Supplementary computer-readable expressions to “Three-loop QCD Corrections to $B_s \to \mu^+\mu^-$”

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The file `cA_NNLO.m` contains computer-readable results for the Standard Model (SM) contributions to the Wilson coefficients C_A that have been calculated in Ref. [1].

The notation used in the file `cA_NNLO.m` is described in the table below, where we use

$$C_A = C_A^{(0)} + \frac{\alpha_s}{4\pi} C_A^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A^{(2)} + \ldots, \qquad (1)$$

$$C_A^{(n)} = C_A^{W,(n)} + C_A^{Z,(n)}, \qquad (2)$$

together with the variables

$$x = \frac{m_t^2(\mu_0)}{M_W^2}, \qquad w = 1 - \frac{1}{x}, \qquad y = \frac{1}{\sqrt{x}}. \qquad (3)$$

The symbol `xTri` labels the fermion triangle contributions to the Z-boson penguin contribution. `MATADMasterIntegralRule` contains MATHEMATICA replacement rules. It has to be applied before evaluating the expressions numerically.

The file `intermediate_results.m` contains results for coefficient functions and renormalization constants which are needed for the matching procedure. In particular, we provide for the one- and two-loop Wilson coefficients $C_A^{W,(n)}$, $C_A^{Z,(n)}$ and $C_A^{E,(n)}$ results including contributions up to order ϵ^2 and ϵ, respectively. In `intermediate_results.m` the following variables are defined:

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dZmt1, dZmt2, dZgSM1, dZEN1, dZEN2, deltaZpsi, Zsb0c, Zsb0t, Zsb1c,
Zsb1t, Zsb2cy, Zsb2cw, Zsb2ty, Zsb2tw, Ktildet, Ktildec, KtildeTrit,
KtildeTric, cAbareWt0, cAbareWc0, cAbareEt0, cAbareEc0, cAbareZt0,
cAbareZc0, cAbareWt1, cAbareWc1, cAbareEt1, cAbareEc1, cAbareZt1,
cAbareZc1, cAbareWt2y, cAbareWt2w, cAbareWc2y, cAbareWc2w, cAbareZc2y,
cAbareZc2w, cAbareZcTriy, cAbareZcTriw, cAbareZt2y, cAbareZt2w,
cAbareZtTriy, cAbareZtTriw, cAZt0withOep2, cAWt0withOep2, cAWc0withOep2,
cAZt1withOep, cAWt1withOep, cAWc1withOep, cAEt0withOep2, cAEc0withOep2,
cAEt1withOep, cAEc1withOep,
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The notation is in analogy to the expressions in Tab. 1 and should be self-explanatory.

cA_NNLO.m	quantity	equation in Ref. [1]
cAWt0	$C_A^{W,t,(0)}$	Eq. (24)
cAWc0	$C_A^{W,c,(0)}$	Eq. (24)
cAWt1	$C_A^{W,t,(1)}$	Eq. (24)
cAWc1	$C_A^{W,c,(1)}$	Eq. (24)
cAWt2log	$C_A^{W,t,(2)}(\mu_0) - C_A^{W,t,(2)}(\mu_0 = m_t)$	Eq. (25)
cAWc2log	$C_A^{W,c,(2)}(\mu_0) - C_A^{W,c,(2)}(\mu_0 = M_W)$	Eq. (25)
cAWc2y	$C_A^{W,c,(2)}(\mu_0 = M_W)$	Eq. (26)
cAWc2w	$C_A^{W,c,(2)}(\mu_0 = M_W)$	Eq. (27)
cAWt2y	$C_A^{W,t,(2)}(\mu_0 = m_t)$	Eq. (28)
cAWt2w	$C_A^{W,t,(2)}(\mu_0 = m_t)$	Eq. (29)
cAZt0	$C_A^{Z,t,(0)}$	Eq. (41)
cAZt1	$C_A^{Z,t,(1)}$	Eq. (41)
cAZt2log	$C_A^{Z,t,(2)}(\mu_0) - C_A^{Z,t,(2)}(\mu_0 = m_t)$	Eq. (42)
cAZ2y	$C_A^{Z,t,(2)}(\mu_0 = m_t) + \text{xTri}\left(C_A^{Z,t,\text{tria.}} - C_A^{Z,c,\text{tria.}}\right)$	Eqs. (43) and (44)
cAZ2w	$C_A^{Z,t,(2)}(\mu_0 = m_t) + \text{xTri}\left(C_A^{Z,t,\text{tria.}} - C_A^{Z,c,\text{tria.}}\right)$	Eqs. (43) and (44)

Table 1: One-, two- and three-loop contributions to the Wilson coefficients $C_A^{W,(n)}$ and $C_A^{Z,(n)}$ as contained in the file `cA_NNLO.m`. In case the variable name ends with "`y`" ("`w`") the corresponding result is expressed in terms of y (w) and is valid for $m_t \gg M_W$ ($m_t \approx M_W$). The ending "`log`" reminds that only the exact dependence on $\log \mu^2$ is contained in the corresponding expression.

References

[1] T. Hermann, M. Misiak and M. Steinhauser, *Three-loop QCD Corrections to $B_s \to \mu^+\mu^-$*, arXiv:1311.1347v1, SFB/CPP-13-83, TTP13-034.