Physics of Strong Interactions

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Exercise Sheet 12

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Equivalent photon approximation

Consider the process in which electrons of very high energy scatter off a target. At leading order in α, the electron interacts with the target via a single photon. If the initial and final energies of the electron are E and E', the photon will carry momentum q with $q^2 \approx -2EE'(1-\cos\theta)$, where θ is the scattering angle. In the limit of forward scattering, whatever the energy loss, the photon momentum approaches $q^2 = 0$; thus, the scattering is highly peaked in the forward direcion. One might expect that in this limit, the virtual photon becomes a real photon. The aim of this exercise is to see in what sense this is true.

1. The matrix element for the scattering process can be written as

$$\mathcal{M} = -ie\bar{u}(p')\gamma^\mu u(p)\frac{-ig_{\mu\nu}}{q^2}\hat{\mathcal{M}}^\nu(q), \tag{1}$$

   where $\hat{\mathcal{M}}^\nu(q)$ represents the coupling of the virtual photon to the target, which in general is very complicated. We therefore focus on the structure of $\bar{u}(p')\gamma^\mu u(p)$. Let $q = (q^0, \mathbf{q})$ and define $\tilde{q} = (q^0, -\mathbf{q})$. The spinor product can then be written as

$$\bar{u}(p')\gamma^\mu u(p) = Aq^\mu + B\tilde{q}^\mu + C\epsilon_1^\mu + D\epsilon_2^\mu, \tag{2}$$

   where A, B, C, and D are functions of the scattering angle and energy loss, and ϵ_i are two unit vectors transverse to q. By dotting this expression with q_μ, show that the coefficient B is at most $\mathcal{O}(\theta^2)$. Then we can ignore this term. The Ward identity implies that $q^\nu\hat{\mathcal{M}}_\nu = 0$, so that the first term is also irrelevant.

2. We will work in the frame where $p = (E, 0, 0, E)$, and in the limit where θ is small, keeping only terms $\mathcal{O}(\theta)$. Compute $\bar{u}(p')\not{\epsilon}_i u(p)$ explicitly, using massless electrons, $u(p)$ and $u(p')$ spinors of definite helicity, and ϵ_1 and ϵ_2 unit vectors perpendicular and parallel to the plane of scattering. Note that the third component of ϵ in the scattering plane also contributes at this order.

3. Now write the expression for the electron scattering cross section in terms of $|\hat{\mathcal{M}}^\mu|^2$ and the integral over phase space on the target side. This expression should be integrated over the final electron momentum p'. The integral over p'^3 is an integral over the energy loss of the electron. Show that the integral over $p'_\perp$ diverges logarithmically as $p'_\perp \to 0$ or $\theta \to 0$.

4. The divergence as $\theta \to 0$ appears because we have ignored the electron mass. Show that reintroducing the electron mass in the expression for q^2,

$$q^2 = -2EE'(1-\cos\theta) + 2m^2 \tag{3}$$

   cuts off the divergence and yields a factor of $\log(s/m^2)$ in its place.

5. Assemble all the factors, assuming that the target cross sections are independent of the photon polarization. Thus, show that the largest part of the electron target scattering cross section is given by considering the electron to be the source of a beam of real photons with energy distribution

$$N_\gamma(x)dx = \frac{dx}{x}\frac{\alpha}{2\pi}\left[1 + (1-x)^2\right]\log(s/m^2), \tag{4}$$

   where $x = E_\gamma/E$. This is the Weizsäcker-Williams equivalent photon approximation. It allows us to study photon-photon scattering using e^+e^- colliders.