle P|T\{J_-^\mu(x) J_+^\nu(0)\}|P\rangle, \tag{2}$$

averaged over the proton spin. Show that the cross section for deep inelastic neutrino scattering can be computed from $W^{\mu\nu(\nu)}$ according to

$$\begin{aligned}\frac{d^2\sigma}{dxdy}(\nu p \to \mu^- X) \quad &= \quad \frac{G_F^2 y}{2\pi^2} \\ &\quad \mathrm{Im}\left[(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k\cdot k' - i\epsilon^{\alpha\beta}_{\mu\nu} k'_\alpha k_\beta) W^{\mu\nu(\nu)}(P,q)\right].\end{aligned} \tag{3}$$

(b) Show that any term in $W^{\mu\nu(\nu)}$ proportional to q^μ or q^ν gives zero when contracted with the lepton momentum tensor in Equation 3. Thus we can expand $W^{\mu\nu(\nu)}$ with three scalar form factors,

$$W^{\mu\nu(\nu)} = -g^{\mu\nu} W_1^{(\nu)} + P^\mu P^\nu W_2^{(\nu)} + i\epsilon^{\mu\nu\lambda\sigma} P_\lambda q_\sigma W_3^{(\nu)} + \ldots, \tag{4}$$

where the additional terms do not contribute to the deep inelastic cross section. Find the formula for the deep inelastic cross section in terms of the imaginary parts of $W_1^{(\nu)}$,$W_2^{(\nu)}$, and $W_3^{(\nu)}$.

(c) Evaluate the form factors $W_i^{(\nu)}$ in the parton model, and show that

$$\mathrm{Im} W_1^{(\nu)} = \pi \left(f_d(x) + f_{\overline{u}}(x)\right), \tag{5}$$

$$\mathrm{Im} W_2^{(\nu)} = \frac{4\pi}{ys} x \left(f_d(x) + f_{\overline{u}}(x)\right), \tag{6}$$

$$\mathrm{Im} W_3^{(\nu)} = \frac{2\pi}{ys} \left(f_d(x) - f_{\overline{u}}(x)\right). \tag{7}$$

Insert these expressions into the formula derived in part (b) and show that the result is

$$\frac{d^2\sigma}{dxdy}(\nu p \to \mu^- X) = \frac{G_F^2 s}{\pi}\left[x f_d(x) + x f_{\overline{u}}(x)(1-y)^2\right]. \tag{8}$$

(d) This analysis motivates the following definition: For a single quark flavor f, let

$$J^\mu_{fL} = \overline{f}\gamma^\mu\left(\frac{1-\gamma^5}{2}\right) f. \tag{9}$$

Define

$$W^{\mu\nu}_{fL} = 2i\int d^4x e^{iq\cdot x}\langle P|T\{J^\mu_{fL}(x)J^\nu_{fL}(0)\}|P\rangle. \tag{10}$$

Decompose this tensor according to

$$W^{\mu\nu}_{fL} = -g^{\mu\nu}W_{1fL} + P^\mu P^\nu W_{2fL} + i\epsilon^{\mu\nu\lambda\sigma}P_\lambda q_\sigma W_{3fL} + \ldots, \tag{11}$$

where the remaining terms are proportional to q^μ or q^ν. Evaluate the W_{iL} in the parton model and show that they are given by

$$\begin{aligned}
\mathrm{Im}W_{1fL} &= \pi\left(f_f(x) + f_{\overline{f}}(x)\right), & (12)\\
\mathrm{Im}W_{2fL} &= \frac{4\pi}{ys}x\left(f_f(x) + f_{\overline{f}}(x)\right), & (13)\\
\mathrm{Im}W_{3fL} &= \frac{2\pi}{ys}\left(f_f(x) - f_{\overline{f}}(x)\right). & (14)
\end{aligned}$$